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ON THE USE OF THE JOHNSON-SCHUMACHER FUNCTION  
TO REPRESENT THE RELATIONSHIP BETWEEN LENGTH AND FISHING  
MORTALITY COEFFICIENTS

by

Prichar SOMMANI



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ABSTRACT

The model is derived under the assumption that within the range of the maximum length and the length at which the fishing mortality is zero, the rate of change in fishing mortality coefficients at any length is proportional to the product of the fishing mortality coefficients and the reciprocal of the squares of the difference between that length and the length at which the fishing mortality is zero. The resultant leads to the asymmetric sigmoid curve that might be referred to as the Johnson-Schumacher function.

Three data sets of three pelagic species and fourteen data sets of five demersal species were used to estimate the three parameters by the Gauss-Newton method. The results of the estimations and the analyses reveal that in most cases the parameters are well or very well determined in the model and all three parameters are equally important in the model, therefore, none of them can be ignored. All but one case indicate that more than 95 per cent of the variations in the fishing mortality coefficients can be explained by the changes in the fish lengths. Thus, the model can be used to represent the relationships between the fish lengths and the fishing mortality coefficients very well.

## INTRODUCTION

In the usual fisheries assessment, it is generally assumed that the growth and mortality parameters are constant. This includes the assumption of constant fishing mortality with age or length. Beverton and Holt (1957), for example, assumed constant fishing and natural mortality over the fishable life span of the fish in their well-known yield per recruit model. They were, however, aware of the variations in fishing mortality. Furthermore, their model was based on the North Sea fishes and fisheries which at that time were approximately in a steady state so that their assumptions held true. In fact, they assumed the constant parameters just for the simplicity of integration over the fishable life span of the fish. Ricker (1958, 1975), on the other hand, did not assume constant growth and mortality in his model for the blue-gill, but rather he allowed them to vary seasonally. He did this by dividing the life span of the fish into very short intervals and computing the yield obtained during each interval and then summing them up to get the total yield from the cohort. Thus, he would not have to worry about constant growth and mortality.

In general, the assumption of constant fishing mortality is not actually true in many cases of fishes and fisheries. The fishing mortality is likely to increase with the size of the animal due to gear selectivity. That is, once a group of fish enters the fishing gear, say trawl gear, while some of them are retained in the gear, others can escape from it. The amount that escape depends on the size of the fish. The smaller the fish, the greater the chance they may escape from the gear and vice versa. In other words, the retention fraction will increase with the size of the fish up to a certain length, then this fraction becomes asymptotically constant at 100 per cent retention.

In my previous report (Sommani, 1987), I described the method of estimating the fishing mortality coefficients from the length composition data and also gave the relationship between the lengths and the coefficients of fishing mortality of the lizard fish (*Saorida elongata*) in the Gulf of Thailand, without an explanation of the details of the model derivation and the method of curve fitting. Since this functional relationship is very useful in the estimation of the yield from a given cohort from the length composition data, in that the lengths could be classified into intervals as small as needed in order to obtain the precise and accurate yield. Furthermore, these intervals may be varied according to the increments of age, which more preferably should be called 'coded age' because the true age is not known as in the case of the fish in the Gulf of Thailand and other tropical fishes.



Therefore, the use of this function in the yield estimation should be superior to the direct use of the individual estimated fishing mortality coefficient. Besides, the plot of the fishing mortality coefficients against the lengths always shows higher variations in the larger fish. This indicates that the variances of the estimated fishing mortality increase as the size of the fish increases. It is, then, necessary to have the smaller length intervals for the larger fish in order to get a precise and accurate estimated yield.

In this paper, it is my attempt to describe the model derivation and the method of curve fitting. This model might be referred to as the positive asymptotic exponential model or the Johnson-Schumacher functional model (For this function, see Grosenbaugh, 1965).

#### BASIC CONCEPTS AND MODEL DERIVATION

It is assumed, as in the other fisheries assessment, that the growth of the fish follows Bertalanffy's growth curve (Bertalanffy 1938); i.e.,

$$L_t = L_{\infty}(1 - e^{-K(t - t_0)}) \quad (1)$$

where  $L_t$  = the length of the fish of age  $t$ ,

$L_{\infty}$  = the maximum or ultimate length of the fish assumed to be constant,

$K$  = the growth coefficient assumed to be constant, and

$t_0$  = the arbitrary age at which the length is zero also assumed to be constant.

