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STRENGTH OF MATERIALS

. Training Department
Southeast Asian Fisheries Development Center

CONTENTS

PREFACE

This reference book contains extracts from a well known text book, "Strength of Materials" by Prof. F.V. Warnock, M.Sc., Ph.D., Mech. E. and extracts of Japanese industrial standard specifications. It is intended specifically for use by the Marine Engineering Course students of the SEAFDEC Training Department and its content is geared to the present curriculum.

A liberal supply of worked examples have been included, as well as exercises which appear at the end of each chapter. It is advisable that these be given considerable attention since they aid the understanding and the application capability of the student.

It is also hoped that this book will be of use to those students who are interested in the subject of design, and that it will assist them in solving the problems related to this aspect encountered in their engineering work.

STRENGTH OF MATERIALS

I. SIMPLE STRESSES AND STRAINS

When an engineer undertakes the design of a machine part or portion of a structure, it is essential that he should fully realize the various forces for which allowances have to be made. He then requires to have at his disposal formulae which will enable him to proportion the design so that fracture will not take place when the part is subjected to the estimated forces. The formulae which follow have been largely arrived at by making use of well-known facts in the study of statics, dynamics, and mathematics. In the derivation of these formulae assumptions have been made, which are very often not exactly realized in practice. The results obtained by using such formulae should be compared, therefore, wherever possible, with those obtained experimentally.

It is advisable that, when attacking a particular problem in design, first one make oneself perfectly familiar with the conditions under which the part has to work. Secondly, before making use of a particular formula, be familiar with the assumptions made in obtaining the formula, and thus see how closely theoretical and working conditions agree. Finally, in making use of the formula, one should incorporate common sense and the results of experience.

1. Load The external forces acting on a piece of material constitute what is called the "load". The following are a few of the forces met with in practice:-

a. A force, due to a load not in motion; an example of which is a load hanging from a crane chain, without being raised or lowered.

b. An inertia force, due to change in velocity of a mass. This is met with in engine practice and is the result of change in velocity of the reciprocating parts.

c. A centrifugal force, which is met with in pulleys and flywheels, and is due to the tendency of a rotating mass to get away from its centre of rotation.

d. A frictional force, resulting from the application of a brake on a drum.

e. A force due to expansion or contraction, which is met with in boiler furnaces.

Forces such as those above mentioned often act at the end of an arm, and cause bending or twisting in the material.

2. Stress No material is perfectly rigid, therefore the application of a load to a piece of material causes a deformation. This deformation may be large and easily measured as in the case of a piece of rubber 1 sq.in. cross-section carrying a pull of 100 lb., or it may be small and require a delicate measuring device where a steel bar of the same cross-section carries a pull of 10 tons. In all cases internal forces are called into play in the material to resist the load and are referred to as "stresses". The intensity of the stress is estimated as the force acting on unit area of cross-section, and is expressed in such units as tons per sq.in., lb. per sq.in., etc.

3. Tensile Stress An example of a body stressed in this manner is shown in Fig. 1 which represents a uniform vertical bar held at its upper end and carrying an axial load W. If the bar is cut at CB, it is easily seen that a force P, acting in the opposite direction

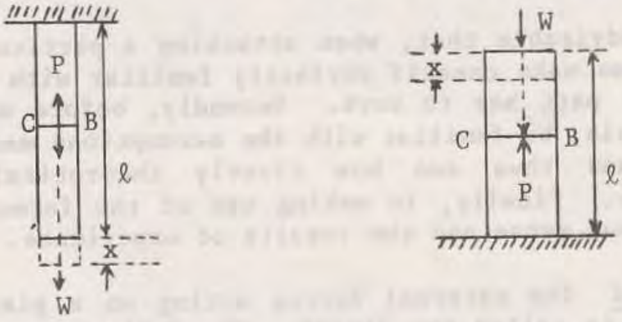


Fig. 1 Fig. 2

to W, and equal in magnitude to W, is required to hold the lower portion in equilibrium. In the normal state of the bar, the force P is supplied by the internal forces in the material. If the section CB is taken at any point between the ends of the bar a similar condition exists (neglecting the weight of the portion of the bar below the section.

The intensity of tensile stress at CB is given by $f_t = \frac{W}{A}$, where A is the cross-sectional area of the bar.

4. Compressive Stress Fig. 2 represents a uniform vertical bar carrying a load W, which in this case causes a stress of opposite nature to that discussed in "tensile stress". The line of action of the load is again assumed to be axial. Considering a section CB, it is seen that a force P equal in magnitude to W, and acting in the

opposite direction to W, is required for equilibrium. Again, neglecting the weight of the portion of the bar above CB, the intensity of the compression stress at CB is given by:-

$$f_c = \frac{W}{A},$$

where A is the cross-sectional area of the bar.

5. Shear Stress A stress of this nature is said to exist on a section of a body if on opposite faces of the section equal and opposite parallel forces exist. Let a rectangular block of metal of cross-sectional area A be soldered to a heavy mass of iron and suppose a force W to be applied, acting as shown in Fig. 3, now consider the section CB. The upper portion H exerts a force W on the face of the lower portion K, which portion in turn exerts an equal and opposite force on the face of the upper portion H. The intensity of the shear stress on the section at CB is given by

$$f_s = \frac{W}{A}.$$

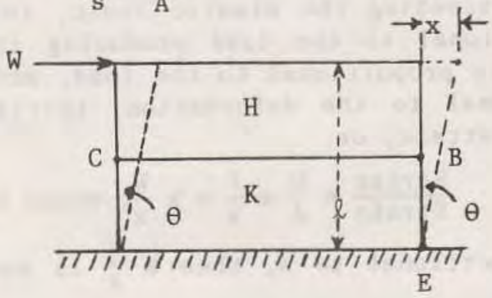


Fig. 3

In each of the three cases considered the force is assumed to be distributed uniformly over the surface. If this is not the case then the intensity of stress at a point in the surface is taken to be equal to the limiting ratio of $\frac{\delta W}{\delta A}$ when each is reduced indefinitely where δW is the force acting on the very small area A.

6. Strain Strain is a measure of the deformation produced by the application of the external forces.

(a) In Fig. 1 it will be observed that the deformation is an elongation of the amount x, and if ℓ is the initial length of the bar, then the tensile strain is given by:

$$e_t = \frac{x}{\ell},$$

or is the elongation per unit of length.

(b) In Fig. 2 the deformation is shown to be a shortening of the bar by the amount x, and if ℓ is the unloaded length, the compressive strain is given by:

$$e_c = \frac{x}{\ell},$$

or the shortening per unit of length.

(c) The state of deformation produced by shear is shown by the dotted lines in Fig. 3. The movement x of the corner is exceedingly small so that EFL may be regarded as a right angled triangle. The shear strain is given by:

$$e_s = \frac{x}{\ell} = \tan\theta.$$

7. Elasticity A material is said to be perfectly elastic if the strain due to loading disappears with the removal of the load, and also if the strain for a given value of load during the unloading process is equal to the strain for the same value of load during the loading process.

A limiting value of load will be found at which the strain does not completely disappear with the removal of the load. The value of the stress corresponding to this load is called the "Elastic Limit" and the residual strain is referred to as a permanent set.

8. Hooke's Law It was discovered by Hooke that if a material be loaded without exceeding the elastic limit, then the deformation produced is proportional to the load producing it. Now the stress caused by the load is proportional to the load, and the corresponding strain is proportional to the deformation, therefore the stress is proportional to the strain, or

$$\frac{\text{Stress}}{\text{Strain}} = \frac{W}{A} \times \frac{\ell}{x} = k\,\frac{W}{x}, \text{ where } k = \frac{\ell}{A},$$

and since x is proportional to W, then $k\,\frac{W}{x}$ is equal to a constant, i.e. $\frac{\text{stress}}{\text{strain}} = N$.

This constant is called the "Modulus of Elasticity", and its magnitude will depend on the material and on the nature of the stress and strain dealt with. Since stress is a force per unit of area, and strain is a number, the modulus of elasticity will also be a force per unit area.

9. The Elastic Constants When a body is subjected to simple tension or compression the modulus of elasticity is usually called "Young's Modulus", and it is invariably denoted by the letter E. In the case of a body subjected to pure shear only the modulus is called the "Shear or Rigidity Modulus", and is usually denoted by the letter C, or sometime G.

If f_t and f_c denote the tensile or compressive stress in a body subjected to pure tension or compression, and e_t and e_c the corresponding strains, then:

$$E = \frac{f_t}{e_t} = \frac{f_c}{e_c}$$

Also if f_s is the shear stress in a body subjected to pure shear and e_s the corresponding strain, then:

$$C = \frac{f_s}{e_s}$$

These constants are related to one another, which relationship will be obtained later.

From a knowledge of the relationship, just obtained, between stress, strain and modulus of elasticity, it will be found that the attack on a problem on strength of materials develops along the following lines:-

1. Assume a deformation (this should be of as simple a character as possible).

2. Calculate the strains.

3. Multiply the strains by the elastic moduli and so obtain the stresses.

4. Multiply the stresses by the areas over which they act, equate to the external forces, and so proceed with the solution of the problem.

Numerous examples of the use of this method of attack will be shown by means of worked examples, and the reader is advised to study each one carefully.

Example 1. Define "stress", "strain", and "modulus of elasticity" in general terms. A metal rod of circular section, 1 in. in diameter, is subjected to stress in a tension-testing machine. It is found that the total extension over a length of 8 in. is 151 scale divisions of an extensometer for a pull of 9,000 lb., the unit of the scale being $\frac{1}{50,000}$ of an inch. Calculate the stress, the strain and the modulus of elasticity for this rod.

Area of cross-section of rod = $0.7854 \times 1^2 = 0.7854$ sq.in.

$$f_t = \frac{W}{A} = \frac{9000}{0.7854} = 11,460 \text{ lb./sq.in.}$$

Extension on 8 in. length = scale division x unit of scale

$$= 151 \times \frac{1}{50,000} \text{ in.}$$

$$e_t = \frac{x}{\ell} = \frac{151}{50,000 \times 8} = 0.0003775$$

$$E = \frac{f_t}{e_t} = \frac{11460}{0.0003775} = 30.37 \times 10^6 \text{ lb./sq.in.}$$ Ans.

Example 2. Design a treble-riveted lap joint in which the pitch of the rivets in the outer rows is twice the pitch of the rivets in the inner row, for plates $\frac{3}{4}''$ in thickness. Determine the pitch and diameter of the rivets so that the tensile stress in the plates is 6 tons per sq.in. and the shearing stress in the rivets is $4\frac{1}{2}$ tons per sq.in. Given $d = 1.2\sqrt{t}$, where d and t are the rivet diameter and plate thickness respectively.

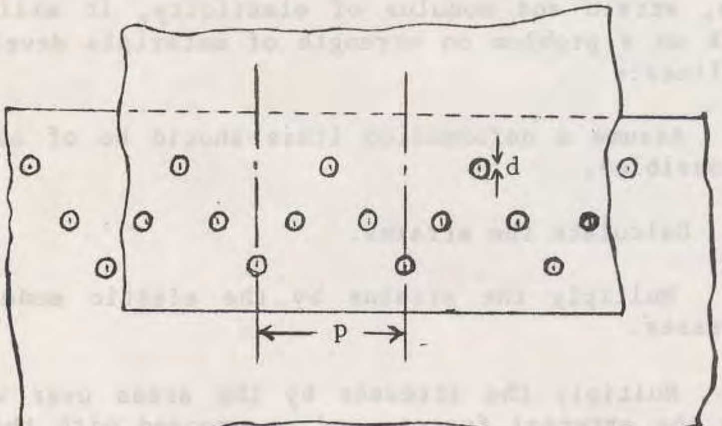


Fig. 4

The arrangement of the joint is shown in Fig. 4.

Diameter of rivets $d = 1.2\sqrt{0.75} \doteqdot 1$ in.

Let n = number of rivets in strip of width equal to p, where p = pitch of rivets.

$$\text{Strength of plate against tearing} = (p-d)t \times f_t$$

$$= (p-1) \times \frac{3''}{4} \times 6$$

$$= 4\frac{1}{2}\,p - 4\frac{1}{2}.$$

$$\text{Strength of rivets against shearing} = \frac{\pi}{4} d^2 \times f_s \times n$$

$$= 0.78 \times 1 \times 4.5 \times 4$$

$$= 14.15 \text{ tons.}$$

Since joint is equally strong in tension and in shear,

$$4\frac{1}{2}\,p - 4\frac{1}{2} = 14.15$$

$p = 4.14$ in, say 4 in. pitch <u>Ans.</u>

<u>Example 3.</u> Two lengths of bar 12 in. wide and   in. thick are to be connected by a double cover butt joint. The diameter of the rivet holes is 1 in. Design a suitable joint. Tensile stress 7 tons/sq.in. Shear stress 5 tons/sq.in. Bearing stress 9 tons/sq.in.

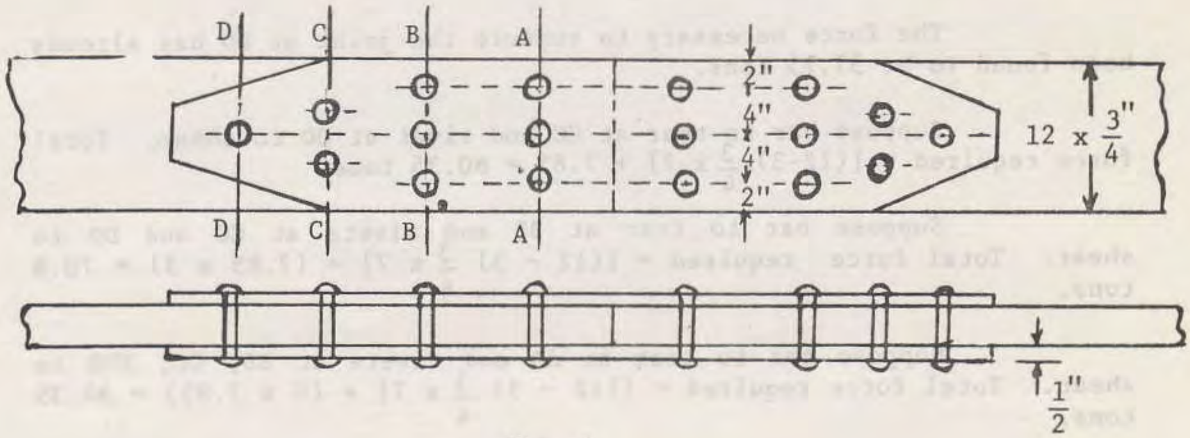


Fig. 5

The rivets in a joint of this type are usually arranged as shown in Fig. 5, the bar is then only weakened by one rivet hole.

Strength of joint against tearing through the outside rivet

$$= (12'' - 1'')\,\frac{3}{4} \times 7 = \underline{57.75} \text{ tons.}$$

Shear strength of one rivet $= \{\frac{\pi}{4} \times (1)^2 \times 5\}\, 2 \rightarrow$ rivet in double shear

$$= \underline{7.85} \text{ tons}$$

Number of rivets required for shear $= \dfrac{57.75}{7.85} = 7.4$, say 8.

Bearing strength of each rivet $= 9 \times 1 \times \dfrac{3}{4} = 6.75$ tons.

Number of rivets required for bearing $= \dfrac{57.75}{6.75} = 8.55$, say 9.

The joint would therefore require 9 rivets in each bar, and would be arranged as shown in Fig. 5.

Each cover plate might be taken to be half the thickness of the bars, but in practice it is usual to make the cover plates $\frac{5}{8}$ the thickness of the bars.

Thickness of each cover plate = $\frac{5}{8} \times \frac{3}{4} = \frac{1}{2}$ in.

It is of interest to further extend this example and find the force necessary to rupture the joint at DD, CC, etc., and hence obtain the efficiency of the joint.

The force necessary to rupture the joint at DD has already been found to be 57.75 tons.

Suppose bar to tear at CC and rivet at DD to shear. Total force required = $[(12-3)\ \frac{3}{4} \times 7] + 7.85 = 60.35$ tons.

Suppose bar to tear at BB and rivets at CC and DD to shear. Total force required = $[(12 - 3)\ \frac{3}{4} \times 7] + (7.85 \times 3) = 70.8$ tons.

Suppose bar to tear at AA and rivets at BB, CC, XDD to shear. Total force required = $[(12 - 3)\ \frac{3}{4} \times 7] + (6 \times 7.85) = 94.35$ tons.

Support cover plates ruptured along AA. Total force required:-

$$= 7(12 - 3) \times 2 \times \frac{1}{2} = 63 \text{ tons.}$$

From this it is clear that the weakest section is at DD.

$$\text{The efficiency of the joint} = \frac{\text{least strength of the joint}}{\text{strength of solid plate}}$$

$$= \frac{57.75 \times 100}{(12 \times \frac{3}{4})\ 7} = 91.6\%$$

<u>Example 4</u>. The diameter of the piston of a diesel engine is 310 mm., and the maximum compression pressure in the cylinder is 500 lb./in^2. The cylinder is held by four bolts whose effective diameter is 2 in. and length 35 in. Estimate the maximum tensile stress in each bolt, and the elongation of each bolt. $E = 30 \times 10^6$ lb./in^2.

$$\text{Maximum force exerted on each bolt} = \left(\frac{310}{25.4}\right)^2 \times 0.7854 \times \frac{500}{4}$$

$$= 14{,}630 \text{ lb.}$$

Maximum stress in each bolt $= f_t = \frac{14630}{0.7854 \times 4} = 4656$ lb./in^2.

Elongation of each bolt $= \frac{f_t \times \ell}{E} = \frac{4656 \times 35}{30 \times 10^6}$

10. Compound Bars and Columns A short compound column, composed of a steel tube fitting loosely inside a copper tube, and carrying an axial load W, is shown in Fig. 8.

It is evident that the strain in each tube will have the same value, but since E is different for each material, the stress will not have the same value for each material.

Let A_c = cross-sectional area of copper tube.

A_s = cross-sectional area of steel tube.

W_c = load carried by the copper tube.

W_s = load carried by the steel tube.

E_c and E_s being the value of Young's modulus for copper and steel, and f_c and f_s being the value of the corresponding stresses in the tubes.

If ℓ is the unloaded length of the column, and x is the amount each tube shortens.

Strain in each $= \frac{x}{\ell}$

but $= \frac{x}{\ell} = \frac{f}{E}$

$$\frac{f_s}{E_s} = \frac{f_c}{E_c}, \quad f_c = f_s \cdot \frac{E_c}{E_s} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

$$\text{Also} \quad W = W_s + W_c = f_s A_s + f_c A_c \quad \ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$\text{From 1}: \quad W = f_s A_s + f_s \cdot \frac{E_c}{E_s} \cdot A_c$$

$$f_s = \frac{W \cdot E_s}{A_s E_s + A_c E_c}$$

$$\text{Similarly } f_c = \frac{W \cdot E_c}{A_s E_s + A_c E_c}$$

11. Temperature Stress When the temperature of a metal changes there is a corresponding change in dimensions. If this change in dimensions is prevented then a stress is set up in the metal.

The linear expansion of the metal is proportional to the change in temperature, and this expansion per unit of length per unit change in temperature is called the linear co-efficient of expansion of the metal.

Suppose a bar be held in such a manner that linear change of dimensions is prevented when heat is applied.

Let ℓ = length of bar.

T = change in temperature.

α = linear co-efficient of expansion.

If the bar was free to expand the increase in length = $\alpha T \ell$
Hence the strain set up due to expansion being prevented = $\frac{\alpha T \ell}{\ell} = \alpha T$.

If f = stress due to the above strain,

E = Young's modulus for the material of the bar.

$$\text{Since } \frac{\text{stress}}{\text{strain}} = \text{modulus of elasticity}$$

$$\therefore \frac{f}{\alpha T} = E$$

$$\text{or } f = \alpha T E.$$

Example 5. A steam pipe is 100 ft. long at a temperature of 15° C. Steam at 180°C is passed through the pipe. What is its length when free to expand? Suppose expansion to be prevented, what stress is induced in the material? $E = 6000$ tons/in^2. $\alpha = 0.000012$ per °C.

Change in temperature = $180 - 15 = 165$°C

Increase in length = $\alpha T \ell = 0.000012 \times 165 \times 100 = 0.198$ ft.

Strain due to prevention of expansion = $\frac{\alpha T \ell}{\ell} = \alpha T = 0.000012 \times 165$

Stress induced in material = $\alpha TE = 0.000012 \times 165 \times 6000$

$= 11.88$ tons/in.^{2}

Example 6. Three rods, each initially of $\frac{1}{2}$ in^2. cross-section and 5 ft. long, support a load of 10 tons. The center rod is made of steel and the outer ones of copper. If the temperature of the rods is increased by 100°C. and the rods are so adjusted that they are extended equal amounts, estimate the load carried by each rod. Es = 30×10^6 lb./in^2. Ec = 12×10^6 lb./in^2. As = 0.000012 per °C. Ac = 0.0000185 per °C.

(Fig. 5A) Let x = the common extension

Then $\alpha T \ell$ = extension due to heating

and $x - \alpha T \ell$ = extension due to loading of each material

and $(\frac{x}{\ell} - \alpha T)$ = strain due to load carried

Stress in each rod = $(\frac{x}{\ell} - \alpha T)$ E .... (1)

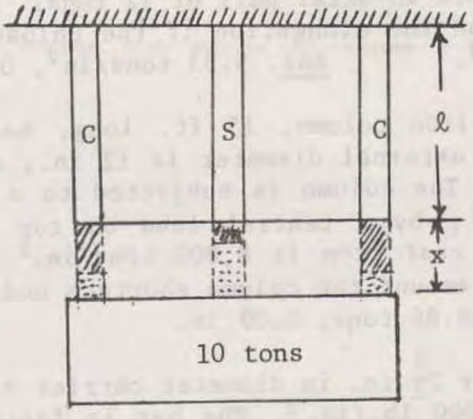


Fig. 5A

= extension due to load 10 tons

= extension due to heating 100°C

Load carried by rod = $(\frac{x}{\ell} - \alpha T)$ EA .... (2)
where A = cross sectional area of rod.

Total load = W = $\Sigma\, (\frac{x}{\ell} - \alpha T)$ EA...................... (3)

$$2240 = \left(\frac{x}{\ell} - 0.000012 \times 100\right) 30 \times 10^6 \times \frac{1}{2} + \left(\frac{x}{\ell} - 0.0000185 \times 100\right)$$
$$12 \times 10^6 \times \frac{1}{2} \times 2.$$

$$\frac{x}{\ell} = \frac{22400 + \frac{100}{2}(0.000012 \times 30 + 0.0000185 \times 24)\ 10^6}{\frac{1}{2} \times 10^6\ (30 + 24)} = 0.002318$$

Load carried by steel rod $= (0.002318 - 0.000012 \times 100)\ 30 \times 10^6 \times \frac{1}{2}$

$= 0.001118 \times 15 \times 10^6$

$= 16{,}770$ lb.

Each copper rod carries a load $= (22400 - 16770)\ \frac{1}{2}$

$= 2815$ lb.

Examples I

1. A flat mild steel bar of rectangular cross-section, 3 in. x $\frac{3}{4}$ in., carries an axial pull of 12 tons. Estimate the tensile stress in the bar and the elongation if the unloaded length is 20 ft. $E = 30 \times 10^6$ lb./in^2. Ans. 5.33 tons/in^2, 0.0956 in.

2. A cast-iron column, 15 ft. long, has a hollow circular cross-section. The external diameter is 12 in., and the thickness of material is $1\frac{1}{4}$ in. The column is subjected to a compressive stress, of 4 tons per in.^{2}, by a central load on top of the column. If Young's modulus for cast iron is 8,000 tons/in.^{2}, estimate the value of the load and the amount the column shortens under the load.
Ans. 168.86 tons, 0.09 in.

3. A tie-bar $2\frac{1}{2}$ in. in diameter carries a load which causes a tensile stress of 8000 lb./in.^{2} The bar is fastened to a cast iron bracket which is held by four bolts. Find the diameter of the bolts at the bottom of the threads if the stress is limited to 5000 lb./in.^{2}
Ans. 0.948 in.

4. If the ultimate shear stress for mild steel is 55,000 lb./in.^{2}, find the force required to punch a $\frac{3}{4}$ in. hole in a mild steel plate $\frac{1}{2}$ in. thick. What is the compressive stress on the punch?
Ans. 28.94 tons, 65.48 tons/in.^{2}

5. A short steel tube 4 in. internal diameter and $\frac{1}{2}$ in. thick is surrounded loosely by a brass tube of the same length and thickness. The tubes carry an axial thrust of $\frac{1}{2}$ ton. Estimate the load carried by each tube, and the amount each tube shortens. Length 3.5 in. Es = 30×10^6 lb./in.^{2}, Eb = 11.8×10 lb./in.

Ans. Ws = 756.2 lb., Wb = 363.8 lb., 1.25×10^{-5} in.

6. Two vertical rods are each rigidly fastened at the upper end at a distance of 24 in. apart. Each rod is 10 ft. long and $\frac{1}{2}$ in. in diameter. A horizontal cross-bar connects the lower ends of the rods and on it is placed a load of 1000 lb. so that the cross-bar remains horizontal. Find the position of the load on the cross-bar and estimate the stress in each rod. One rod is of wrought iron for which E = 28×10^6 lb./in.^{2}, and the other of bronze for which E = 9×10^6 lb./in.^{2}

Ans. f_{wi} = 3855 lb./in.^{2}, $_b$ = 1239 lb./in.^{2}, 5.8 in. from the w.i. bar.

7. A steel rail is 30 ft. long at a temperature of 15°C. Estimate the elongation when the temperature increases to 85°C. If no allowance is made for expansion calculate the stress in the rail. α = 0.000012 per °C. E = 30×10^6 lb./in.^{2}

Ans. 0.302 in; 25,200 lb./in.^{2}

8. A weight of 20 tons is supported by three short pillars each 1 in.^{2} in section. The center pillar is of steel and the two outer ones of copper. The pillars are so adjusted that at a temperature of 15°C each carries one-third of the total load. The temperature is then raised to 115°C. Estimate the stress in each pillar at 15°C and at 115°C. Es = 30×10^6 lb./in.^{2}, Ec = 12×10^6 lb./in.^{2}, αs = 12×10^{-6} per °C., αc = 18.5×10^{-6} per °C.

Ans. 6.66 tons/in.^{2}, c = 8.6 tons/in.^{2}, s = 2.8 tons/in^2

9. A mild steel bar $\frac{3}{4}$ in. diameter and 12 in. long is place inside a tube having an external diameter of 1 in. and an internal diameter of $\frac{3}{4}$ in. The combination is then subjected to an axial thrust of 5 tons. The modulus of elasticity of the metal of the tube is 11,500,000 lb./in.^{2}, and of the steel 30,000,000 lb./in.^{2} Find: (a) the stress in the tube and in the rod. (b) the shortening of the rod. (c) the work done in compression.

Ans. f_r = 8.72 tons/in., f_{tube} = 3.34 tons/in.^{2}, 78.12 x 10^{-14}in., 43.75 in.lb.

10. A bar of copper, 1.5 in. diameter, is completed enclosed in a steel tube, 2.5 in. external diameter. A pin, 0.75 in. diameter, is fitted transversely to the axis of the bar near each end, to secure the bar to the tube. Calculate the intensity of shear stress induced in the pin when the temperature of the whole is raised 100°F. Ec = 6.5×10^3 tons/in.^{2} Es = 13×10^3 tons/in.^{2} αc = 9.5×10^{-6} per °F. αs = 6.2×10^{-6} per °F.

Ans. 3.48 tons/in.^{2}

11. Design a riveted joint for joining two lengths of flat tie-bar. Double cover plates. Find the efficiency of the joint. Load 80 tons. Diameter of rivets $1\frac{1}{8}$ in. Clearance in rivet holes 0.05 in. Tensile stress not to exceed 5 tons/in.^{2} Shearing stress not to exceed 4 tons/in.^{2} Bearing stress not to exceed 6 tons/in. The width should be about 15 times the thickness.

Ans. 88.8%

12. Two thick plates made of an aluminium alloy are held together in contact by copper bolts. Find the increase in the tensile stress (in tons/in.^{2}) in the bolts due to a rise in temperature from 60°F to 80°F, neglecting any compressive strain in the aluminium alloy. Modulus of direct elasticity for copper = 18×10^6 lb./in.^{2} Coefficient of expansion of copper per °F = 0.0000093. Coefficient of expansion of aluminium alloy per °F = 0.0000126.

Ans. 0.534 tons/in.^{2}

II. MECHANICAL PROPERTIES OF METALS

12. Properties of Metals: Properties associated with metals, and of great importance from an engineering standpoint are elasticity, ductility, brittleness, plasticity, and malleability.

Several of these properties are in opposition, and thus a particular metal can not possess all of them simultaneously. Mild steel possesses elasticity, copper ductility, cast iron is brittle, whilst lead is plastic, and wrought iron malleable. In a simple tensile test on a material, carried to destruction, sufficient information is obtained to form an opinion of the strength, elasticity and ductility of the material. In design these properties are of premier importance, the strength of the material must be known in order to calculate the dimensions, and the remaining properties are desirable, since, for various reasons, high local stress sometimes occur, and thus it is essential that a certain amount of elastic deformation should take place in order to relieve these local stresses.

13. Behaviour of Metals in Tension: When the ductile metals are subjected to a gradually increasing tensile load, the tensile stress is proportional to the tensile strain until a certain point is reached, after which proportionality no longer holds, and slightly before reaching this stage it is found that the elasticity of the material has broken down. The elongation so far has been exceedingly small, and very delicate measuring devices called extensometers are required in order to measure the small changes in length.

If the load be increased a small amount, it will be found that the material "flows or yields" a large amount, and the elongation can be measured with a scale and a pair of dividers. The material is now in a semi-plastic state, and further applications of load cause extensions which increase with time. Each application of load causes the strains to increase more and more rapidly with the stresses, until, finally, a value of load is reached at which the material stretches locally and a "waist" is formed. At this stage, owing to the decrease in cross-section, a smaller load than that at which the waist formed is required to continue the elongation. With the continuance of elongation the cross-section becomes smaller and smaller. Hence the load necessary will be gradually reduced until fracture occurs.

Fig. 6 represents a stress-strain diagram for a ductile material tested, in tension, to destruction. That portion of the test up to the point at which yield occurs in shown plotted to a larger scale. The stress are all calculated using the original cross-sectional area of the material, and are called Nominal Stresses, since

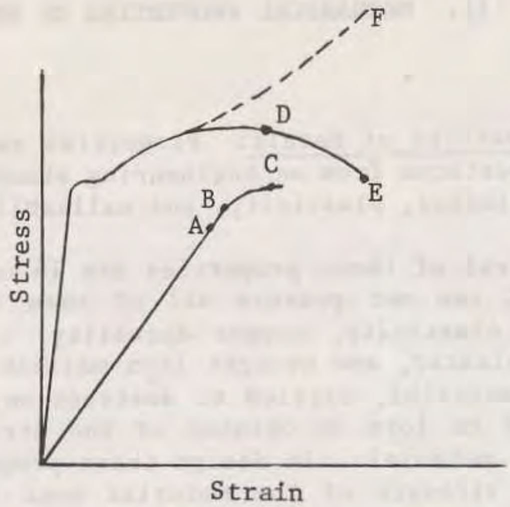


Fig. 6

the actual cross-sectional area does not remain constant. If the acutal cross-sectional area was calculated for each load, then the "actual stress" diagram is that shown in Fig. 6. In the nominal stress diagram the stress at the breaking point appears much less than the maximum stress carried, but on examination of the actual stress diagram it is found to be the greatest stress.

14. Important Stages in the Test: All the important stages have been marked on the nominal stress-strain curve shown in Fig. 6, and a particular name is usually applied to the stress at each stage, as follows:-

At the point A the elasticity has broken down, and the stress corresponding to the load at A is called the "Elastic Limit".

At the point B the proportionality of stress and strain has ceased, and the stress at B is called the "Limit of Proportionality".

At the point C the material has yielded a large amount, and the corresponding stress is known as the "Yield Stress".

These 3 stresses are exceedingly close together, and great care and delicate measurements are required to distinguish one from the other.

At the point D local yielding takes place, and the waist begins to form. The material is then carrying the maximum load, and the corresponding stress is known as the "Maximum Stress". It should be remembered, however, that this is maximum norminal stress.

At the point E fracture takes place, and the stress at this point is known as the "Breaking Stress".

15. Working Stresses. The "Tenacity" or "Ultimate Stress" of a material is the greatest load required to fracture the material divided by the area of the original cross-section at the point of fracture. It is important that the working stress shall be well below the elastic limit, and the ultimate stress is usually divided by a number called the "Factor of Safety" in order to obtain the working stress. The factor of safety has also been referred to as a factor of ignorance. It would seem more reasonable to use a smaller factor of safety and the elastic limit when calculating the working stress.

Factors of safety depend on how the stress is applied. An alternating stress requires a larger factor of safety than a constant stress, and a stress caused by shock will require the factor of safety to be much larger still. Since the various working stresses are calculated from factors of safety based on the ultimate stress, it can be readily understood that the ultimate stress is the most important stress in a commercial test, the elastic limit being only asked for on rare occasions.

16. The Measurement of Ductility. The elongation on a test bar, up to the maximum load, is practically distributed uniformly over the bar, after which local yielding occurs and the cross-section decreases at the point of yielding. Two methods are used to estimate the ductility, one being based on the total elongation, and the other on the total reduction in cross-sectional area. The former is the more common and from it we obtain the "Percentage Elongation". If L is the length of the test bar at fracture, and ℓ the length before stress is applied, i.e. the original length, then:-

$$\text{Percentage elongation} = \frac{L - \ell}{\ell} \times 100$$

The percentage elongation depends on the length of the bar owing to the local yielding before fracture. If the test bar be marked off in inches, then the percentage elongation of the inch containing the fracture will be very large. If, however, the result is calculated on a 2-inch length containing the fracture, the local extension does not play so important a part, and the importance of its effect becomes less as the length of the test bar increases. It is of the utmost importance, therefore, always to state the length of the test bar on which the percentage elongation has been calculated. A common length of test bar is 8 in. The following table serves to show how the percentage elongation varies with the length of the specimen. The results are taken from a test on a mild steel bar.

Original length (inches)	0.5	1.5	2.5	3.5	4.5	5.5	6.5	8
Final length (inches)	0.9	2.2	3.48	4.65	5.88	7.08	8.28	10.1
Percentage Elongation %	80	46.7	39.2	32.8	30.6	28.7	27.4	26.2

The percentage elongation is usually calculated without taking into account the cross-sectional area of the test bar, although the results are not strictly comparable when the cross-sectional dimensions differ for a given gauge length. Where bars have different cross-sectional areas, it has been shown that comparative resûlts may be obtained if the bars are geometrically similar.

The total extension of a bar is made up of a uniformly distributed extension and a local extension, the former being proportional to the length of the bar, and the latter almost independent of the length.

Thus if x is the total extension, y the local extension, and ℓ the gauge length, then: $x = y + z\ell$.

Now y is nearly proportional to the square root of the cross sectional area of the bar. Hence if A is the cross-sectional area

$$y = s\ \sqrt{A}$$

$$\therefore\ x = s\ \sqrt{A} + z\ell$$

Thus, if s and z are known, a rough idea can be obtained of the elongation of another bar of different dimensions and the same material.

Example 1. A bar of wrought iron 0.875 in. dia. and 8 in. gauge length was subjected to a static tensile test. The result gave yield load 8.7 tons, max. load 13 tons, and load at instant of fracture 11 tons. The total elongation was 2.3 in., of which 1.8 in. was elongation up to the point of max. load. The area of reduced section at fracture was 0.315 in.^{2} From the above calculate the results of a tensile test on a bar of the same material 1 in. dia. and 6 in. gauge length, giving the elongation per cent, and the load at instant of fracture.

Total extension $x = s\sqrt{A} + z\ell$

and $s\sqrt{A} = 2.3 - 1.8 = 0.5$ in.

$\therefore s = \frac{0.5}{\sqrt{0.6013}} = \frac{0.5}{0.7754} = 0.6449$

also $z\ell = 1.8$ in.

$\therefore z = \frac{1.8}{8} = 0.225$

Total extension on 6 in. gauge length and 1 in. diameter specimen

$= 0.6449\sqrt{0.7854} + (0.225 \times 6)$

$= 0.5715 + 1.35$

$= 1.9215$ in.

Percentage elongation $= \frac{L - \ell}{\ell} 100 = \frac{1.9215}{6} \times 100 = 32.025\%$

Yield load on 1 in. specimen = Yield stress x area of section

$= \frac{8.7}{0.6013} \times 0.7854$

$= 11.36$ tons.

Up to the maximum load the elongation is uniform along the bar, and the area of the cross-section of bar at this load is found from:-

Length x area of section = original length x original area of section.

$\therefore$ Area of section of original bar at max. load $= \frac{8 \times 0.6013}{9.8}$

$= 0.491$ in.^{2}

Area of section of 1 in. diameter bar at max. $= \frac{6 \times 0.7854}{7.35}$

$= 0.641$ in.^{2}

Maximum load on 1 in. dia. specimen = max. stress x area of section

$= \frac{13}{0.491} \times 0.641$

$= 16.97$ tons.

Percentage reduction of area of original test bar $= \frac{0.6013 - 0.315}{0.6013} \times 100$

$= 47.62\ \%$

Reduction in area of 1 in. dia. specimen $= 0.4762 \times 0.7854$

$= 0.3740\ \text{in.}^2$

$\therefore$ Area at fracture $= 0.7854 - 0.3740$

$= 0.4114\ \text{in.}^2$

Load at instant of fracture on 1 in. dia. specimen = breaking stress x area of section

$= \frac{11}{0.315} \times 0.4114$

$= 14.35$ tons.

The second method of calculate the ductility of a metal depends on the change in the cross-sectional dimensions, and from it is obtained the "Percentage Reduction in Area".

If A = area of cross-section before applying load.

A = area of cross-section after fracture.

then percentage reduction in area $= \frac{A-A_1}{A} \times 100$

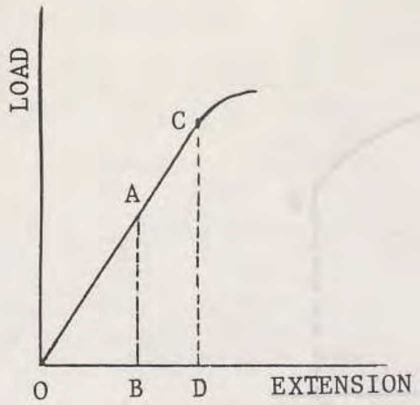
---

17. Resilience: The term "resilience" is usually restricted to mean the amount of energy stored up in an elastic strained body per unit volume. In the case of a body strained, within the elastic range, in simple tension we have already seen that the load is proportional to the extension. The work done on the bar, which is equal to the strain energy, is given by:

$$\frac{\text{load x extension}}{2} = \frac{AB \times OB}{2} \quad \text{(Fig. 7)}$$

The term "proof resilience" is often adopted to represent the greatest strain energy per unit volume which can be stored, inside the elastic range, and is represented by the area

$$\frac{OCD}{\text{Volume of bar}} = \frac{CD \times OD}{2\ \text{Velume}}.$$



Let A = cross-sectional area of bar

ℓ = length of bar

V = volume of bar

W = load applied

x = extension due to W

E = young's modulus

f = tensile stress in bar due to W.

Fig. 7

$$\text{Work done in straining bar} = \frac{W \times x}{2}$$

$$= \frac{fA \times x}{2}$$

$$= \frac{fA}{2} \cdot \frac{f\ell}{E}$$

$$= \frac{f^2}{2E} \cdot V$$

$$\text{Resilience} = \frac{f^2}{2E} \text{ (units of Work per unit volume of bar).}$$

If $f$max. is the greatest stress that can be applied within the elastic range, then:

$$\text{Proof Resilience} = \frac{1}{2} \cdot \frac{(f\,\text{max})^2}{E}$$

The expression $\frac{(f\text{max})^2}{2E}$ is sometimes called the "Modulus of Resilience".

18. Total Work Done up to Fracture: The total work done up to fracture is given by the area OCDEF (Fig. 8). The portion OCH represents the work done during the elastic stage, and the remaining portion that done during the plastic stage.

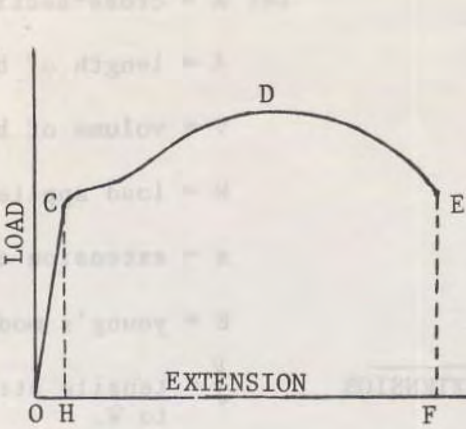


Fig. 8

19. Suddenly-applied Loads: Let a tensile load W be applied suddenly to a bar and produce an extension x. The work done will be given by W.x. Suppose W_1 to be a load which increases gradually from zero and produces the same extension, the work done is now given by $\frac{1}{2}W_1x$ (Fig. 9).

Since the bar is strained an equal amount in each case, the strain energy is the same in each case, and consequently the applied energies will be equal.

$$\therefore \frac{1}{2} W_1 x = W.x$$

$$W_1 = 2W.$$

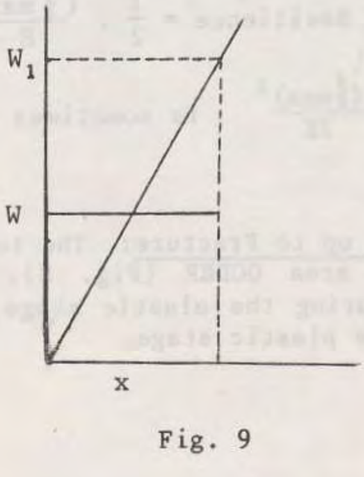


Fig. 9

That is, the suddenly-applied load required to produce a given strain is just half the magnitude of that required, for the same strain, when gradually applied. This may also be stated in the form that the strain, and consequently the stress, produced by a given suddenly-applied load is exactly twice that produced by the same load when gradually applied. This result makes it clear that great danger arises from the application of a suddenly-applied load to a machine part unless the part has been specially designed to withstand this type of load.

If a dead load of magnitude W_1 be carried by a bar of cross-section A and a suddenly-applied load of magnitude W_2 be applied, then the instantaneous stress is given by $\frac{W_1 \pm 2W_2}{A}$, the upper sign being taken when the loads act in the same direction, and the lower sign when they act in opposite directions.

20. Stress due to Shock: Fig. 10 represents a weight W, which can fall freely on to a collar or anvil carried at the lower extremity of a vertical rod, the upper end of the rod being fixed. If the weight is allowed to fall through a height h and impinge on the collar, we can calculate the maximum stress in the rod when the following assumptions are made:-

1. That the weight W falls freely.
2. That the support is rigid.
3. That there is no loss of energy by straining of the striking surfaces. (This assumption is not true in practice).

The energy possessed by W at the height h is therefore expended in straining the rod.

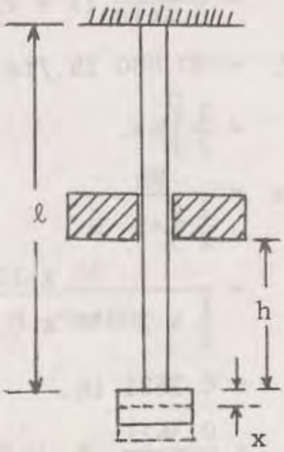


Fig. 10

Let x = the elongation of the rod

f = the maximum stress induced in the rod

A = cross-sectional area of rod

$$W(h + x) = \frac{1}{2} f.A.x.$$

$$W\left(h + \frac{f\ell}{E}\right) = \frac{1}{2} f^2 . \frac{A.\ell}{E}$$

$$\text{or } f^2 \frac{A\ell}{2E} - f\frac{W\ell}{E} - Wh = 0$$

$$f = \frac{\frac{W\ell}{E} + \sqrt{\frac{W^2\ell^2}{E^2} + 2\,\frac{WhA\ell}{E}}}{\frac{A\ell}{E}}$$

$$\text{or} \qquad f = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{W\ell}}\right)$$

If $h = o$, that is if the load is suddenly applied, then $f = \frac{2W}{A}$, a result already obtained in the previous paragraph.

<u>Example 2</u>. A weight of 20 lb. falls freely through 10 ft. and is then suddenly checked by the reaction of a bar of steel $\frac{3}{4}$ in. diameter, and 30 ft. long. Find the maximum stress and strain induced in the bar. $E = 30 \times 10^6$ lb./in.^{2}

$$f = \frac{W}{A}\left(1 + \sqrt{1 + \frac{2hAE}{W\ell}}\right)$$

$$= \frac{20}{0.4418}\left(1 + \sqrt{1 + \frac{2 \times 120 \times 0.4418 \times 30 \times 10^6}{20 \times 360}}\right)$$

$$= 30{,}080 \text{ lb./in.}^2$$

$$W(h + x) = \frac{1}{2} f A x.$$

$$x = \frac{Wh}{\frac{1}{2} f A - W}$$

$$= \frac{20 \times 120}{\frac{1}{2} \times 30080 \times 0.4418 - 20}$$

$$= 0.3621 \text{ in.}$$

$$\text{Strain} = \frac{0.3621}{360} = 0.001005$$

Examples II

1. Calculate the extension produced and the work expended in extension when a bar of steel 15 in. long and $1\frac{1}{2}$ in. diameter is subjected to a pull of 6 tons. Over what length must the bar be reduced to 1 in. diameter in order that the extension produced on the 15 in. length by the 6 ton load shall be increased by 50%. $E = 30 \times 10^6$ lb./in.^{2}

Ans. 0.003817 in., 25.65 in.lbs./6.05 in.

2. What do you understand by the term "Modulus of Resilience" of a material? A bar 12 in. long is 1.25 in. in diameter for 5 in. of its length, and 1 in. in diameter for the remainder. The bar receives a blow in an axial direction, which induces in it a maximum stress of 15 tons/in.^{2} Calculate the stress that would be induced in a bar of the same material 12 in. long and 1 in. in diameter through out, if subjected to the same blow.

Ans. 13.82 tons/in.^{2}

3. Two round bars A and B, of the same material, are each 10 in. long. A is 1 in.^{2} in section for 4 in. of its length, and the remainder is 1.5 in.^{2} in section. B is 1 in.^{2} in section from end to end. If B receives an axial blow which induces a stress of 5 tons/ in. , find the maximum stress produced by the same blow on A. Compare the "proof resilience" of the two bars.

Ans. 5.58 tons/in.^{2}, $\frac{A}{B} = \frac{8}{13}$

4. Explain how a load, suddenly applied, but without impact, will produce an intensity of stress twice as great as the intensity of stress produced by the same load applied gradually. A bar, 20 ft. long, is suspended vertically, and a collar is fixed to the lower end. A weight of 200 lb. is threaded on the bar, and falls a distance of 6 in.^{2} before engaging with the collar. Determine the diameter of the bar if the intensity of stress induced is not to exceed 12,000 lb./in.^{2} given $E = 30 \times 10^6$ lb./in.^{2}

Ans. 1.64 in.

5. Find the cross-sectional area of a vertical suspended steel tie-bar 6 ft. long, subjected to a load of 0.5 ton, which is allowed to fall 0.25 in. on to a collar at the bottom end of the bar. The ratio of the extension to the original length must not exceed $\frac{1}{1800}$. $E = 12,000$ tons/in.^{2}

Ans. 1.09 in.^{2}

6. A sliding weight of 4000 lb. is dropped down a vertical rod which is suspended from the top, and is provided with a collar at the bottom end. The length of the rod is 12 ft. and the diameter is 2 in. In order to reduce the shock a helical buffer spring is placed on the collar; the spring will compress 1 in. per 1000 lb. dead load. Taking account of the work done in compressing the spring and in stretching the bar, find approximately the height, measured from the top of the uncompressed spring, from which the weight must be dropped in order to produce a momentary stress of 10,000 lb./in.^{2} in the bar. $E = 30 \times 10^6$ lb./in.^{2}

Ans. 92 inches.

7. A crane chain whose sectional area is 1 in.^{2} carries a load of 2000 lb., which is being lowered at a uniform rate of 120 ft./min. When the length of the chain unwound is 30 ft. the chain suddenly jams on the pulley. Estimate the stress induced in the chain (neglecting the weight of the chain) due to the sudden stoppage. Modulus of direct elasticity = 30×10^6 lb./in.^{2}, $g = 32.2$ ft./sec.^{2}

Ans. 15800 lb./in.^{2}

III. THIN CYLINDERS, SPHERES.

21. Thin Cylinders: Vessels such as steam boilers and large pipes are subjected to an internal fluid pressure which is uniformly distributed over the internal surfaces. Such vessels have large cross-sectional dimensions in comparison with the thickness of their plates. In general there is a principal stress acting in the direction tangential to the circumference, called the "hoop stress", a principal stress acting in the direction of the axis, and a principal stress acting in a radial direction. The last mentioned stress can be neglected, however, where the thickness of the plates in small compared to the cross-sectional dimensions, also the first principal stress which is not uniform, over a section from the internal surface to the external surface, may be taken as uniform under this condition. These assumptions lead to the following easy method of attack:-

(a) Hoop Stress: In Fig. 11A let r be the internal radius of the cylinder, subjected to a uniform internal fluid pressure of intensity p. Consider the equilibrium of a length of the cylinder between the parallel sections AA and BB.

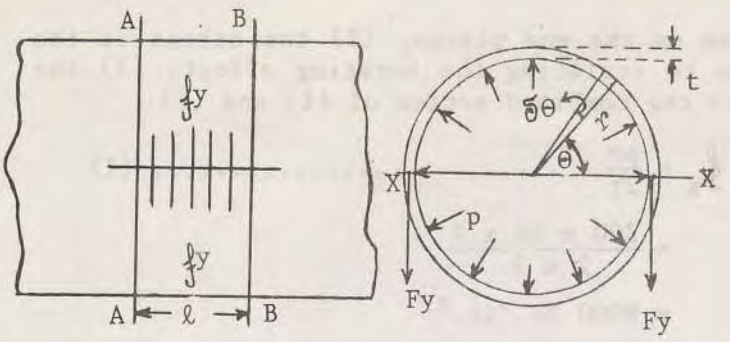


Fig. 11A

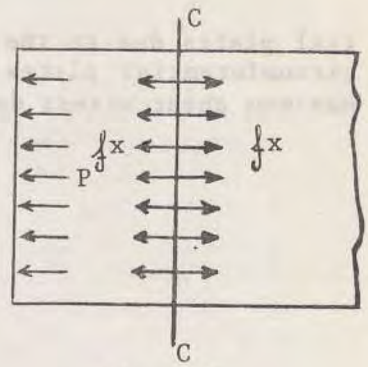


Fig. 11B

The radial force acting on the small area subtended by the angle $\delta\theta$ = $p.\ell.r.\delta\theta$, and the component of this force perpendicular to a diameter is $p\ell r \sin\theta\delta\theta$.

The total force perpendicular to a diameter = $\int_0^{\pi} p\ell r \sin\theta.d\theta. = 2p\ell r.$

If f_y is the intensity of the "hoop stress", then the:-

Resisting force = 2 Fy = $2 f_y \ell \times t$.

Hence for equilibrium of the material:-

$$2 f_y \ell \times t = 2p\ell r.$$

$$f_y = \frac{pr}{t} \quad \text{(hoop stress)} \ldots\ldots\ldots\ldots\ldots (1)$$

(b) Stress in the direction of the axis: If the ends of the cylinder are only constrained by the material, there will be a stress f_x in the shell acting in the direction of the axis, as shown in Fig. 11B. The total force on the ends, whether plane or dished, acting axially due to the internal fluid pressure will be $\pi r^2 p$, and the resisting force = $2\pi.r.t.f_x$. Therefore for equilibrium of the material:

$$2\pi.r.t.f_x = \pi r^2 p$$

$$f_x = \frac{pr}{2t} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Thus the intensity of stress, in the shell, acting in the direction of the axis is only half the intensity of the hoop stress, and these are the principal stresses in the material.

Example 1. A steam boiler, 10 ft. internal diameter, with flat ends (not stayed), plates $\frac{3}{4}$ in. thick, sustains an internal pressure of 200 lb./in.^{2} Calculate (1) the stress in the circumferential plates due to the load on the end plates, (2) the stress in the circumferential plates due to resisting the bursting effect, (3) the maximum shear stress due to the combined action of (1) and (2).

$$f_x = \frac{pr}{2t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$= \frac{200 \times 60 \times 4}{2 \times 3}$$

$$= 8000 \text{ lb./in.}^2$$

$$f_y = \frac{pr}{t} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

$$= 2\, f_x$$

$$= 16{,}000 \text{ lb./in.}^2$$

The maximum shear stress is given by $q = \frac{f_y - f_x}{2} = \frac{16000 - 8000}{2}$

$$= 4000 \text{ lb./in.}^2 \ldots (3)$$

---

22. Modification for Built-up Shell: The foregoing argument assumes that the shell and end plates are of jointless uniform material. Steam boilers and large pipes are, however, built up of plates joined by riveted joints, and hence the preceeding equations require slight modification for such cases.

The resisting force of the boiler shell will be reduced by an amount depending on the efficiency of the joint, and the new resisting force is now equal to the previous value multiplied by e where e is the joint efficiency. The equations (1) and (2) now give:

$$f_y = \frac{hr}{te} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

and $$f_x = \frac{pr}{2te} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

23. Thin Spherical Shell:

Let r = radius of shell

t = uniform thickness of the material

p = internal pressure in shell

The shell will tend to fail along a diametral plane such as EE, Fig. 12. The force acting on a thin ring of internal surface ABCD = $2\pi r.\cos\theta.r.\delta\theta.p$.

The vertical component of this force $= 2\pi p.r^2 \sin\theta\cos\theta.\delta\theta$

$= \pi r^2 p \sin 2\theta.\delta\theta.$

The total force tending to cause failure along EE

$$= \pi r^2 p \int_0^{\frac{\pi}{2}} \sin 2\theta . d\theta$$

$$= \pi r^2 p.$$

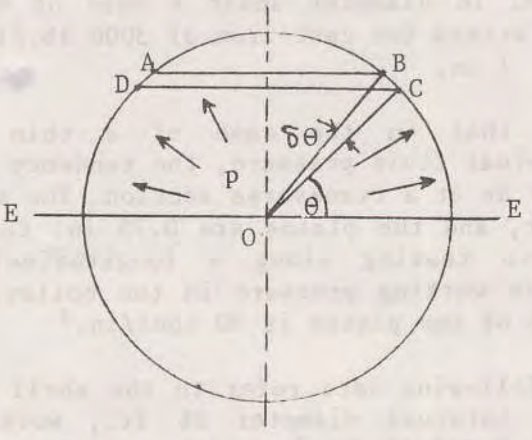


Fig. 12

This force is resisted by a force $= 2\pi r t f_z$, where f_z is the stress in the material.

$$2\pi r t f_z = \pi r^2 p$$

$$\text{and} \qquad f_z = \frac{pr}{2t} \text{ .... (1)}$$

Thus the stress in a thin spherical shell is equal to the stress in the direction of the axis of a cylindrical shell. The principal stress at any point in the shell are $\frac{pr}{2t}$, $\frac{pr}{2t}$, and a radial principal stress which may be neglected.

24. <u>Modification for Built-up Shell</u>: When the spherical shell is built of plates joined by riveted joints the resisting force is lessened in proportion to the joint efficiency just as in the case of the cylindrical shell.

A similar argument gives the modified formula as

$$f_z = \frac{pr}{2te} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ (2)$$

where e is the efficiency of the joint.

Examples III

1. What thickness of metal would be required for a cast-iron water pipe 36 in. in diameter under a head of 400 ft.? Assume a limiting tensile stress for cast-iron of 3000 lb./in.^{2}

Ans. 1 in.

2. Show that in the case of a thin cylindrical shell subjected to internal fluid pressure, the tendency to burst lengthwise is twice as great as at a transverse section. The shell of a boiler is 6 ft. in diameter, and the plates are 0.75 in. thick; the efficiency of the joints to tearing along a longitudinal seam being 72%. Calculate the safe working pressure in the boiler, assuming that the tensile strength of the plates is 30 tons/in.^{2}

3. The following data refer to the shell of a boiler of the Scottish type: Internal diameter 16 ft., working steam pressure (above atmospheric) 210 lb./in.^{2}, thickness of shell plates $1\frac{19}{32}$ in., diameter of rivet holes $1\frac{5}{8}$ in. The longitudinal joint is a treble riveted double strap but joint, the pitch of the rivets in the outer row being $10\frac{1}{2}$ in. The circumferential joint is a double lap joint, the pitch of the rivets being $4\frac{1}{2}$ in. Calculate the maximum stress in the boiler plate (a) longitudinally, and (b) circumferentially.

Ans. Hoop stress 15,000 lb./in.^{2}, in direction of axis 10,000 lb./in.^{2}

IV. BENDING MOMENTS AND SHEARING FORCES

25. Definitions. Consider the horizontal beam, Fig. 13, which is in equilibrium under the forces R_1 and R_2 acting vertically upwards, and the vertical down ward forces W_1, W_2, and W_3. The forces

keeping the portion, to the right-hand side of a section at A, in equilibrium are $(R_2 - W_3)$, and the forces exerted by the left-hand portion of the beam across the section at A on the right-hand portion.

Thus force exerted by right-hand portion of beam on left-hand portion = $(R_2 - W_3)$, and similarly the force exerted by the left-hand portion of beam on right-hand portion = $W_1 + W_2 - R_1$.

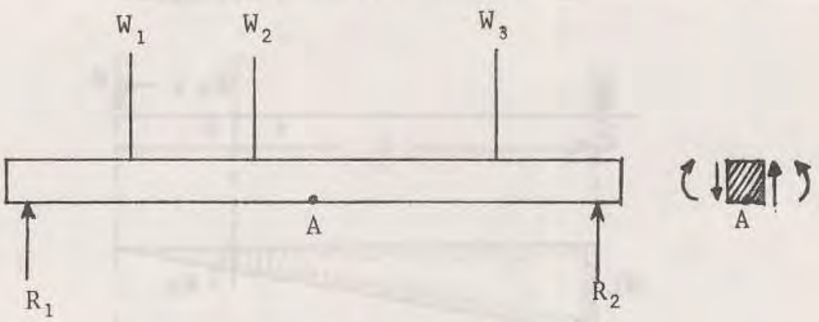


Fig. 13

A thin longitudinal slice of beam as at A containing the point A is thus kept in equilibrium by two opposite parallel forces, and since there are no other forces acting on the slice (neglecting its weight), the parallel forces must be equal in magnitude. Thus the slice is in a state of shear.

The shearing force at any point along a loaded beam is the algebraic sum of all the vertical forces acting to one side of the point.

The forces R_2 and W_3 have a resultant moment acting on the thin slice in an anti-clockwise direction, and the forces W_1, W_2, and R_1 have a resultant moment acting on the slice in a clockwise direction. Since the slice in the beam has no rotary, the two resultant moments must be equal in magnitude.

The bending moment at any point along a loaded beam is the algebraic sum of the moments of all the vertical forces acting to one side of the point about the point.

26. Bending Moment and Shearing Force Diagrams: In order to design a beam to carry a given load system it is necessary to know the value of the bending moment and shearing force at any point along the span. These are usually calculated for various points, and diagrams are then plotted showing the bending moment and shearing force as ordinates, and the span as abscissae. A number of typical cases will now be considered.

(1) Cantilever with concentrated load at free end (Fig. 14). The bending moment at a point A distance x from the free end is given by:

$$MA = W.x.$$

Thus the bending moment increases gradually from zero at the free end where x=o to a maximum at the fixed end where x = L. The bending moment diagram is therefore a triangle.

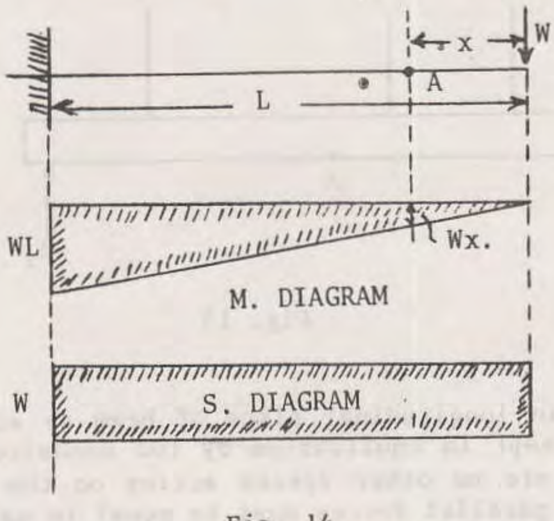


Fig. 14

The shearing force at A = W, hence the shearing force is uniform from end to end of the beam, and the diagram is therefore rectangular.

(2) Cantilever with several concentrated loads (Fig. 15). The bending moment and shearing force diagram can be drawn separately for W_1, W_2, W_3, then add together the ordinates of each bending moment diagram to get the bending moment diagram for the combined system, and similarly add together the ordinates of the shearing force diagrams in order to get the shearing force diagram for the system. This follows directly from the definition of bending moment and shearing force.

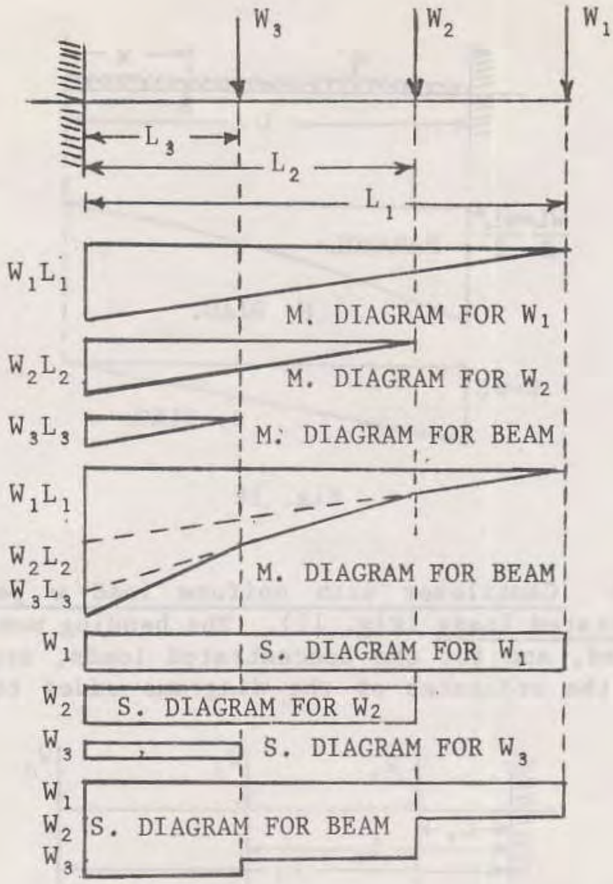


Fig. 15

(3) Cantilever with uniform load w per unit run (Fig. 16). The load on the small piece of span of length x, measured from the free end of the cantilever, is wx, and the distance of its center of gravity from A $= \frac{x}{2}$.

$$\therefore MA = (wx)\,\frac{x}{2} = \frac{wx^2}{2}$$

Plotting bending moment against x, we obtain a parabola, and the maximum value of M is obtained when x=L, and is given by:

$$M_{max} = \frac{wL^2}{2} = \frac{WL}{2}$$

Considering the forces to the right - hand side of A, the shearing force at A is wx. The shearing force diagram is therefore triangular, and $S_{max} = wL = W$, and occurs at the constraint.

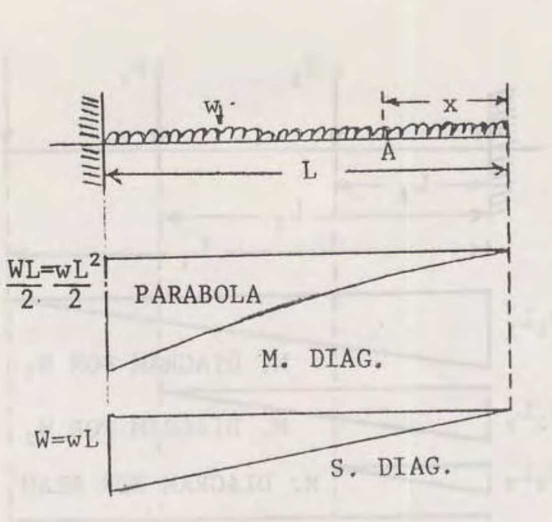


Fig. 16

(4) Cantilever with uniform load w per unit run and several concentrated loads (Fig. 17). The bending moment diagrams for the uniform load, and for the concentrated loads, are obtained as in Fig. 15 & 16, the ordinates of the diagrams added together give the

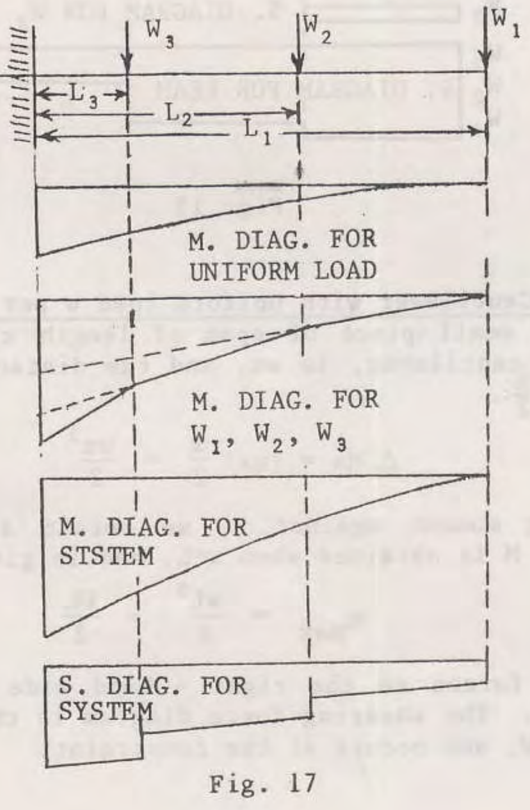
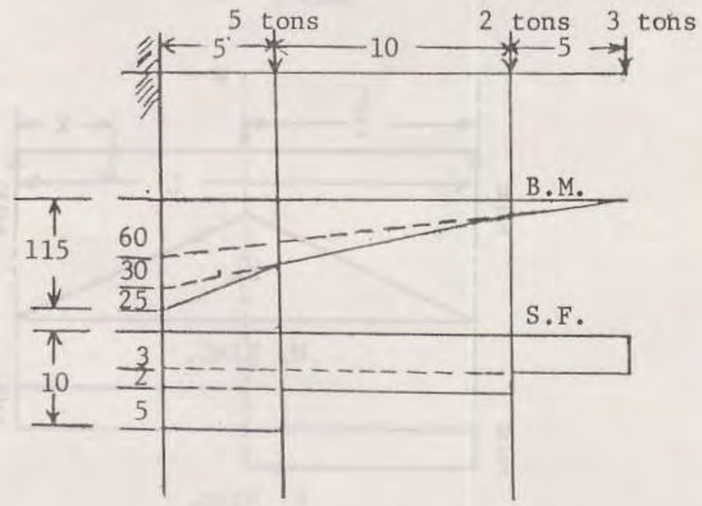


Fig. 17

bending moment diagram for the load system. Similarly, the shearing force diagrams for the uniform load, and the concentrated loads, are drawn, and the ordinates of these diagrams added together give the shearing force diagram for the system.

Example 1. A cantilever 20 ft. long carries loads of 5, 2, and 3 tons at distances of 5, 15 and 20 ft. from its fixed end respectively. Estimate the maximum bending moment and shearing force on the cantilever, and the intensity of a uniformly distributed load which would cause the same bending moment.



The max. bending moment and shearing force occur at the fixed end.

$$M_{max} = (5 \times 5) + (2 \times 15) + (3 \times 20) = 115 \text{ ton.ft.}$$

$$S_{max} = 5 + 2 + 3 = 10 \text{ tons.}$$

Let w be the intensity of the uniformly distributed load, which gives the same maximum bending moment,

$$\text{then } M_{max} = \frac{wL^2}{2} = \frac{w}{2} \times 20^2$$

$$\therefore \frac{w}{2} \times 400 = 115$$

$$w = \frac{230}{400}$$

$$= 0.575 \text{ ton./ft.run.}$$

---

(5) Freely supported horizontal beam with concentrated load at mid-span (Fig. 28). From symmetry the reaction at each support is $\frac{W}{2}$. Considering the right-hand half of the span, the bending moment at A, a distance x from the right-hand support is given by:

$$MA = \frac{W}{2} x$$

The bending moment, therefore, gradually increases from zero at the support, where x is zero, to $\frac{W}{2} \cdot \frac{L}{2} = \frac{WL}{4}$ where x is $\frac{L}{2}$. If we consider the left-hand half of the span, it is found that the bending moment at the left-hand support is zero and increases gradually to $\frac{WL}{4}$ at mid-span. Hence the maximum bending moment occurs at mid-span, and is given by:

$$M_{max} = \frac{WL}{4}$$

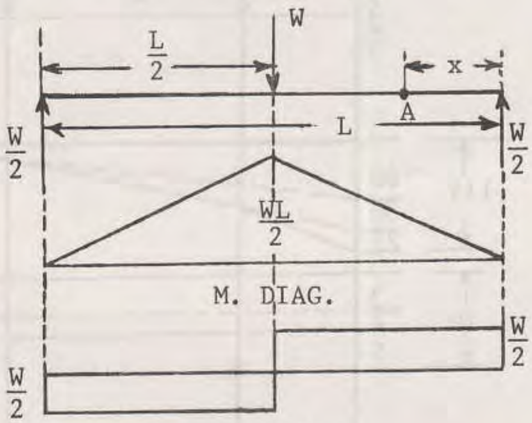


Fig. 18

The shearing force at x is $\frac{W}{2}$ and is uniform over the right-hand half of the span. After passing mid-span the shearing force changes sign, but is still uniform over the left-hand half of the span and equal to $\frac{W}{2} - W = -\frac{W}{2}$.

(6) Freely supported beam with concentrated load not at mid-span (Fig. 19). Let the load devide the span into two portions a and b. Since the load is not at mid-span, the reaction at each end will be different. Taking moments about B, we have:-

$$R_1L = Wb$$
$$R_1 = \frac{Wb}{L}$$
$$\text{and } R_1 + R_2 = W$$
$$R_2 = W - R_1$$
$$R_2 = W - \frac{Wb}{L} = W\left(\frac{L-b}{L}\right) = \frac{Wa}{L}$$

Considering the right-hand portion of the span, the bending moment at A is given by:

$$MA = R_1.x = \frac{Wb}{L}.x$$

Thus the bending moment diagram for the right-hand half of the span will be triangular, increasing from zero at x=o to $\frac{Wb}{L}\cdot a$ where x=a.

The bending moment at D, a distance x_1 to the left of W, is given by:

$$MD = R_1\ (a + x_1) - Wx$$
$$= \frac{Wb}{L}\ (a + x_1) - Wx_1$$
$$= W\ \left\{\frac{ab}{L} + x_1\left(\frac{b}{L} - 1\right)\right\}$$
$$= W\ \left\{\frac{ab}{L} + x_1\left(\frac{b-L}{L}\right)\right\}$$
$$= W\ \left\{\frac{ab}{L} - x_1\frac{a}{L}\right\}.$$

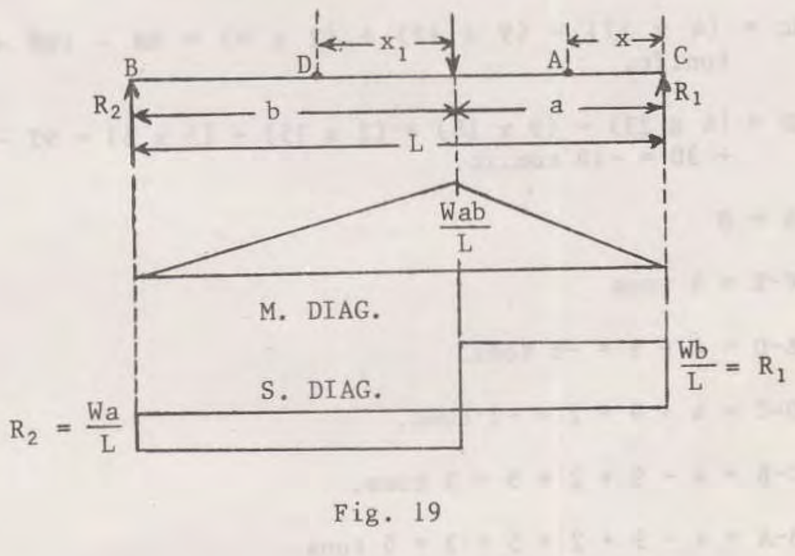


Fig. 19

This gives a triangular bending moment diagram for the left-hand portion of the span, the bending moment decreasing from $\frac{Wab}{L}$ where x is zero to:

$$MB = W\left(\frac{ab}{L} - \frac{ab}{L}\right) = 0,$$

where $x_1 = b$. The maximum bending moment is thus given by:

$$M_{max} = \frac{W.a.b}{L}$$

The shearing force over the right-hand portion of the span is uniform and equal to $R_1 = \frac{Wb}{L}$. After passing W the shearing force becomes $\frac{Wb}{L} - W = -\frac{Wa}{L}$, and is uniform over the left-hand portion of the span.

(7) <u>Horizontal freely supported beam with concentrated loads and overhanging end</u> (Fig. 20). The dimensions and load distribution are given in figure.

$$20R_1 + 20 = 6 + 60 + 54 = 120$$

$$R_2 = \frac{100}{20} = 5 \text{ tons}$$

$$R_1 + R_2 = 3 + 5 + 2 + 4 = 14$$

$$R_1 = 14 - 5 = 9 \text{ tons.}$$

MF = 0

ME = 4 x 5 = 20 ton.ft.

MD = (4 x 8) - (9 x 3) = 32-27 = 5 ton.ft.

Mc = (4 x 17) - (9 x 12) + (2 x 9) = 68 - 108 + 18 = -22 ton.ft.

MB = (4 x 23) - (9 x 18) + (2 x 15) + (5 x 6) = 92 - 162 + 30 + 30 = -10 ton.ft.

MA = 0

SF-E = 4 tons

SE-D = 4 - 9 = -5 tons.

SD-C = 4 - 9 + 2 = -3 tons.

SC-B = 4 - 9 + 2 + 5 = 2 tons.

SB-A = 4 - 9 + 2 + 5 + 3 = 5 tons.

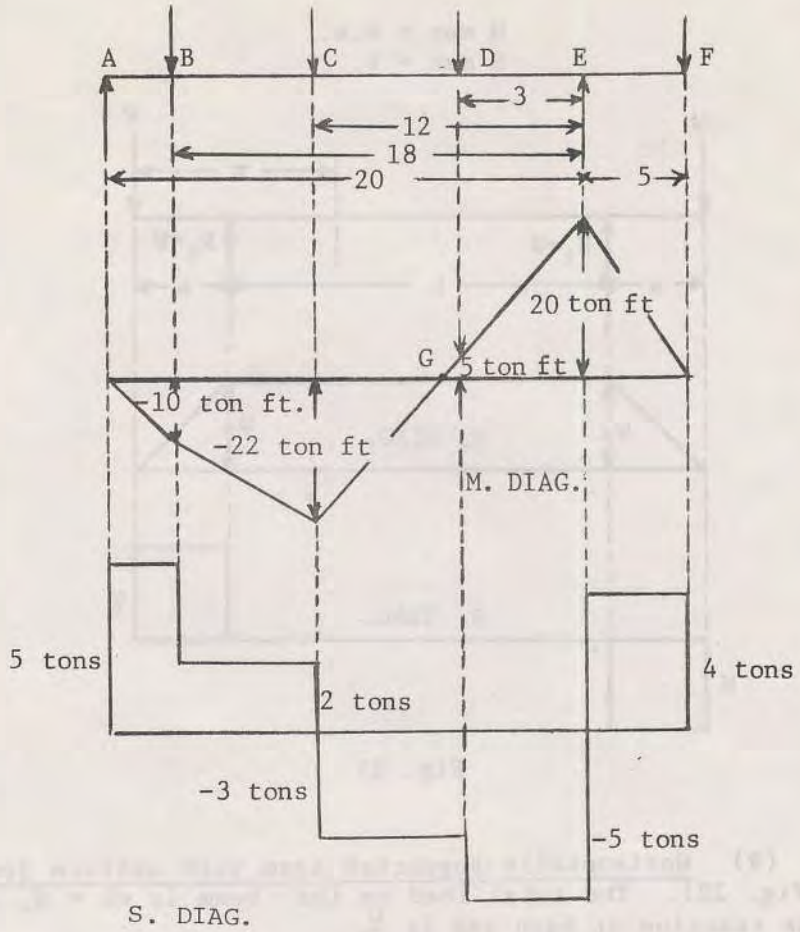


Fig. 20

The bending moment and shearing force diagrams are shown in Fig. 20.

The point G on the beam where the bending moment is zero, and, as shall be found later where the curvature changes sign, is called a "point of inflection or point of contraflexure".

(8) <u>Horizontally supported beam with equal overhanging ends at which isolated loads act</u> (Fig. 21). An important example of this case is a wagon axle. The support or wheels are R and R , and the load at each bearing is W.

The diagrams show the bending moment being uniform between the supports or wheels, and the shearing force being zero between the same points.

M max = W.a.
S max = W.

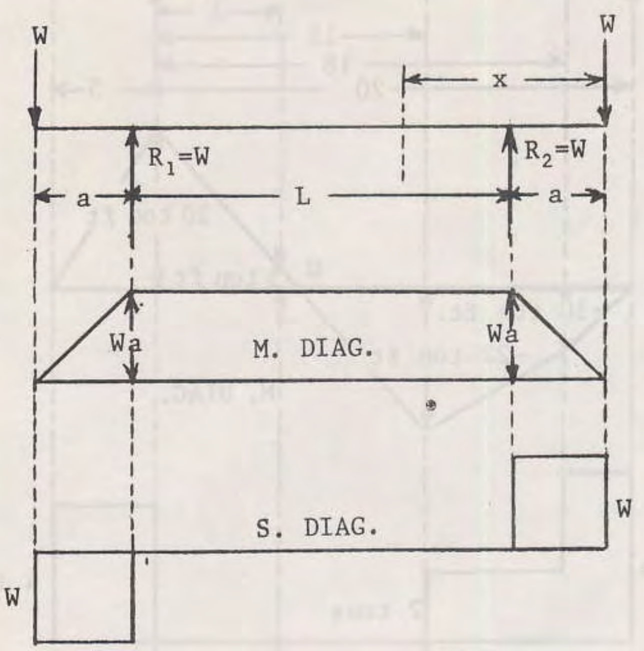


Fig. 21

(9) <u>Horizontally supported beam with uniform load w per unit run</u> (Fig. 22). The total load on the beam is $wL = W$, and from symmetry the reaction at each end is $\frac{W}{2}$.

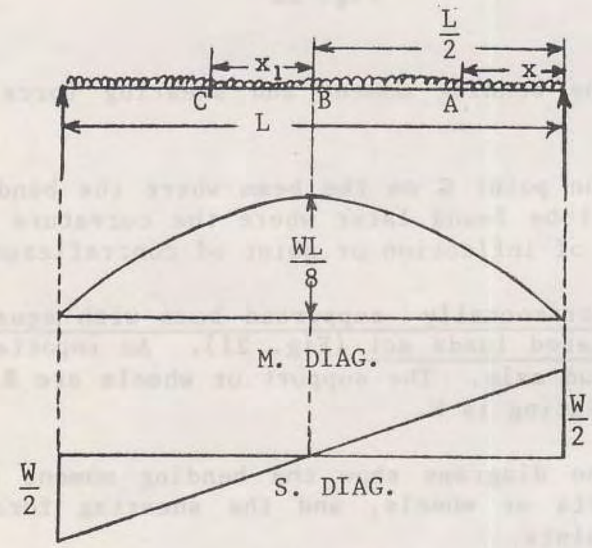


Fig. 22

$MA = \frac{W}{2}x - wx\,\frac{x}{2} = \frac{W}{2}x - \frac{wx^2}{2}$. Thus the bending moment diagram is parabolic in form, increasing from zero where x = o to a value $\frac{W}{2}.\frac{L}{2} - \frac{wL}{8} = \frac{WL}{4} = \frac{WL}{8} = \frac{WL}{8}$ where $x = \frac{1}{2}$.

$$MC = \frac{W}{2}\left(\frac{L}{2} + x_1\right) - \frac{w}{2}\left(\frac{L}{2} + x_1\right)^2$$

This expression also gives a parabolic bending moment diagram for the left-hand side of the diagram, decreasing from $\frac{WL}{8}$ where x_1 is zero, to zero where x_1 has the value $\frac{L}{2}$.

$M_{max} = \frac{WL}{8}$ and occurs at mid-span

$SA = \frac{W}{2} - wx$. The shearing force diagram is thus triangular for the right-hand side of the span, decreasing from $\frac{W}{2}$ where x = 0 to zero at mid-span where $x = \frac{L}{2}$.

$SC = \frac{W}{2} - w\left(\frac{1}{2} + x_1\right)$. This gives a triangular diagram for the left-hand side of the diagram, the shearing force increasing from zero where $x_1 = 0$ to $-\frac{W}{2}$ Where $x_1 = \frac{L}{2}$.

$S_{max} = \frac{W}{2}$ at each end of the span.

(10) <u>Horizontally supported beam with equal overhanging ends, and a uniform load w per unit run, over the beam</u> (Fig. 23). Each overhanging end acts as a cantilever, and the maximum bending moment at the supports is $\frac{wa^2}{2}$. If the load was removed from b. the bending moment diagram would be PQRT. The diagram for the portion of span b. is a parabola whose maximum ordinate is $\frac{wb^2}{8}$. This bending moment, due to the load on b, is of opposite sign to that due to the two end loads; hence, as the diagram due to b is drawn on TR as base, we obtain the resultant diagram shown by Fig. 23.

The value of $MB = \frac{wb^2}{8} - \frac{wa^2}{2}$

The value of $MA = \frac{wa^2}{2}$

The resultant bending moment at the center will be zero when MB = 0, i.e. $\frac{wa^2}{2} = \frac{wb^2}{8}$

or $a = \frac{b}{2}$

When a increase, MA increases and MB decreases. Thus when the bending moment at mid-span is zero, the bending moment at each support has the value $\frac{wb^2}{8}$; the point V would then just touch the line PQ.

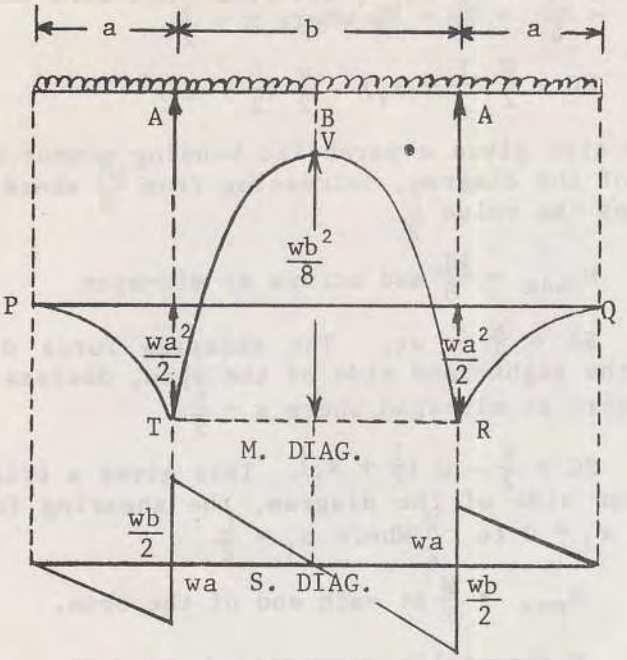


Fig. 23

The least bending moment on the beam is obtained MA is equal to MB.

$$\text{or} \quad \frac{wa^2}{2} = \frac{wb^2}{8} - \frac{wa^2}{2}$$

$$\therefore a = \frac{b}{2\sqrt{2}} = 0.354\ b$$

Example 2. A beam 40 ft. long resting on two supports 24 ft. apart, center to center, projects to the extent of 6 ft. and 10 ft. beyond the supports at either end, and it is loaded uniformly with 0.5 ton per foot. Find the position and magnitude of the greatest bending moment on the beam and the points of contraflexure. Make a dimensioned sketch bending moment diagram for the beam.

Let R_1 and R_2 be the reactions of the supports (Fig. 24).

Then $R_1 \times 24 = 40 \times 0.5 \times 10$

$R_1 = 8\frac{1}{3}$ tons.

The bending moment at A = $-6 \times 0.5 \times 3 = -9$ ton.ft.

The bending moment at B distant x from A

$$= (8\tfrac{1}{3} \times x) - (6 + x)\,0.5\,(\tfrac{6 + x}{2})$$

$$= 8\tfrac{1}{3}\,x - \tfrac{1}{4}\,(36 + 12x + x^2)$$

$$= -\tfrac{1}{4}\,x^2 + 5\tfrac{1}{3}\,x - 9$$

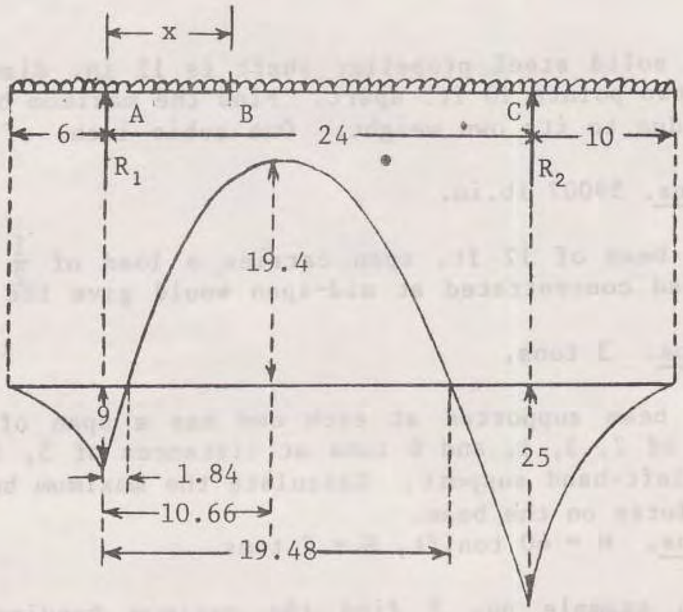


Fig. 24

For maximum bending moment between supports $\frac{dM}{dx} = 0$.

$$\therefore -\tfrac{1}{2}\,x + 5\tfrac{1}{3} = 0$$

$$\text{or } x = 10\tfrac{2}{3} \text{ ft.}$$

$$\therefore M_{max} = 19.4 \text{ ton.ft.}$$

At points of contraflexure M = 0,

$$\tfrac{1}{4}\,x^2 - 5\tfrac{1}{3}\,x + 9 = 0$$

$$x = \frac{5\tfrac{1}{3} \pm \sqrt{(5\tfrac{1}{3})^2 - 9}}{\tfrac{1}{2}}$$

$$= 1.84 \text{ and } 19.48 \text{ ft.}$$

$$MC = 10 \times 0.5 \times 5$$

$$= 25 \text{ ton.ft. and is greatest on the beam.}$$

Examples IV

1. A solid steel propeller shaft is 12 in. diameter, and is supported at two points 10 ft. apart. Find the maximum bending moment on the shaft due to its own weight. One cubic inch. of steel weighs 0.29 lb.

   Ans. 59007 lb.in.

2. A beam of 12 ft. span carries a load of $\frac{1}{2}$ ton per foot run. What load concentrated at mid-span would give the same bending moment?

   Ans. 3 tons.

3. A beam supported at each end has a span of 30 ft., and carries loads of 2, 3, 1, and 6 tons at distances of 5, 10, 20 and 25 ft. from the left-hand support. Calculate the maximum bending moment and shearing force on the beam.

   Ans. M = 40 ton.ft, S = 7 tons.

4. In example no. 3 find the maximum bending moment and shearing force if the beam is increased in length, so that there is an overhang of 5 ft. at each support, and a load of 2 tons at the end of the left-hand overhang, and a load of $\frac{1}{2}$ ton per foot run spread over the right-hand overhang.

   Ans. M = 32.5 ton.ft., S = 6.875 tons.

5. A wheel weighing 60 lb. rotates at a uniform speed of 300 r.p.m., and is keyed to a shaft which projects 6 in. beyond a bearing. If the centre of gravity of the wheel is $\frac{1}{4}$ in. from the axis of rotation, find the greatest bending moment on the shaft.

   Ans. 594 lb.in.

V. STRESSES IN BEAMS

27. Neutral Axis: The bending moment and shearing force, at various points along a loaded beam, introduce stresses in the beam, and, with certain assumptions, the connection between the stresses, the bending moment, the curvature of the beam, the dimensions of the beam and the elasticity of the beam can be obtained. The nature of these stresses may be studied by reference to Fig. 25. A beam of uniform section is shown resting, in the unloaded state, at (a). When under a given load system the shape of the beam, greatly exaggerated, is given by (b). If we suppose the beam to be of wood, and attempt to

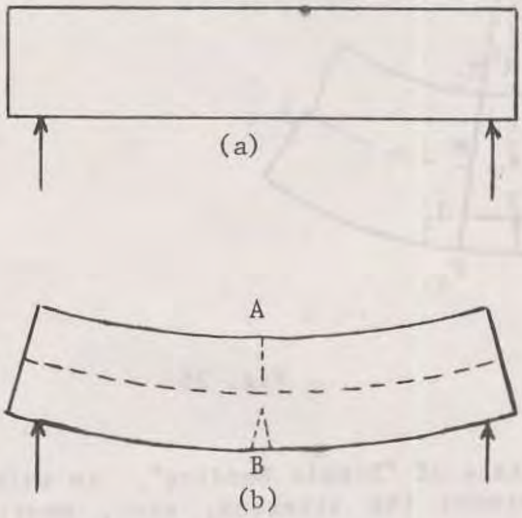


Fig. 25

make a saw cut in the direction A to B, it is a matter of common experience that the beam will eventually close in on the saw, thus showing that a state of compression exists at one side of the beam. If the saw cut is made in the direction B to A, it is also well known that the cut opens out rapidly, thus showing that the other side of the beam is in a state of tension. The fibres at one side of the beam will be shortened, as shown by (b), owing to the state of compression existing, and those at the other side will be lengthened owing to the state of tension existing. At a point between the top and bottom of the beam a layer of fibres will be found which suffer no stress, and consequently remain their original length. This layer of fibres forms what is known as the neutral surface and the trace of this surface on a cross-section (N.A.) is called the "Neutral Axis".

28. Assumptions in the Simple Bending Theory: When a beam is bent, due to the application of a constant bending moment, i.e. by couples applied to its ends, without being subjected to shear, it is

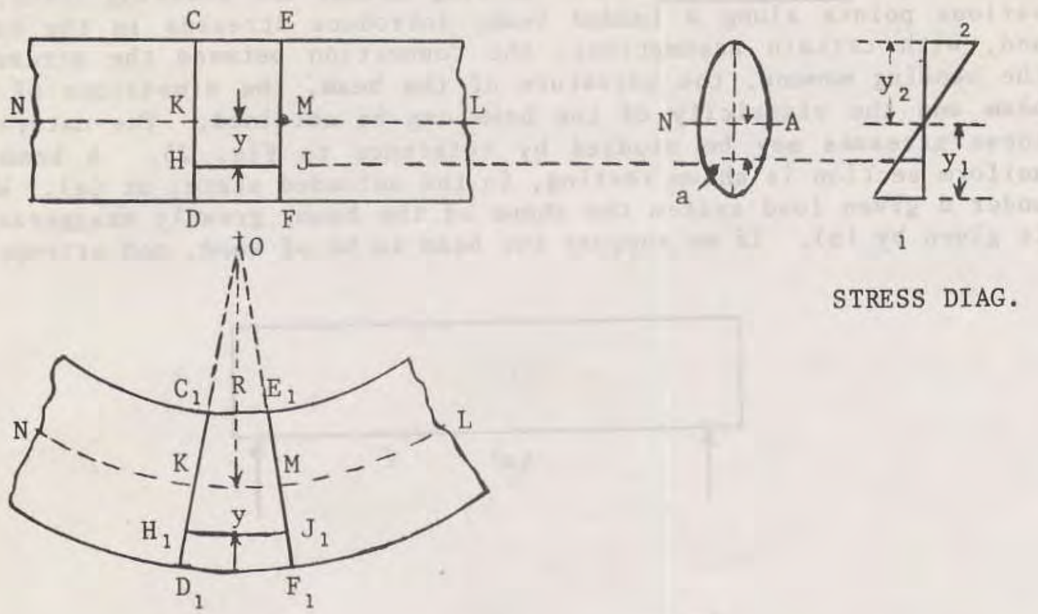


Fig. 26

said to be in a state of "Simple Bending". In this case the relationship existing between the stresses, etc., mentioned previously in paragraph 27, are obtained readily with the aid of the following assumptions:

(1) That Young's modulus has the same value, for the material of the beam, in tension as in compression, and that the stress is proportional to the strain.

(2) That a transverse section of the beam, which is a plane before bending, will remain a plane after bending.

(3) That the radius of curvature of the beam, before bending, is very large in comparison to the transverse dimensions of the beam.

(4) That the resultant pull or thrust across a transverse section of the beam is zero.

(5) That the transverse section of the beam is symmetrical about an axis, passing through the centroid of the section, and parallel to the plane of bending.

In Fig. 26 a portion of a uniform beam, subjected to simple bending, is shown. In the unstrained state let HJ be a portion of a fibre at the distance y from the neutral surface; its length being determined by the two parallel planes CD and EF. After bending, the planes assume the position shown, being inclined at the angle θ and intersecting at the point O. Let R be the radius of the neutral surface, the radius of the curved fibre is (R+y). Since the fibre is not at the neutral surface, its length is altered to H_1J_2.

$$\text{Now } \frac{H_1J_1}{KM} = \frac{(R+y)O}{RO} = \frac{R+y}{R}$$

Also the strain in the fibre is given by:

$$\frac{H_1J_1 - HJ}{HJ} = \frac{H_1J_1 - KM}{KM} = \frac{H_1J_1}{KM} - 1 = \frac{R+y}{R} - 1 = \frac{y}{R}.$$

If f is the intensity of the stress in the fibre then:

$$\frac{f}{\frac{y}{R}} = E$$

$$\text{or} \quad \frac{f}{y} = \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(1)}$$

This may also be expressed in the form $f = \frac{E}{R}.y$, and for a given beam under a given load system we may write $f = ky$, or, the stress in the fibres of a beam, at a distance y from the neutral surface, is directly proportional to the distance of the fibre from the neutral surface.

Since the material in proximity to the neutral surface carried no stress, and, therefore, lends no assistance to resist the applied couple, a beam should be such that the greatest possible of its area is as far away from the neutral surface as possible. We see this exemplified in the well known sections adopted in engineering practice, such as H, T, and channel sections, etc.

From (1) it will be observed that, since the beam is of uniform section, R is constant and hence NL will form an arc of a circle.

29. <u>Position of the Neutral Axis</u>: Consider an element of area a at the distance y from the neutral axis. We have that the total force on the element = f x a,

$$\text{but } \frac{f}{y} = \frac{f_1}{y_1}$$

$\therefore$ Total force on element $= a \frac{f_1}{y_1} y = \frac{f_1}{y_1}$ a.y.

and total force on transverse section below N.A. $= \Sigma \frac{f_1}{y_1}$ a.y.

$= \frac{f_1}{y_1} \Sigma$ a.y.

Similarly, it may be shown that if a and y be chosen on the upper side of N.A., the total force on the transverse section above N.A. $= \frac{f_2}{y_2} \Sigma$a.y.

From assumptions (1) and (2), paragraph 28, we have that:-

$$\frac{f_1}{y_1} = \frac{f_2}{y_2}$$

Also $\frac{f_1}{y_1} \Sigma ay$ for the area below N.A. is the total tension on the section, and $\frac{f_2}{y_2} \Sigma ay$ for the area above N.A. is the total compression on the section. By assumption (4), paragraph 28, the resultant of these is zero. Therefore Σ a.y. for the area below N.A. $= \Sigma$ a.y. for the area above N.A. and the signs are different, hence the value of Σ a.y. for the whole section (or the first moment of the section) about N.A. is zero. From the theory of centroid we see that for this to occur, N.A. must pass through the centroid of the section. We have, therefore, the important rule that, in cases of simple bending, the neutral axis passes through the centroid of the section.

30. Moment of Resistance: The moment of the force, on the small element a, about N.A. $= \frac{f_1}{y_1}$ .a.y.^{2}, and the total moment of all the forces, acting on the various small elements composing the cross-section, forms a couple which is equal to the bending moment. This total moment is called the "Moment of Resistance" = M.

$$M = \frac{f_1}{y_1} \Sigma \text{ a.y.}^2$$

but Σay^2 is equal to the second moment or moment of inertia, I, of the section about N.A.

$$M = \frac{f_1}{y_1} I$$

$$\text{or} \quad \frac{M}{I} = \frac{f_1}{y_1} = \frac{f}{y} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(2)}$$

$$\text{or} \quad \frac{M}{I} = \frac{f_2}{y_2} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(3)}$$

this may also be put in the form:

$$M = f_1 \frac{I}{y_1} = f_2 \frac{I}{y_2}$$

$\frac{I}{y_1}$ and $\frac{I}{y_2}$ are called the tension and compression section moduli respectively, and are denoted by Z_1 and Z_2.

$$M = f_1 Z_1 = f_2 Z_2 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

It is usual to combine equations (1) and (2) into what is called the bending equation, which gives:

$$\frac{M}{I} = \frac{f}{y} = \frac{E}{R} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Too much stress cannot be laid on the importance of this equation, and the student is strongly advised to make himself perfectly familiar with it. The units of the various quantities are as follows:

M = bending moment lb.in. (or ton.in).

I = moment of inertia of section about the neutral axis in inch units.

E = Young's modulus in lb./in^2 (or tons/in.^{2})

R = radius of curvature of neutral line in inches.

f = stress in lb./in^2. (or tons/in.^{2}) due to bending at a distance y from the neutral axis.

31. Application of the Bending Equation to Practical Cases: In practice it is usually found that the bending moment on a beam varies from point to point along the span, and that the bending moment is accompanied by a shearing force. Thus it would appear that the bending equation, which deals with a constant bending moment unaccompanied by a shearing force, is not strictly applicable to such a case.

It will be found, however, that in a great number of practical cases, the bending moment is maximum when the shearing force is zero, and that when the shearing force is maximum the bending moment is almost negligible. Thus the conditions of simple bending are approximated to at the point of maximum bending moment, and hence it seems justifiable to apply the bending equation at this point. The stresses due to the maximum bending moment are usually the most important stresses in beam, and, therefore, if a beam is designed by the aid of the bending equation to resist the maximum bending moment, its strength will more than suffice at other points along the span where different conditions hold.

Example 1. A rolled joist of I-section has the following dimensions: Flanges 5 in. wide, 0.575 in. thick, web 0.375 in. thick, depth overall 8 in. It is used as a beam freely supported at each end, and covering a clear span of 12 ft. It carries a load of 9 tons uniformly distributed over the span. Calculate the maximum stress produced in the material of the girder due to bending.

$$Ixx = \frac{5 \times 8^3}{12} - \frac{4.625 \times 6.85^3}{12} = 89.33 \text{ in. units.}$$

$$M = \frac{WL}{8} = \frac{9 \times 12 \times 12}{8} = 162 \text{ ton-inches.}$$

$$\frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} . y$$

$$= \frac{162}{89.33} \times 4$$

$$= 7.25 \text{ tons/in.}^2 \quad \underline{\text{Ans.}}$$

32. Reinforced Concrete Beams: Concrete is a material which is strong in compression, but comparatively weak in tension. A beam of such material, therefore, would fail under fairly light loads owing to the tensile stresses due to bending. Additional strength is given to such a beam by embedding in it iron or steel bars in such a position that the bars take the tensile forces. Several theories have been developed to cover the bending of reinforced concrete beams, but the following, called the "No Tension Theory" is most generally accepted.

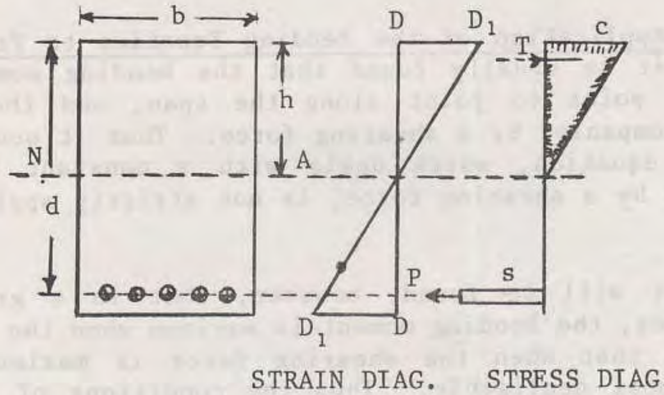


Fig. 27

The following additional assumptions to the simple bending theory are made:-

(1) That there is perfect adhesion between the concrete and the reinforcement.

(2) That all the tensile stress is carried by the reinforcement.

(3) That, in the concrete, the stress is proportional to the strain.

(4) The area of reinforcement is so small that the stress may be assumed constant over it.

A reinforced beam of rectangular section is shown by Fig. 27, with the strain and stress diagrams. The neutral axis of such a beam does not pass through the centroid. Let h be the distance of N.A. from the top of the beam, Ec the value of Young's modulus for concrete, and Es the value for steel. The ratio $\frac{Es}{Ec}$ is usually denoted by m.

Since a plane before bending remains a plane after bending, DD will take up the position $D_1 D_1$, and hence the strain is proportional to the distance from the neutral axis.

$$\frac{\text{strain in concrete at top of beam}}{\text{strain in the steel}} = \frac{h}{d-h} \quad \ldots\ldots\ldots\ldots (1)$$

Also if f_c denotes the stress in the concrete at top of beam,

and if f_s denotes the stress in the steel, we have:

$$\text{strain in concrete at top of beam} = \frac{f_c}{Ec}$$

$$\text{strain in steel} = \frac{f_s}{Es}$$

$$\therefore \text{by substitution in (1)}: \quad \frac{h}{d-h} = \frac{f_c}{Ec} \times \frac{Es}{f_s} = m\frac{f_c}{f_s} \quad \ldots\ldots\ldots\ldots (2)$$

$$\text{or } hf_s = mf_c d - mf_c h$$

$$h = \frac{mf_c d}{f_s + mf_c} \quad \ldots\ldots\ldots\ldots (3)$$

An expression which enables us to find the distance h if f_s and f_c are known.

Let As be the area of the reinforcement, the total pull P exerted by it is $f_s As$. Since the concrete carries no tension, the stress diagram is triangular and the thrust T exerted by the concrete is given by:

$$T = \frac{f_c}{2}.b.h. \quad \ldots\ldots\ldots\ldots (4)$$

Since the conditions of simple bending are assumed to hold, then: $P = T$

$$\text{and } f_s As = \frac{f_c}{2} b.h.$$

and substituting the value of $\frac{f_c}{f_s}$ as found from (2), we have:

$$As = \frac{h}{m(d-h)2} \times bh.$$

$$\text{or } bh^2 + 2\,m.As.h. - 2mdAs = 0 \quad \ldots\ldots\ldots\ldots (4a)$$

a guadratic equation which fixes the value of h, and depends only on known quantities.

The thrust T acts at a distance $\frac{h}{3}$ from the top of the beam, and hence the resisting moment of the beam is given by

$$M = T\ (d - \frac{h}{3}) = \frac{f_c}{2}\ b.h.\ (d - \frac{h}{3})\ \ldots\ldots\ (5)$$

$$\text{or} \qquad M = P\ (d - \frac{h}{3}) = f_s.As\ (d - \frac{h}{3})\ \ldots\ldots\ldots\ (6)$$

$$\text{or} \qquad M = T\ (\frac{2}{3}\ h) + P\ (d - h)\ \ldots\ldots\ldots\ldots\ldots\ (7)$$

These reinforced beams are often made of T section, and it is very common practice to design the beam so that the neutral axis coincides with the bottom line of the flange of the section. If the neutral axis is lower than this line, we use a similar method of attack to that just used.

<u>Example 2</u>. A concrete beam 15 ft. long x 12 in. broad x 15 in. deep, is reinforced by six $\frac{3}{4}$ in. round steel bars, having their centers 2 in. from the bottom of the beam. Assuming that all the compression is carried by the concrete and all the tension by the steel, determine the uniform loading that may be applied without the stress in the concrete exceeding 600 lb./in.^{2}, and ascertain the resulting stress in the steel. Take the steel-concrete modular ratio as 15 and the concrete stress strain curve as straight.

$$\text{Area of reinforcement} = \frac{\pi}{4} \cdot x\ \frac{3}{4}\ x\ \frac{3}{4}\ x\ 6 \ = \ 2.6488\ \text{in.}^2$$

$$bh^2 + 2mAs.h - 2mdAs \quad = 0$$

$$12h^2 + 79.47\ h - 1033.11 = 0$$

$$h = \frac{-79.47 \pm \sqrt{(79.47)^2 + (48\ x\ 1033.11)}}{}$$

$$= 6.55\ \text{in.}$$

$$M = \frac{f_c}{2}\ b.h\ (d - \frac{h}{3})$$

$$= \frac{600\ x\ 12\ x\ 6.55}{2}\ (13 - \frac{6.55}{3})$$

$$= 255{,}000\ \text{lb.in.}$$

(4) The area of reinforcement is so small that the stress may be assumed constant over it.

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$$\text{or } h f_s = m f_c d - m f_c h$$

$$h = \frac{m f_c d}{f_s + m f_c} \quad \ldots\ldots\ldots\ldots\ldots (3)$$

An expression which enables us to find the distance h if f_s and f_c are known.

Let As be the area of the reinforcement, the total pull P exerted by it is f_sAs. Since the concrete carries no tension, the stress diagram is triangular and the thrust T exerted by the concrete is given by:

$$T = \frac{f_c}{2}.b.h. \quad \ldots\ldots\ldots\ldots\ldots (4)$$

Since the conditions of simple bending are assumed to hold, then: $\quad P = T$

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and substituting the value of $\frac{f_c}{f_s}$ as found from (2), we have:

$$As = \frac{h}{m(d-h)2} \times bh.$$

$$\text{or } bh^2 + 2\,m.As.h. - 2mdAs = 0 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4a)$$

a guadratic equation which fixes the value of h, and depends only on known quantities.

The thrust T acts at a distance $\frac{h}{3}$ from the top of the beam, and hence the resisting moment of the beam is given by

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These reinforced beams are often made of T section, and it is very common practice to design the beam so that the neutral axis coincides with the bottom line of the flange of the section. If the neutral axis is lower than this line, we use a similar method of attack to that just used.

Example 2. A concrete beam 15 ft. long x 12 in. broad x 15 in. deep, is reinforced by six $\frac{3}{4}$ in. round steel bars, having their centers 2 in. from the bottom of the beam. Assuming that all the compression is carried by the concrete and all the tension by the steel, determine the uniform loading that may be applied without the stress in the concrete exceeding 600 lb./in.^{2} , and ascertain the resulting stress in the steel. Take the steel-concrete modular ratio as 15 and the concrete stress strain curve as straight.

$$\text{Area of reinforcement} = \frac{\pi}{4} \cdot x \frac{3}{4} \times \frac{3}{4} \times 6 \ = \ 2.6488 \text{ in.}^2$$

$$bh^2 + 2mAs.h - 2mdAs \quad = 0$$

$$12h^2 + 79.47\ h - 1033.11 \ = 0$$

$$h = \underline{-79.47 \pm \sqrt{(79.47)^2 + (48 \times 1033.11)}}$$

$$= 6.55 \text{ in.}$$

$$M = \frac{f_c}{2}\ b.h\ (d - \frac{h}{3})$$

$$= \frac{600 \times 12 \times 6.55}{2}\ (13 - \frac{6.55}{3})$$

$$= 255{,}000 \text{ lb.in.}$$

but $M = \frac{WL}{8}$, where W = total load uniformly distributed

$$= \frac{W \times 15 \times 12}{8}$$

$$\therefore W = \frac{255{,}000 \times 8}{15 \times 12} \text{ lb.} = 5.05 \text{ tons.}$$

$$\therefore f_s . A_s = \frac{f_c}{2} bh$$

$$f_s = \frac{600}{2} \times \frac{12 \times 6.55}{2.649} = 8900 \text{ lb./in.}^2$$

33. Bending Combined with Direct Stress: Numerous cases occur in which the applied load causes not only stress due to bending, but also exerts a pull or thrust on the section. The resultant stress at any point in the section may be found by calculating the stress due to bending, and superimposing on it the stress due to the direct force; due regard being given to the sign of each stress.

If M = bending moment

A = area of the section

P = pull or thrust on the section

Z = the section modulus.

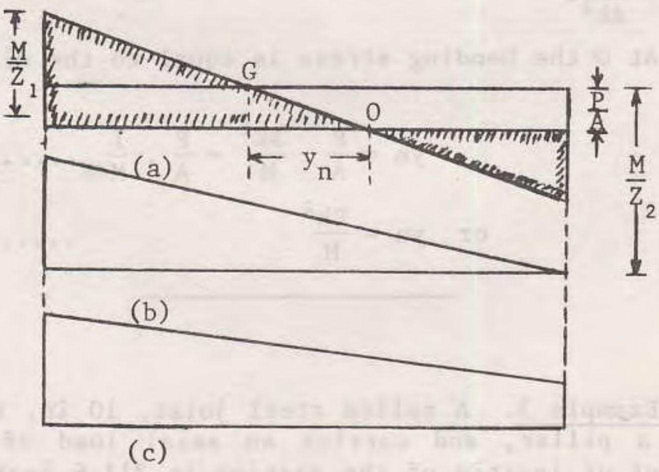


Fig. 28

The resultant stress at any point in the section is given by:

$$f = \frac{P}{A} \pm \frac{M}{Z} \quad \dots\dots\dots\dots\dots\dots\dots\dots (1)$$

the plus sign being used when the bending stress is similar to $\frac{P}{A}$ and the negative sign when it differs from $\frac{P}{A}$. Also, if the section is not symmetrical about a line through the centroid and parallel to the neutral axis, then Z will have the two values Z_1 and Z_2 (paragraph 30) A diagram of combined stress is shown by (a), Fig. 28. The centroid of the section is at G, and, if the bending stress acted alone, the neutral axis would pass through G, but, owing to the presence of the direct stress, the point of no stress, and hence the neutral axis, passes through O.

The diagram has been drawn for a case where P is tensile and it may have three possible forms depending on the magnitude of M, Z, P and A.

When $\frac{M}{Z_2}$ is greater than $\frac{P}{A}$ the resultant diagram is given by (a)

When $\frac{M}{Z_2}$ is equal to $\frac{P}{A}$ the resultant diagram is given by (b)

When $\frac{M}{Z_2}$ is less than $\frac{P}{A}$ the resultant diagram is given by (c)

The neutral axis, distant Yn from G, is found as follows:

The stress at O due to bending = $\frac{Myn}{I}$, where I is the moment of inertia of the section about an axis through G, or if k is the radius of gyration of the section about this axis then the bending stress at O = $\frac{Myn}{Ak^2}$.

At O the bending stress is equal to the direct stress, and $\frac{Myn}{Ak^2} = \frac{P}{A}$

$$yn = \frac{P}{A} \cdot \frac{Ak^2}{M} = \frac{P}{A} \cdot \frac{I}{M} \quad \dots\dots\dots\dots (2)$$

$$\text{or} \quad yn = \frac{Pk^2}{M} \quad \dots\dots\dots\dots (3)$$

Example 3. A rolled steel joist, 10 in. x 6 in. section, is used as a pillar, and carries an axial load of 25 tons. The maximum moment of inertia of the section is 211.6 inch units, and the area is 12.36 in.^{2} A bracket is bolted to a flange of the pillar and supports a vertical load of 6 tons, which acts in the plane of the major axis of the section, and at a distance of 3 in. from the face of the flange. Calculate the maximum and minimum intensity of stress in a section of the pillar.

The bending moment due to eccentric loading is given by

$$M = 6 \times 8 = 48 \text{ ton-inches.}$$

The section modulus $Z = \frac{I}{y} = \frac{211.6}{5} = 42.3$

Then assuming compressive stress as positive, the resultant stress is

$$f = \frac{P}{A} \pm \frac{M}{Z}$$

$$= \frac{31}{12.36} \pm \frac{48}{42.3}$$

$$= 2.508 \pm 1.134$$

Maximum stress $= 3.642$ tons/in.^{2} in compression

Minimum stress $= 1.374$ tons/in.^{2} in compression.

Example 4. A cylindrical masonry column is 6 ft. in diameter and the maximum wind pressure upon it may be assumed to be equivalent to 20 lb. per sq.ft. of diametrical longitudinal section. If the masonry weighs 140 lb. per cu.ft., to what height can the column be built without causing tension in the cross-section at the base?

Let h ft. be the limiting height of the column. Weight of column $= \frac{\pi}{4} \times 36 \times h \times 140 = 3960\,h$ lb.

Total pressure due to wind $= 6 \times h \times 20 = 120\,h$ lb.

The column may be looked on as a cantilever carrying a direct thrust and a uniformly distributed load, the latter being due to the wind pressure.

Bending moment due to wind pressure $= 120\,h \times \frac{h}{2}$

$$= 60\,h^2 \text{ lb.ft.}$$

Resultant tensile stress $= \frac{3960h}{28.3} - \frac{60h^2 \times 3}{63.7} = 0$

(area at base) $\left(\frac{\pi r^4}{4}\right)$

$$\therefore h = \frac{3960 \times 63.7}{180 \times 28.3}$$

$$= 49.5 \text{ ft.}$$

34. Riveted Joint with Eccentric Load: Fig 29. represents a riveted joint on which the load P has its line of action at a horizontal distance x from the centroid of the joint. By applying two equal and opposite forces passing through the centroid G, equal in magnitude to P and parallel to its line of action, it will be observed that the joint is subjected to a couple of magnitude P.x and a vertical load of magnitude P. The couple causes shearing forces whose directions are shown by the arrows in the diagram, and the vertical load will cause, in each rivet, a vertical shear of magnitude $\frac{P}{n}$, where n is the number of rivets in the joint. The total load on a rivet is the vector sum of the loads acting on it.

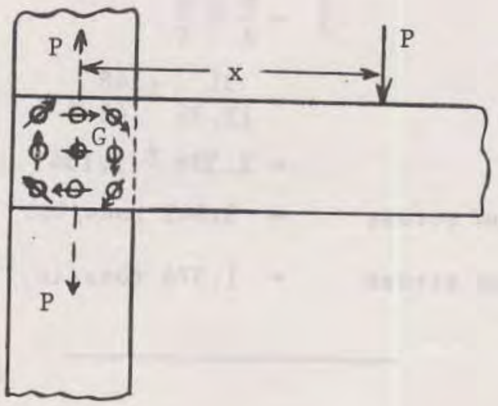


Fig. 29

If it is assumed that the load on a rivet, due to the couple, is proportional to the relative displacement between the plates, then since this displacement is proportional to the distance of the rivet from G, it follows that the load on a rivet, and hence the stress, is proportional to the distance of the rivet from G.

Let Fs be the shearing force on a rivet at distance r from G, and let f_s be the stress due to Fs, then:-

$$f_s = kr \quad \text{.........................} \quad (1)$$

and since Fs = Af_s where A is the cross-sectional area of a rivet,

$$\begin{aligned} \text{then} \quad P.x. &= \Sigma Fs.r. \\ &= \Sigma f_s A.r \\ &= \Sigma k.A.r^2 \\ \therefore k &= \frac{P.x.}{A\Sigma r^2} \end{aligned}$$

Hence : $\quad f_s = \dfrac{P.x.r}{A\Sigma r^2}$ .... (2)

and $\quad Fs = \dfrac{P.x.r}{\Sigma r^2}$ .... (3)

Example 5. The bracket shown in Fig. 30 is attached to a plate by means of six $\frac{3}{4}$ in. diameter rivets, and carries a load of 12 tons in the direction shown. Estimate the stress on each rivet. If safe working shear stress of each rivet is 8 tons/in.^{2}, which rivet is liable to breakage by shear?

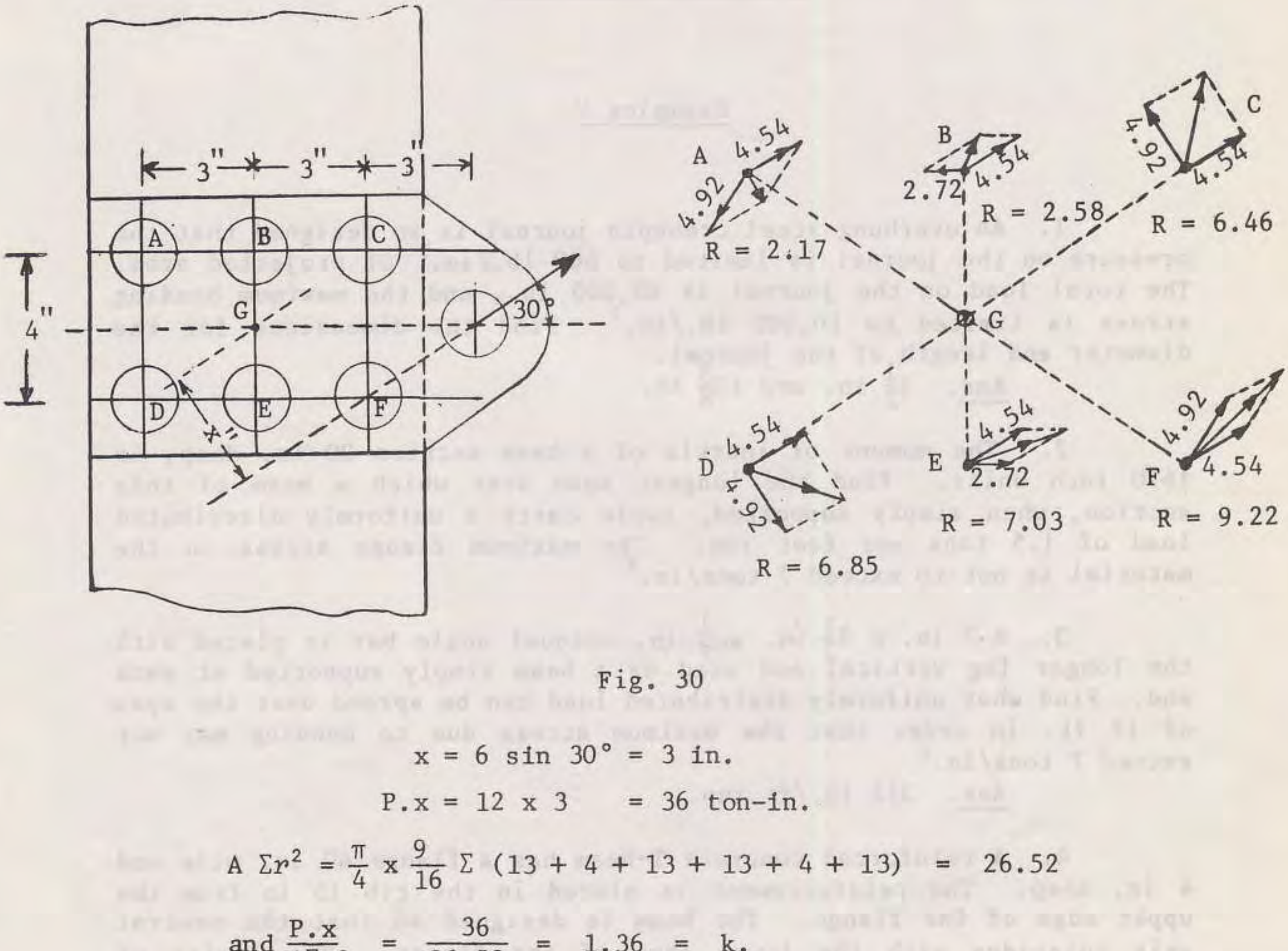


Fig. 30

$$x = 6 \sin 30° = 3 \text{ in.}$$

$$P.x = 12 \times 3 \quad = 36 \text{ ton-in.}$$

$$A\,\Sigma r^2 = \frac{\pi}{4} \times \frac{9}{16}\,\Sigma\,(13 + 4 + 13 + 13 + 4 + 13) \quad = \quad 26.52$$

$$\text{and } \frac{P.x}{A\Sigma r^2} \quad = \quad \frac{36}{26.52} \quad = \quad 1.36 \quad = \quad k.$$

$\therefore$ for A, C, F and D, $f_s = 1.36 \times 3.61 = 4.92$ tons/in.^{2}

and for B and E: $\quad f_s = 1.36 \times 2 \quad = 2.72$ tons/in.^{2}

The shear stress on each rivet due to P acting through G is given by $\dfrac{12}{6 \times \frac{\pi}{4} \times \frac{9}{16}} = 4.54$ tons/in.^{2}

The resultant stress on each rivet is found, as shown in Fig. 30, in both magnitude and direction. These are as follows:-

Rivet	A	B	C	D	E	F
Resultant Shear Stress tons/in^2.	2.17	2.58	6.46	6.85	7.03	9.22

Examples V

1. An overhung steel crankpin journal is so designed that the pressure on the journal is limited to 600 lb./in.^{2} of projected area. The total load on the journal is 60,000 lb., and the maximum bending stress is limited to 10,000 lb./in.^{2} Find the dimensions for the diameter and length of the journal.

Ans. $7\frac{1}{2}$ in. and $13\frac{3}{8}$ in.

2. The moment of inertia of a beam section 20 in. deep, is 1670 inch units. Find the longest span over which a beam of this section, when simply supported, could carry a uniformly distributed load of 1.5 tons per foot run. The maximum flange stress in the material is not to exceed 7 tons/in.^{2}

3. A 7 in. x $3\frac{1}{2}$ in. x $\frac{1}{2}$ in. unequal angle bar is placed with the longer leg vertical and used as a beam simply supported at each end. Find what uniformly distributed load can be spread over the span of 12 ft. in order that the maximum stress due to bending may not exceed 7 tons/in.^{2}

Ans. 312 lb./ft.run.

4. A reinforced concrete T-beam has a flange 60 in. wide and 4 in. deep. The reinforcement is placed in the rib 15 in from the upper edge of the flange. The beam is designed so that the neutral axis coincides with the lower edge of the flange. The limits of stress are for steel 16,000 lb./in.^{2}, and for concrete 600 lb./in.^{2} The ratio $\frac{E_s}{E_c}$ is 15. Calculate:

(a) the area of the reinforcement,

(b) the moment of resistance of the beam,

(c) the actual maximum stress in the steel and in the concrete.

Ans. 2.9 in^2, 636000 lb.in., f_s = 16000 $lb./in.^2$, f_c = 388 $lb./in.^2$

5. A short circular column, external diameter 10 in., thickness of metal 1 in., carries a load of 50 tons, the line of action of which coincides with the axis of the column. It also carries a second load of 15 tons, whose line of action is 6 in. from the center. Find the maximum and minimum stresses in the column, and show by a diagram how the stresses vary across the section.
Ans. 3.84 $tons/in.^2$ comp., 0.744 $tons/in.^2$ comp.

6. A steel plate chimney, 4 ft. in diameter and 80 ft. high, has a cylindrical bottom ring $\frac{3}{8}$ in. thick. The weight of the structure may be considered as that of a cylinder of this thickness, and the internal stiffening devices increase the second moment of area of the section 60 per cent. Find the greatest stress in the plates when the chimney is subjected to a uniform wind pressure of 40 $lb./ft.^2$ of projected area. Take steel as weighing 480 $lb./ft.^3$
Ans. 6,100 $lb./in.^2$

7. A short stanchion is made of I-section, 6 in. wide and 8 in. deep. Find the greatest and least intensity of compressive stress on a section if the load of 50 tons has its center in the center line of the section which bisects both flanges, but 1.6 in. from the center line which is parallel to the outer edges of the flanges. The moment of inertia of the cross-section about a central axis parallel to the outer edges of the flanges is 110.5 $in.^4$ and the area of the cross-section is 10.29 $in.^2$
Ans. 7.755 and 1.965 $tons/in.^2$ compression.

8. A horizontal chain passes over a pulley carried by a bracket attached to a stanchion by six $\frac{3}{4}$ in. rivets as shown in Fig. 30A. If the rivets are in single shear and the shear stress is limited to $5\frac{1}{2}$ $tons/in.^2$, find the load W which can be carried from the free end of the chain.
Ans. 1.6 tons.

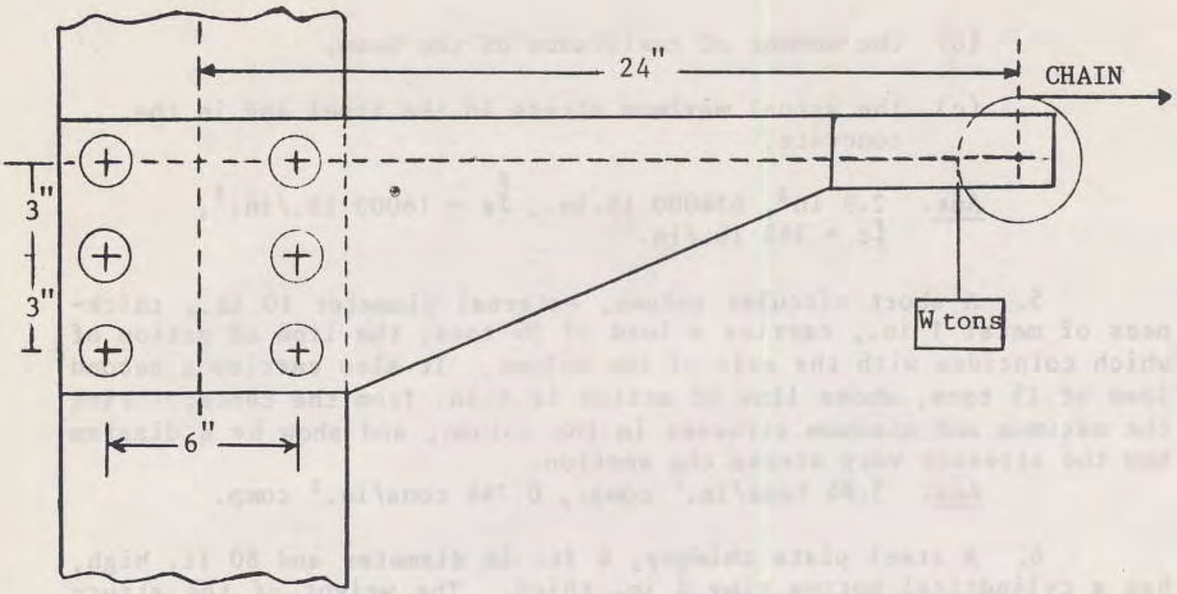


Fig. 30 A

VI. THE TORSION OF SHAFTS

35. The Relation between Stress, Strain, and Angle of Twist: A cylindrical rod is said to be subject to pure torsion when the torsion when the torsion is caused by a couple, applied so that the axis of the couple coincides with the axis of the rod. The state of stress, at any point in the cross-section of the rod, is one of pure shear, and the strain is such that one cross-section of the rod moves relative to another.

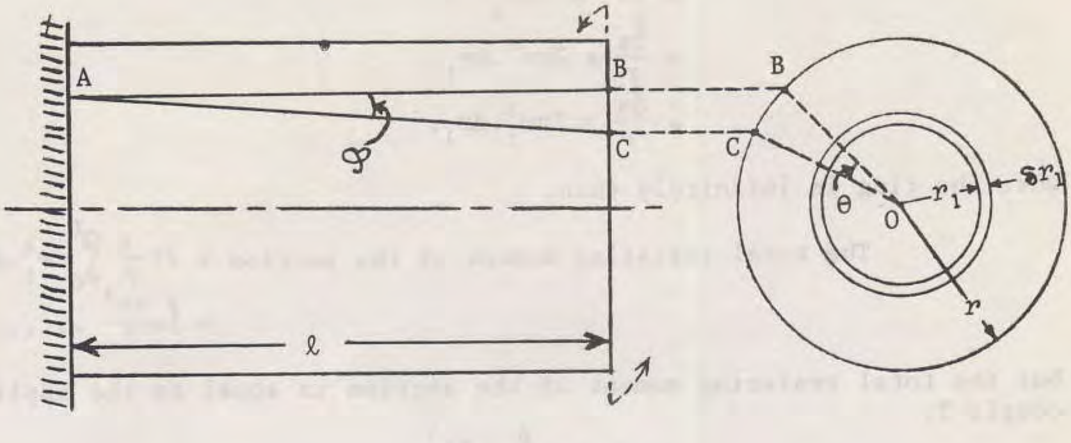


Fig. 31

Considering the cylindrical rod of length ℓ and radius r, shown in Fig. 31, a couple of magnitude T is applied to one end, and the other end of the rod is held, or constrained, by a balancing couple of equal magnitude. A line AB, on the surface of the rod, which is parallel to the axis before strain, takes up the form of a long helix AC after strain; the angle ϕ being the shear strain of the material at the surface, and since this angle is small:

$$BC = \ell.\phi.$$

$$\text{or } \phi = \frac{BC}{\ell} = \frac{r.\theta}{\ell} \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (1)$$

but $\phi = \frac{f_s}{C}$, where f_s is the shear stress in the material at the surface of the rod, and $\hat{BOC}$ is the angular movement of a radius OB due to strain in the length ℓ.

$$f_s = \phi C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (2)$$

$$\text{or } f_s = \frac{r\theta}{\ell}.C \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3)$$

which may also be put in the form: $f_s = kr$, where $kr = \frac{\theta}{\ell}.C$. hence, if f_s^1 is the shear stress at a radius r_1, we have

$$\frac{f_s^1}{r_1} = \frac{f_s}{r}$$

36. <u>Relation between Twisting Couple and Shear Stress:</u> Considering a thin ring in the cross-section at A, of radius r_1, and thickness δr_1, on which the shear stress is f_s^1, the total force on the ring = $f_s^1 \times 2\pi.r_1.\delta r_1$ and the moment of this, about the axis of the rod,

$$= \mathfrak{f}s^1 \times 2\pi r_1^2 . r_1 .$$
$$= \frac{\mathfrak{f}s}{r} \times 2\pi r_1^3 . dr_1 .$$
$$= \frac{\mathfrak{f}s}{r} \times 2\pi r_1^3 . dr_1 .$$

when the ring is infinitely thin.

The total resisting moment of the section $= 2\pi \frac{s}{r} \int_0^r r_1^3 . dr_1$

$$= \mathfrak{f}s \frac{\pi r^3}{2} \quad .... (5)$$

but the total resisting moment of the section is equal to the applied couple T.

$$\therefore T = \mathfrak{f}s . \frac{\pi r^3}{2} \quad .... (6)$$

Multiplying the right-hand side above and below by r.

$$T = \frac{\mathfrak{f}s}{r} . \frac{\pi r^4}{2}$$

and $\frac{\pi r^4}{2}$ is the the polar moment of inertia of the section Ip.

$$\therefore T = \frac{\mathfrak{f}s}{r} . Ip. \quad .... (7)$$

and from (3) and (7) we get:

$$\frac{T}{I_p} = \frac{\mathfrak{f}s}{r} = \frac{C\theta}{\ell} \quad .... (8)$$

This is usually called the torsion equation. The usual units of measurement for the various quantities are:-

T = Twisting moment in lb.in.

I_p = Polar moment of inertia of cross section in inch. units.

$\mathfrak{f}s$ = Shear stress in lb./in.^{2} at a radius r in.

C = Rigidity modulus in lb./in.^{2}

θ = Angle of twist in radians in a length of ℓ in.

Example 1. Calculate the size of a shaft, which will transmit 50 h.p. at 110 rpm. The shearing stress to be limited to 3 tons/in.^{2}, and the twist of the shaft is not to exceed 1 degree in $7\frac{1}{2}$ ft. of length of shaft. The modulus of rigidity, C, is 5000 tons/in.^{2} Assume the twisting moment to be uniform. (1 ton = 2240 lb.)

$$\therefore T = 63{,}000\,\frac{H.P.}{N}$$

$$= \frac{63{,}000 \times 50}{110}\ \text{lb.in.}$$

$$\text{Resisting moment} = f_s.\frac{\pi r^3}{2} = \text{torque.}$$

$$\therefore r^3 = \frac{63{,}000 \times 50}{110} \times \frac{2}{3 \times 2240 \times \pi}.$$

$$\text{and } r = 1.394 \text{ in. for safe stress.}$$

$$\therefore \frac{T}{I_p} = C\frac{\theta}{\ell}$$

$$\therefore r^4 = \frac{2T\ell}{\pi C\theta} = \frac{2 \times 63{,}000 \times 50 \times 90 \times 57.3}{110 \times \pi \times 5000 \times 2240 \times 1}$$

$$\therefore r = 1.7 \text{ for safe twist.}$$

$$\therefore \text{Dia. of shaft} = 3.4 \text{ in.}$$

$$\doteq 3\frac{7}{16} \text{ in.}$$

---

37. Hollow Circular Shafts: In equation (3), paragraph 35, we proved that $f_s = kr$; thus, in a solid shaft of large diameter, a great deal of the material towards the axis will carry very little stress, and consequently will offer very little resistance to the applied couple. In a hollow shaft the average intensity of stress will be greater than that for a solid shaft of the same diameter, and hence a greater resistance to the applied couple will be exerted.

Let R = outer radius of a hollow shaft

r = inner radius of a hollow shaft

f_s = shear stress at radius R

f^1_s = shear stress at radius r.

Then the value of the resisting moment, which is equal to the applied couple T, is given by:

$$T = \frac{f_s.\pi R^3}{2} - f_s^1.\frac{\pi r^3}{2}, \text{ and } f_s^1 = f_s.\frac{r}{R}.$$

$$T = \frac{f_s\pi}{2}(R^3 - \frac{r^4}{R}) = \frac{f_s\pi}{2}(\frac{R^4 - r^4}{R}) \ldots\ldots. (1)$$

the value of Ip in the twisting equation being given by:

$$Ip = \frac{\pi}{2}(R^4 - r^4) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

Comparing the strength of a hollow shaft with that of a solid shaft of the same material, weight, and length.

Let R and r be the outer and inner radii of the hollow shaft, and f_s the maximum shear stress; also let R_1 be the radius of the solid shaft.

$$T \text{ hollow} = \frac{f_s\pi}{2}(\frac{R^4 - r^4}{R})$$

$$T \text{ solid} = \frac{f_s.\pi}{2}.R_1^3$$

If the maximum stress is the same for each shaft.

$$\frac{T \text{ hollow}}{T \text{ solid}} = \frac{R^4 - r^4}{RR_1^3}$$

$$= \frac{(R^2 + r^2)(R^2 - r^2)}{RR_1^3}$$

$$= \frac{R^2 + r^2}{RR_1}, \text{ since } R_1{}^2 = R^2 - r^2 \text{ (same cross sectional area)}$$

$$= \frac{R}{R_1}(1 + \frac{1}{n^2}), \text{ where } r = \frac{R}{n}.$$

Substitute $R_1 = \sqrt{R^2 - r^2}$ and $r = \frac{R}{n}$,

$$\frac{Th}{Ts} = \frac{5}{2\sqrt{3}} = 1.44$$

38. <u>Shaft Diameter for a Given Horse-Power:</u>

If T = the mean twisting moment in lb.in.

N = the spm. of the shaft.

h.p. = the horse power to be transmitted.

then: $h.p. = \frac{T \times 2\pi N}{12 \times 33{,}000}$ .... (1)

$T = \frac{12 \times 33{,}000 \text{ h.p.}}{2\pi N} = 63000 \frac{\text{h.p.}}{N}$ (or $h.p. = \frac{Tw}{550}$, where T is in ft./lb. and w in radians/sec.) .... (2)

If the twisting moment is variable, then the maximum twisting moment Tmax. will be equal to a constant multiplied by the value in (2), and from (6), paragraph 36.

$$T = f_s \frac{\pi r^3}{2} = 63{,}000 \frac{h.p.}{N}$$

$$d = 2r = \sqrt[3]{\frac{2 \times 63000}{\pi f_s}} \cdot \sqrt[3]{\frac{h.p.}{N}}$$

$$d = k \sqrt[3]{\frac{h.p.}{N}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

where d is the diameter of the shaft and f_s the maximum shear stress.

From equation (3), we see that, for a given value of horse power, the diameter of the shaft will be smaller for a high speed than that required for a low speed. As an example, we may take the case of a De Laval steam turbine, which develops 15 h.p. on a $\frac{3}{8}$ in. diameter shaft, at 33,000 rpm. The diameter of shaft for the same horse power, at 100 rpm., is given by:-

$$\frac{d^3}{(\frac{3}{8})^3} = \frac{33{,}000}{100}$$

$$d = 2.59 \text{ in.}$$

Example 2. The external and internal diameters of a hollow steel shaft are 15 in. and 9 in. respectively. Determine what horse-power it will transmit when the speed is 90 rpm. The maximum intensity of shear stress is not to exceed 8000 lb./in.^{2} Compare the strength of this shaft with a solid one of the same material and weight.

$$T = \frac{f_s \pi}{2} \left(\frac{R^4 - r^4}{R}\right) = \frac{8000\pi}{2} \left(\frac{7.5^4 - 4.5^4}{7.5}\right)$$

$$= \frac{8000\pi}{2} \left(\frac{2755}{7.5}\right) \quad \text{lb.in.}$$

$$h.p. = \frac{T.N}{63{,}000} = \frac{8000\pi}{2} \left(\frac{2755}{7.5}\right) \times \frac{90}{63000}$$

$$= 6595$$

$$\frac{\text{Strength of hollow shaft}}{\text{Strength of solid shaft}} = \frac{n^2 + 1}{n\sqrt{n^2 - 1}}, \text{ where } n = \frac{R}{r}$$

$$= \frac{15^2 + 9^2}{9^2 \times \frac{15}{9} \sqrt{\frac{15^2 - 9^2}{9^2}}}$$

$$= 1.7$$

---

39. Horse-power of Turbine-driven Ships: In order to determine the power developed by the turbine, we make use of instruments called "Torsion Meters", mounted on the shaft. The torsion meter is able to measure the angle of twist θ in a given length ℓ of the shaft, while rotation of the shaft is taking place.

From (1), paragraph 38:

$$\text{h.p.} = \frac{T \times 2\pi N}{12 \times 33{,}000}$$

$$= \frac{2\pi}{12 \times 33000} \cdot \frac{C\theta}{\ell} \cdot Ip.N.$$

and for a given shaft C, Ip, and ℓ are constant.

$$\text{h.p.} = x.\theta.N. \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

Thus, if x is known and θ and N obtained, we can calculate the power being transmitted along the propeller-shaft.

The value of C for the shaft can be found by mounting the shaft in a lathe, locking one end, and applying various torques to the other end, which is free. If the torsion meter be mounted on the shaft, the angle of twist for each torque can be read off. Torque and angle of twist are then plotted, and taking a reading from the graph, and substituting in $\frac{T}{Ip} = C\frac{\theta}{\ell}$, we get the value of C and hence x.

Example 3. A propeller shaft, diameter $10\frac{1}{2}$ in. is to be subjected to a torque for the calibration of a torsion meter. Two long pointers are fixed to the shaft at a distance of 8 ft. apart. Arrangements are made to apply a torque which is to be 30 per cent in excess of the twisting moment due to 12,500 h.p. at 500 rpm. The movements of the ends of the pointers are observed by means of verniers reading to 0.01 in. What length of pointer is necessary if, at maximum test torque, the margin of error due to vernier reading is limited to 2 per cent. Assume a provisional value for the torsion modulus of the shaft to be 12 millions in lb.in. units.

$$\text{Applied torque} = \frac{63000 \times 12500}{500} \times \frac{130}{100} = 2{,}047{,}500 \text{ lb.in.}$$

$$\text{Angle of twist due to this torque} = \frac{T\ell}{IpC} = \frac{2{,}047{,}500 \times 96 \times 2}{\pi \times (5.25)^4 \times 12 \times 10^6}$$

$$= 0.01372 \text{ radians.}$$

Let ℓ in. = length of pointers. Correct movement of verniers = $0.01372 \times \ell$".

$$\therefore \frac{0.01}{0.01372\,\ell} = \frac{2}{100}$$

$$\ell = 36.44 \text{ in.}$$

40. Torsional Resilience of Circular Shafts: If a solid shaft be subjected to a torque which increases gradually from zero to a value T, and θ is the resultant angle of twist, then the energy stored in the shaft, or the "Resilience" of the shaft:

$$\begin{aligned}\text{Resilience} &= \frac{1}{2}T\theta \\ &= \frac{1}{2}\,\frac{f_s.I_p}{r} \times \frac{f_s}{r}.\frac{\ell}{C} \\ &= \frac{\frac{1}{2} f_s^2}{C}\,\frac{I_p}{r^2}.\ell \\ &= \frac{1}{2}\,\frac{f_s^2}{C}.\frac{\pi r^2 \ell}{2} \\ &= \frac{1}{4}\,\frac{f_s^2}{C}.(\text{volume of shaft}) \;\ldots\ldots\ldots\ldots\; (1)\end{aligned}$$

In the case of a hollow shaft of radii R and r, we have:

$$\begin{aligned}\text{Resilience} &= \frac{1}{2}T\theta \\ &= \frac{1}{2}.\frac{T^2}{I_p}.\frac{\ell}{C} \\ &= \frac{\frac{1}{2}\left\{\frac{\pi f_s}{2}\left(\frac{R^4 - r^4}{R}\right)\right\}^2}{\frac{\pi(R^4 - r^4)}{2}.C} \\ &= \frac{1}{4}.\frac{f_s^2}{C}.\frac{R^2 + r^2}{R^2}.\pi(R^2 - r^2)\ell \\ &= \frac{1}{4}\,\frac{f_s^2}{C}.\frac{R^2 + r^2}{R^2}.(\text{volume of shaft}) \;\ldots\ldots\; (2)\end{aligned}$$

Examples VI

1. Determine the diameter of a solid steel shaft which will transmit 450 h.p. at 300 rpm. The twist must not exceed 1 degree in 7 ft. length, nor the maximum shear stress 5500 lb./in.^{2} Modulus of rigidity 11.5×10^6 lb./in.^{2} Ans. 4.44 in. or $4\frac{1}{2}$ in. approx.

2. A steel shaft is required to transmit 80 h.p. at 60 rpm., and the maximum twisting moment is 30% greater than the mean. Find the diameter for a shear stress of 8000 lb./in.^{2}, also the twist of the shaft on a length of 10 ft. C = 5,200 tons/in.^{2}

Ans. d = 4.1 in., θ = 2.35°.

3. A vessel having a single propeller shaft 12 in. in diameter, and running at 160 rpm. is re-engined with turbines driving two equal propeller shafts at 750 rpm. and developing 60% more horse power. If the working stresses of the new shafts are 10% greater than that of the old shaft, find their diameters.

Ans. 6.45 in., or $6\frac{1}{2}$ in. approx.

4. In calculating the size of shafts to transmit power, the following formula is sometimes used.

$$d = \sqrt[3]{\frac{60 \times h.p.}{rpm}}$$

What shear stress is allowed in the shaft by this rule? A shaft is to be made up of two lengths connected by a flanged coupling taking 8 bolts on a pitch circle diameter which may be assumed to be $2\frac{1}{2}$ in. greater than the shaft diameter. If 400 h.p. is to be transmitted at 80 rpm., determine suitable diameters of shafts and bolts. The shear stress in the bolts is to be 20% in excess of that in the shaft, which is to be fixed by the above rule.

Ans. $6\frac{3}{4}$ in. and 1 in. approx.

5. Calculate the dimensions of a hollow steel shaft to transmit 2100 h.p. at a speed of 120 rpm.; the maximum twisting moment being 1.25 times the mean. The internal diameter of the shaft is 60% of the outside diameter, and the greatest intensity of shear stress in the steel is limited to 3 tons/in.^{2}

Ans. D = $10\frac{5}{8}$ in., d = $6\frac{3}{8}$ in.

6. In determining the horse-power transmitted by a turbine driven propeller shaft by means of a torsion meter, it is found that the angle of twist, measured on a length of 20 ft., is 1.2 degrees. The external & internal diameters of the shaft are 10 in. and 7 in. respectively, and the speed is 250 rpm. Find the horse-power being transmitted by the shaft, and the maximum intensity of shear stress induced in the material. C = 5200 tons/in.^{2}

Ans. 3039 h.p., 5150 lb./in.^{2}

VII. JAPANESE INDUSTRIAL STANDARD

UDC 620. 115. 8:620. 172:669

JIS

Tension Test Pieces for Metallic Materials Z 2201-1968

(Reaffirmed 1971)

1. Scope

This standard specifies standard test pieces, hereinafter referred to as the "test piece", to be used for tension test of metallic materials. Which type of these test pieces is to be used shall comply with the respective standards for materials.

2. Definition of Terms

The meaning of chief terms used in this standard shall be as follows:

(1) Parallel Portion is the part with constant section in the middle of the test piece.

(2) Gauge Length is the distance between the two points marked on the parallel portion, being used as the basic length in the determination of elongation.

(3) Gripped Portion is the part to be gripped by the gripping device of a testing machine.

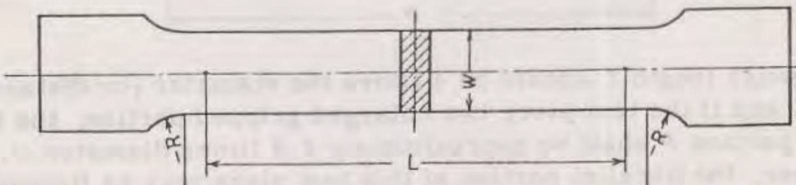
(4) Radius of Shoulder is the radius of curvature provided between the parallel portion and the gripped portion so that the stress in the parallel portion might be uniformly distributed.

3. Classification of Test Piece

The test pieces shall be classified into No. 1 to No. 14 test piece in accordance with the shape and size, and the standard dimensions of these test pieces shall comply with the following:

(1) No. 1 Test Piece This test piece shall be principally used for tension test of steel plates, steel flats and steel sections.

Fig. 1



Gauge length L=200 mm

Length of parallel portion P=approx. 220 mm

Radius of shoulder R=25 mm or more

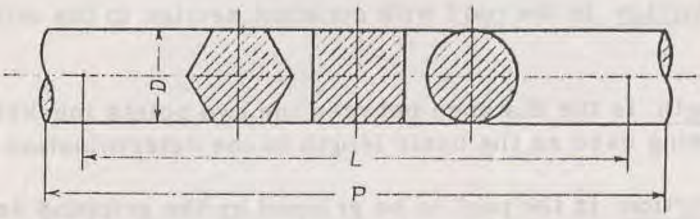
The thickness shall be as the original size

Unit: mm

Division of test piece	Width W
1 A	40 (or 38 may be used)
1 B	25

(2) No. 2 Test Piece This test piece shall be used for tension test of steel bars not more than 25 mm in nominal diameter (or distance across flats) of the material.

Fig. 2

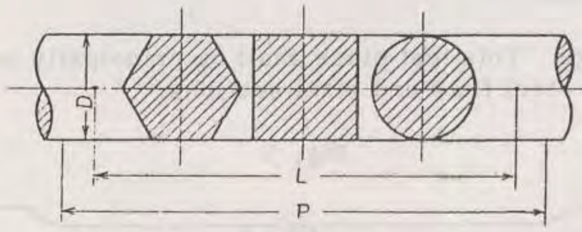


The gauge length L should be 8 times the diameter (or distance across flats) D, and if the test piece has enlarged gripped portion, the length of parallel portion P shall be approximately 9 times diameter D.

Further, the parallel portion of this test piece may be finished by machining.

(3) No. 3 Test Piece This test piece shall be used for tention test of steel bars over 25 mm in nominal diameter (or distance across flats) of the material.

Fig. 3

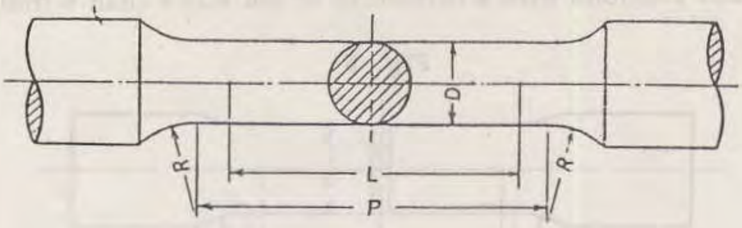


The gauge length L should be 4 times the diameter (or distance across flats) D, and if the test piece has enlarged gripped portion, the length of parallel portion P shall be approximately 4.5 times diameter D.

Further, the parallel portion of this test piece may be finished to a diameter of 25 mm and over by machining.

(4) No. 4 Test Piece This test piece shall be principally used for tension test of steel castings, steel forgings, rolled steel, malleable iron castings and nodular graphite iron castings. And it shall also be used for tension test of bars and castings of non-ferrous metal (or alloy thereof).

Fig. 4



Gauge length	L=50 mm
Length of parallel portion	P=approx. 60 mm
Diameter	D=14 mm
Radius of shoulder	R=15 mm or more

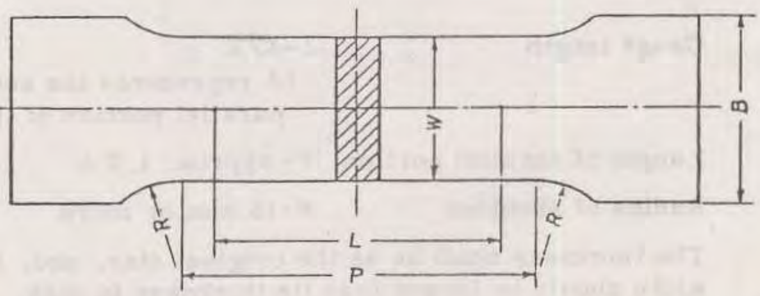
This test piece is required that the section of the parallel portion is finished to a circle, but it should not be finished, as a rule, for malleable castings. Where the test piece of dimensions specified above can not be obtained for the reason of the material, the diameter of the parallel portion and the gauge length shall be determined by the following formula. In this case, the gauge length may be rounded up to an integer.

$$L=4\sqrt{A} \quad =3.54\ D$$

where, A represents the sectional area of parallel portion of the test piece

(5) No. 5 Test Piece This test piece shall be principally used for tension test of pipes and tubes, steel sheets and non-ferrous metal (or alloy thereof) sheets and sections.

Fig. 5



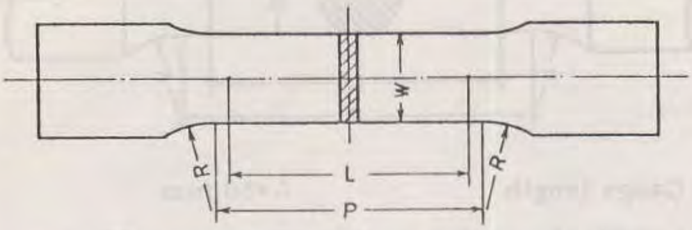
Gauge length	L=50 mm
Length of parallel portion	P=approx. 60 mm
Width	W=25 mm
Radius of shoulder	R=15 mm or more

The thickness shall be as the original size. For thin steel sheets only, the following shall be applied.

Radius of shoulder	R=20 ~ 30 mm
Width of gripped portion	B=30 mm or more

(6) No. 6 Test Piece This test piece shall be principally used for tension test of sheets and sections with a thickness of not more than 6 mm.

Fig. 6

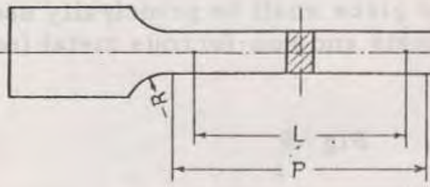


Gauge length	$L=8\sqrt{A}$ (*A* represents the sectional area of parallel portion of the test piece).
Length of parallel portion	$P=L+$ approx. 10 mm
Width	$W=15$ mm
Radius of shoulder	$R=15$ mm or more

The thickness shall be as the original size.

(7) No. 7 Test Piece This test piece shall be principally used for tension test of steel flats, steel plates and square steel of high tensile strength.

Fig. 7

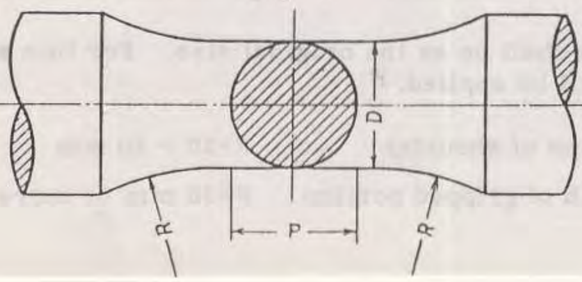


Gauge length	$L=4\sqrt{A}$ (*A* represents the sectional area of parallel portion of the test piece).
Length of parallel portion	$P=$ approx. 1.2 L
Radius of shoulder	$R=15$ mm or more

The thickness shall be as the original size, and, as a rule, the width should be larger than its thickness in size.

(8) No. 8 Test Piece This test piece shall be principally used for tension test of general iron castings. It shall be made out of the sample with the dimensions given in the Table, and the parallel portion shall be finished to the diameter D.

Fig. 8

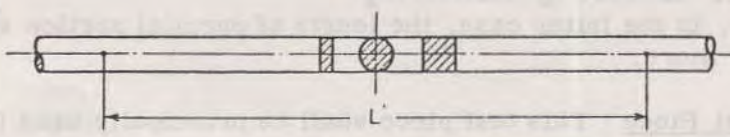


Unit: mm

Division of test piece	Size of casted sample (dia.)	Length of parallel portion P	Dia. D	Radius of shoulder R
8 A	Approx. 13	Approx. 8	8	16 or more
8 B	Approx. 20	Approx. 12.5	12.5	25 or more
8 C	Approx. 30	Approx. 20	20	40 or more
8 D	Approx. 45	Approx. 32	32	64 or more

(9) No. 9 Test Piece This test piece shall be principally used for tension test of steel wires and non-ferrous metal (or alloy thereof) wires.

Fig. 9

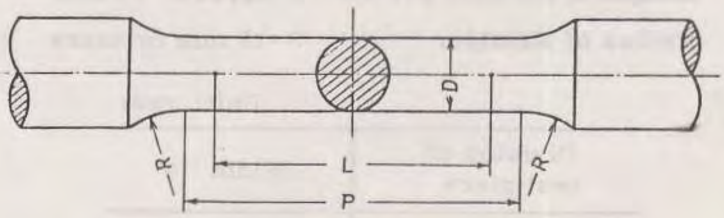


Unit: mm

Division of test piece	Gauge length L
9 A	100
9 B	200

(10) No. 10 Test Piece This test piece shall be principally used for tension test of deposited metal, steel forgings, steel castings and rolled steel.

Fig. 10

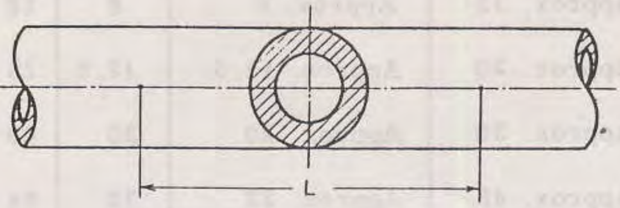


Gauge length	L=50 mm
Length of parallel portion	P=approx. 60 mm
Diameter	D=12.5 mm
Radius of shoulder	R=15 mm or more

In case of deposited metal, the whole parallel portion should be made of deposited metal.

(11) No. 11 Test Piece This test piece shall be used for tension test of pipes and tubes, where the test is carried out on a specimen of tubular form.

Fig. 11



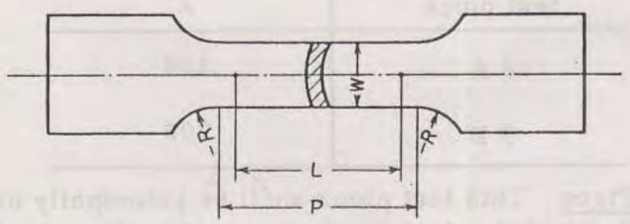
Gauge length L = 50 mm

The cross-section of this test piece shall be left same as cut from the original material, and its both gripped ends should be either inserted with mandrels or flattened by hammering.

Further, in the latter case, the length of parallel portion should be 100 mm or more.

(12) No. 12 Test Piece This test piece shall be principally used for tension test of pipes and tubes, where the test is not carried out on a specimen of tubular form.

Fig. 12



Gauge length L=50 mm

Length of parallel portion P=approx. 60 mm

Radius of shoulder R=15 mm or more

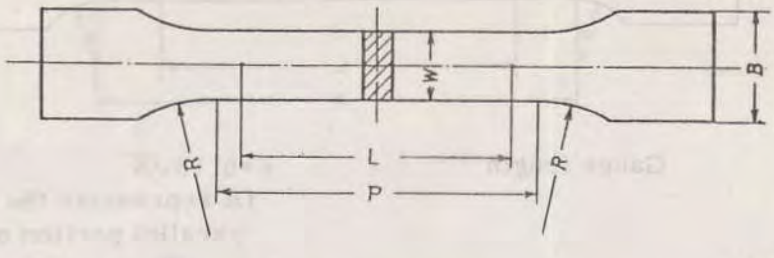
Unit: mm

Division of test piece	Width W
12 A	19
12 B	25
12 C	38

Both gripped ends of the test piece may be flattened by hammering at cold state.

(13) No. 13 Test Piece This test piece shall be principally used for tension test of plates.

Fig. 13



Unit: mm

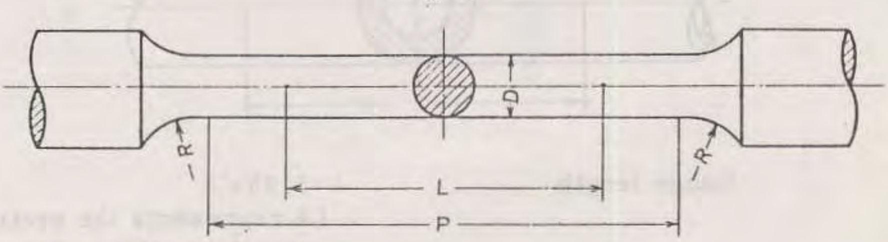
Division of test piece	Width W	Gauge length L	Length of parallel portion P	Radius of shoulder R	Width of gripped portion B
13 A	20	80	Approx. 120	$20 \sim 30$	-
13 B	12.5	50	Approx. 60	$20 \sim 30$	20 or more

The thickness shall be as the original size.

(14) No. 14 Test Piece

(a) No. 14 A Test Piece This test piece shall be principally used for tension test of steel.

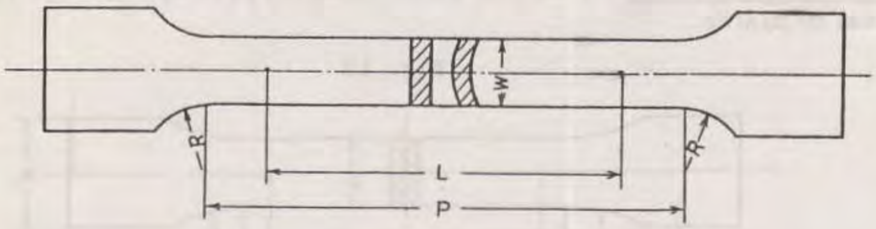
Fig. 14



Gauge length	$L=5\ D$
Length of parallel portion	$P=(5.5 \sim 7)\ D$, preferably $7\ D$
Diameter	D shall comply with the specification of the standards for materials.
Radius of shoulder	$R=15$ mm or more

(b) No. 14 B Test Piece This test piece shall be principally used for tension test of steel, pipes and tubes where the test is not carried out on a specimen of tubular form.

Fig. 15



Gauge length $L = 5.65\sqrt{A}$
(A represents the sectional area of parallel portion of the test piece).

Length of parallel portion $P = L + (\frac{W}{2} \sim 2W)$, preferably $L + 2W$

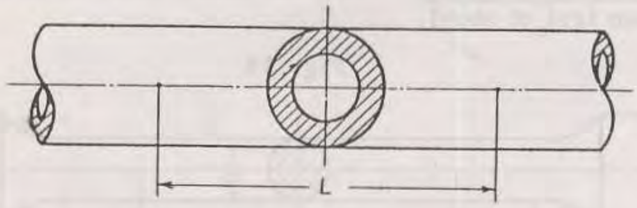
Width W shall comply with the specification of the standards for materials. But, W should be not more than four times the thickness.

Radius of shoulder $R = 15$ mm or more

The thickness shall be as the original size.

(c) No. 14 C Test Piece This test piece shall be principally used for tension test of pipes and tubes where the test is carried out on a specimen of tubular form.

Fig. 16



Gauge length $L = 5.65\sqrt{A}$
(A represents the sectional area of parallel portion of the test piece).

4. Tolerance on Nominal Dimensions of Parallel Portion of Test Piece

For machined parallel portion of the test piece, machining error within tolerance given in the following table shall be allowed on the nominal dimension.

Unit: mm

Range of dimension	Tolerance
Over 4 (excl.) up to 16 (incl.)	± 0.5
Over 16 (excl.) up to 63 (incl.)	± 0.7

5. Variation of Dimension of Parallel Portion of Test Piece

The variation of dimension (difference between the maximum value and the minimum value) of the parallel portion of a test piece finished by machining shall conform to Table 1 and Table 2.

Table 1. In Case of Circular Section

Unit: mm

Diameter of machined parallel portion	Variation
Over 3 (excl.) up to 6 (incl.)	0.03
Over 6 (excl.) up to 16 (incl.)	0.04
Over 16 (excl.)	0.05

Table 2. In case of Rectangular Section

Unit: mm

Thickness and width of machined parallel portion	Variation
Over 3 (excl.) up to 6 (incl.)	0.06
Over 6 (excl.) up to 16 (incl.)	0.08
Over 16 (excl.)	0.10

6. If necessary, the parallel portion of the test piece may be narrowed with a taper toward the middle within the value of variation of dimension prescribed above.

JAPANESE INDUSTRIAL STANDARD — J I S

Method of Tension Test for Metallic Materials

Z 2241-1968 (Reaffirmed 1971)

1. Scope

This standard covers the method of tension test for metallic materials.

2. Definition of Terms

The definitions of principal terms used in this standard are as follows:

(1) Tension Test is a test to determine all or a part of the properties, i.e. yield point, proof stress, tensile strength, elongation and reduction of area by gradually pulling a test piece on a testing machine.

(2) Parallel Portion of Tension Test Piece is the portion having a uniform section in the middle part of the test piece.

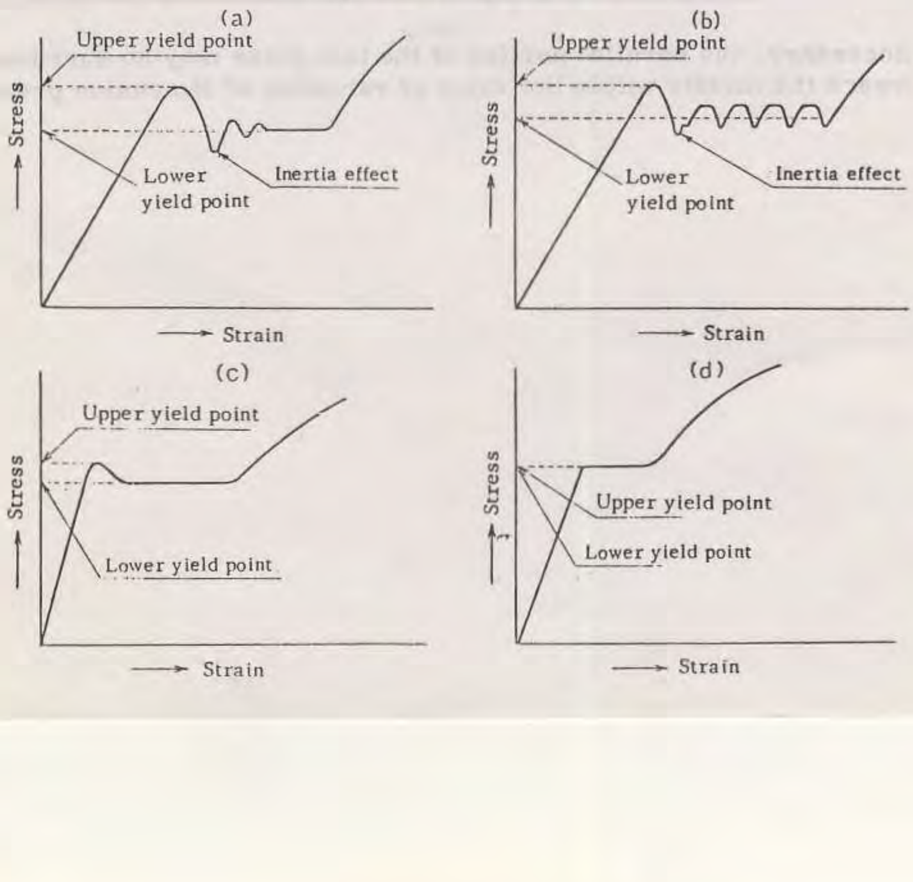
(3) Gauge Length of Test Piece means the distance between two points marked on the parallel portion of test piece over which elongation is to be determined.

(4) Yield point includes an upper yield point and a lower yield point, however, when there is no fear of confusion, the upper yield point may be called the yield point.

(a) Upper Yield Point is the quotient (kg/mm^2) obtained by dividing the maximum load (kg) exhibited before the parallel portion of the test piece begins to yield, by the original cross-sectional area (mm^2) of the parallel portion during the tension test as shown in Fig. 1.

(b) Lower Yield Point is the quotient (kg/mm^2) obtained by dividing the load (kg) indicating nearly constant after the parallel portion of the test piece begins to yield, by the original cross-sectional area (mm^2) of the parallel portion during tension test as shown in Fig. 1.

Fig. 1. Yield Point



(5) Proof Stress is the quotient (kg/mm^2) obtained by dividing the load (kg) at which the specified permanent elongation occurs in the tension test, by the original cross-sectional area (mm^2) of the parallel portion of the test piece. Unless otherwise specified, the value of the permanent elongation or "offset shall be 0.2 %.

(6) Tensile Load is the maximum load (kg) under which the test piece withstands during the tension test.

(7) Tensile Strength is the quotient (kg/mm^2) obtained by dividing the tensile load (kg) by the original cross-sectional area (mm^2) of the parallel portion of test piece.

(8) Total Elongation is the difference of the gauge length and the length between the gauge marks while the load is still applied, expressed as percentage of the gauge length.

(9) Permanent Elongation is the difference between the gauge length and the length taken between the gauge marks after the removal of the load in the tension test, expressed as percentage of the gauge length.

(10) Elongation after Fracture (may be referred to as called simply as "elongation") is the permanent elongation after fracture of the test piece.

(11) Reduction of Area is the difference between the minimum cross-sectional area of the test piece after fracture and the original area in the tension test, expressed as percentage of the original cross-sectional area of the test piece.

3. Test Piece

3.1 The test piece shall be prepared in accordance with JIS Z 2201-Tension Test Pieces for Metallic Materials, unless otherwise specified.

3.2 The sampling and preparation of the test piece shall be carried out in accordance with the product specification of respective materials, and deformation or heating such as to change the quality of the test piece, shall be avoided. This is especially important in the determination of the upper yield point, the lower yield point or the proof stress.
When rough working shall be given to the material by shearing, punching, etc., the part worked shall be removed by cutting, followed by finishing of the parallel portion.

3.3 Straightening of a wire shall preferably be avoided, and if necessary, it shall be done in such a manner as not to affect the quality of the wire , for example, by hand or the like.

3.4 The gauge marks shall be usually marked by a punch or a scriber. However, for the material of test piece being sensitive to a surface flaw or having an extreme hardness, it shall be marked by a scriber on the painted surface thereof.

4. Testing Machine

4.1 The testing machine used for the tension test shall comply with JIS B 7721-Tension Tester.

4.2 The testing machine shall be installed on a rigid foundation and operated so that the line joining the grips should be correctly vertical or horizontal.

4.3 The testing machine shall be thoroughly inspected when it is over-hauled and reassembled, modified or replaced, and it shall be used after assuring it in compliance with JIS B 7721.

4.4 Even in the cases other than those specified above, the accuracy of the testing machine shall be confirmed periodically at fixed interval, depending on the frequency of use.

5. Test

5.1 The test shall be carried out using grips suitable for the shape of the test piece, so that the load is applied only axially to the test piece during the test.

5.2 Preferably the speed of loading is as uniform as possible, and one of the following methods shall be specified:

(1) Increasing rate of stress

(2) Increasing rate of strain

(3) Lapsed time

5.2.1 When the speed of loading is considered to affect significantly the result of test, the speed shall be adjusted in accordance with the requirements of the product specification of the material. Unless otherwise specified, the restriction under 5.2.2 and 5.2.3 shall generally be observed, and attension shall be paid to enable accurate determination of the load and deformation to be made.

5.2.2 On determination of the upper yield point, the lower yield point or the proof stress, the load may be applied at a suitable speed until it reaches 1/2 of the load corresponding to the specified value, and as for steel, after being over the 1/2 load, the average increasing rate of stress up to the upper yield point, the lower yield point or the proof stress shall be 1 to 3 $kg/mm^2/s$.

5.2.3 On determination of tensile strength when the upper yield point, the lower yield point or the proof stress is not required to be determined, the load may be increased to a suitable speed until it reaches 1/2 of the load corresponding to the specified value of tensile strength, and as for steel, after being over the 1/2 load, the load shall be increased to such a speed that the increasing rate of strain in the parallel portion of the test piece is 20 to 80 %/min. This shall be kept also when the tension test is carried out subsequently to the determination of the upper yield point, the lower yield point or proof stress.

5.3 The test temperature shall usually be within the range from 5 to 35 °C, and if necessary, it shall be recorded. However, for the material sensitive to the variation of temperature, the test temperature shall be 20 ± 2 °C, as a rule, and shall comply with the requirements of the product specification of the material.

6. Calculating Method for Obtaining Original Cross-Sectional Area of the Parallel Portion of Test Piece, Gauge Length, Yield Point, Proof Stress, Tensile Strength, Elongation after Fracture (Elongation) and Reduction of Area

6.1 The original cross-sectional area of the parallel portion of the test piece shall be the mean value of the areas taken at three points, i.e. both ends and centre of the gauge length. When it is necessary, however, to use a tapered test piece, the area measured at the minimum cross-section, shall be considered as the original cross-sectional area. For the determination of each cross-sectional area, the diameter or the width and the thickness shall be measured at least to 0. 5 % of the specified dimension by means of a suitable measuring instrument. For the dimensions not more than 2 mm, the measurement may be taken to 0.01 mm. The diameter for determining a circular cross-sectional area shall be the mean value measured in two directions intersecting normal to each other.

6.2 The gauge length shall be measured at least to the value of 0.1 % of the specified dimension by using a suitable measuring instrument. For the dimensions less than 100 mm, the measurement may be taken to 0.1 mm. In case of an extensometer having the fixed gauge length is used, the measurement specified above, shall be made previously on the extensometer.

6.3 The upper and the lower yield point shall be calculated using the following formula:

For the upper yield point $\sigma_{SU}=\frac{P_{SU}}{A_0}$

For the lower yield point $\sigma_{SL}=\frac{P_{SL}}{A_0}$

where, σ_{SU} : upper yield point (kg/mm^2)

σ_{SL} : lower yield point (kg/mm^2)

P_{SU} : ตัวอย่าง refer to 6.3.1

P_{SL} : refer to 6.3.2

A_0 : original cross-sectional area (mm^2) specified in 6.1

Both σ_{SU} and σ_{SL} may be written as σ_S , if there is no fear of confusion.

6.3.1 The upper yield point shall be determined from the maximum load exhibited before the parallel portion of the test piece begins to yield under the load added gradually on the test piece. When a testing machine of the dial indicator type is used, the maximum load P_{SU}(kg) just before the pointer halts for a time or moving back, shall be recorded.

6.3.2 The lower yield point shall be determined, from the load reached nearly constant after the parallel portion of the test piece begins to yield under the load added gradually on the test piece. If the testing machine is of the dial indicator type, the load P_{SL}(kg) at which the pointer halts temporalily, or halts for a time after moving back, shall be recorded.

6.4 The proof stress shall be calculated using the following formula:

$$\sigma_E=\frac{P_E}{A_0}$$

where, σ_E : proof stress (kg/mm^2)

P_E : load (kg) obtained from the following procedure: Plot a curve which shows the relation between the load and the amount of elongation measured by an extensometer, and draw a line parallel to the initial straight portion of the curve, through the point on the elongation axis corresponding to the specified permanent elongation (ε).
The point of intersection of the line and the curve represents the load (kg).

A_0 : original cross-sectional area (mm^2) specified in 6.1

When it is required only to ascertain that the proof stress satisfies the minimum specified value, it may be determined from the result whether the permanent elongation is under specified value or not, after the load equal to the specified proof stress multiplied by the original cross-sectional area has been applied for 15 seconds to the test piece, and removed.

Remarks: The formula for the proof stress given above shall be expressed as follows when the value of the specified permanent elongation $\varepsilon=0.2\%$:

$$\sigma_{0.2}=\frac{P_{0.2}}{A_0}$$

6.5 When the total elongation λ % under the load which produces the specified permanent elongation ε % is known, the proof stress may be determined by the following method instead of that specified in 6.4.

$$\sigma_\varepsilon(\lambda)=\frac{P_\lambda}{A_0}$$

where, $\sigma_\varepsilon(\lambda)$: proof stress (kg/mm^2) obtained by total elongation method

P_λ : load (kg) at which the total elongation under load measured by an extensometer is λ %.

A_0 : original cross-sectional area (mm^2) specified in 6.1

6.6 The tensile strength shall be calculated using the following formula:

$$\sigma_B=\frac{P_{\max}}{A_0}$$

where, σ_B : tensile strength (kg/mm^2)

$P_{\max}$: tensile load (kg)

A_0 : original cross-sectional area (mm^2) specified in 6.1

6.7 The reading of the load to determine the upper yield point, the lower yield point, the proof stress or the tensile strength shall be made at least to 0.5 % of the value. The numerical value of the yield point, the proof stress and the tensile strength shall be rounded off to a whole number. But, when the obtained value of the stress is under 10 kg/mm^2, the numerical value shall be so rounded off as to have two significant figures.
The numerical values shall be all rounded off in accordance with JIS Z 8401-Rules for Rounding off of Numerical Values.

6.8 The elongation after fracture (elongation) shall be calculated through the following formula:

$$\delta=\frac{l-l_0}{l_0}\times 100$$

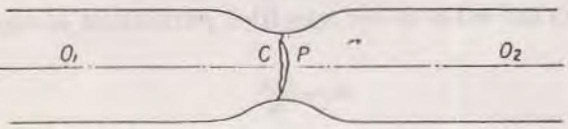
where, δ : elongation after fracture (elongation) (%)

l : length (mm) between the gauge marks measured according to 6.2, when the two broken parts of the test piece are brought into contact on their fractured surfaces, keeping the centre line of each piece in a straight line

l_0 : gauge length (mm)

Remarks: When a clearance (CP) is found in the middle of fractured joint of flat test piece (see Fig. 2), the dimension CP shall not be deducted in measuring the distance between the gauge marks O_1 and O_2 in calculating the elongation.

Fig. 2



6.9 The numerical value of the elongation after fracture (elongation) shall be rounded off to a whole number in accordance with JIS Z 8401-Rules for Rounding off of Numerical Values. But, where the gauge length is over 100 mm, the value may be expressed more minutely.

6.10 For the determination of the reduction of area, a test piece of round cross-section shall be used.

6.11 The reduction of area shall be calculated by the following formula:

$$\varphi = \frac{A_0 - A}{A_0} \times 100$$

where, φ: reduction of area (%)

A: minimum cross-sectional area (mm^2) measured according to 6.1, bringing carefully the fractured surfaces of the test piece into contact

A_0: original cross-sectional area (mm^2)

6.12 The numerical value of the reduction of area shall be rounded off to a whole number in accordance with JIS Z 8401-Rules for Rounding off of Numerical Values.

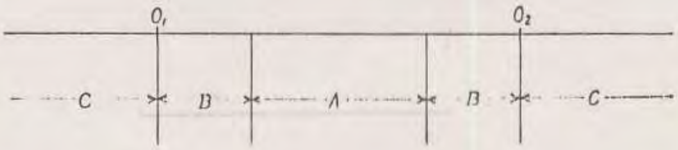
6.13 The result of the tension test shall be discriminated by adding the following symbol depending on the position of fracture of the test piece.

A : when the fracture occurs at a position within 1/4 the gauge length from the middle point between the gauge marks (Fig. 3, portion A)

B : when the fracture occurs at a position over 1/4 the gauge length from the middle point between the gauge marks and within them (Fig. 3, portion B)

C : when the fracture occurs at a position outside the gauge marks (Fig. 3, portion C)

Fig. 3



The division A, B and C may be considered on the length between the gauge marks after fracture of the test piece.

6.14 The results of the test shall be indicated together with the type of the test piece used.

Reference

Assumption of Value in Fractured Elongation (Elongation)

When the actual fractured position of a test piece is in (B) as defined in 6.13, the value of fractured elongation at the mid-point between the gauge marks (O_1) and (O_2) may be assumed from the actual fractured position by the following procedure:

Fig. 4

(1) The length between the gauge marks (O_1) and (O_2) is predivided into suitable equal fractions and graduated.

(2) After the test, fractured ends of the fractured test piece are butted to each other and an appropriate point (A) is marked on the longer piece, symmetrical to point (O_1) on the shorter piece, from the fractured point (P), thus to determine the length between (O_1) and (A).

(3) The length between gauge mark (O_2) and point (A) being already divided into fractions n, and when n is even number, graduation $n/2$, from (A) to (O_2) is taken as point (B), and when n is an odd number, the mid-point between $(n - 1)/2$ and $(n + 1)/2$ graduation is taken as point (B), to determine the length between (A) and (B).

(4) Such an assumed value is calculated by the following formula and should be noted as "assumed value".

$$\text{Assumed value} = \frac{O_1A + 2\ AB - \text{Gauge length}}{\text{Gauge length}} \times 100\ \%$$