Let us consider the changes of the fishing mortality coefficients of the fish whose sizes range from the length, which will be denoted by  $\gamma$ , at which the fish mortality is zero and the ultimate length. This is the range of lengths that we can use to follow the changes in fishing mortality. Within this range of sizes, the fishing mortality coefficients will firstly increase as the lengths increase and then become asymptotically constant at the ultimate length. Assuming further that the rate of change of the fishing mortality coefficients declines curvilinearly of the following form,

$$\frac{dF}{dL} = \frac{\beta F}{(L_t - \gamma)^c} \quad (2)$$



where  $\frac{dF}{dL}$  = the rate of change of the fishing mortality,

$F$  = the fishing mortality coefficient,

$\beta$  = the constant, and

$c$  = the constant which is not unity.

As the first approximation, it will be assumed that 'c' is equal to 2, then equation (2) becomes,

$$\frac{dF}{dL} = \frac{\beta F}{(L_t - \gamma)^2} \quad (3)$$

It is known that when the length of the fish has reached the ultimate length, it stops growing. Therefore,  $L_t$  will never be greater than  $L_{\infty}$ . According to (2) and (3), it is obvious that the rate of decrease in fishing mortality will become constant at the ultimate length. By integrating equation (3), the following function is obtained;

$$F = \alpha e^{-\beta/(L_t - \gamma)} \quad (4)$$

where  $\alpha$  is constant and the others are as previously defined.

The expression, as illustrated in equation (4), might be referred to as the Johnson-Schumacher function. It is almost identical to the Johnson-Schumacher function as expressed by Grosenbaugh (1965) except that in his expression the sign in the denominator of the exponent is the positive sign but in our equation (4) it is the minus sign. In fact, if his parameter 'C' is negative the two expressions are identical. According to him, the exponent expression must be negative. In equation (4), since the lengths must be within the range of lengths which originate at  $\gamma$  and end at  $L_{\infty}$ , this requirement is therefore fulfilled. The Johnson-Schumacher function is the asymmetric sigmoid curve.

According to equation (4), the actual upper limit of the fishing mortality coefficient is equal to  $\alpha$  which would be reached when the length of the fish becomes infinite. However, since the length of the fish has its upper limit at the maximum or the ultimate length, the highest possible fishing mortality coefficient is obtained when the length of the fish is equal to the ultimate length. Therefore, the highest possible fishing mortality coefficient must be less than  $\alpha$ .

The highest possible fishing mortality coefficient will be called the ultimate fishing mortality coefficient, denoted by  $F(L_{\infty})$ . The magnitude of the difference between the upper limit,  $\alpha$  and the ultimate fishing mortality coefficient depends upon the difference between the ultimate length ( $L_{\infty}$ ) and the length at which the fishing mortality is zero ( $\gamma$ ). That is, if the difference between  $L_{\infty}$  and  $\gamma$  is very large one would expect the ultimate fishing mortality coefficient to be very close to the actual upper limit,  $\alpha$  and vice versa. It is clearly seen from equation (4) that the fishing mortality coefficient is zero at  $\gamma$ , it increases rapidly until it reaches the inflection point and then the rate of increase becomes smaller and smaller until it has attained the ultimate fishing mortality coefficient. This ultimate fishing mortality coefficient is equal to  $\alpha e^{-\beta/(L_{\infty} - \gamma)}$ .

#### METHOD OF CURVE FITTING

Hereafter, for simplicity, the subscript  $t$  will be ignored and the subscript  $i$  will be used instead, where  $i$  denotes the  $i^{\text{th}}$  pair of observations and  $i = 1, 2, 3, \dots, n$ . But one must keep in mind that  $L$  is still the function of  $t$ . By using the natural logarithmic transformation, equation (4) becomes,

$$\ln F_i = \ln \alpha - \frac{\beta}{(L_i - \gamma)} \quad (5)$$

If  $\gamma$  is known, this is a straight line when  $\ln F_i$  is plotted against the reciprocal of  $(L_i - \gamma)$ . In practice, however, this parameter is not known but must be estimated. Thus, equation (5) contains three unknown parameters; namely,  $\gamma$ ,  $\ln \alpha$  and  $\beta$ . These three parameters could be estimated by varying  $\gamma$  until the result of the plot is a straight line and the estimates of these parameters could be computed by the least squares method. If the input value of  $\gamma$ , say  $L_0$ , results in the minimum sum of squares or the maximum coefficient of correlation; this  $L_0$  is the estimate of  $\gamma$ , and the y-intercept and the negative value of the slope are the estimates of  $\ln \alpha$  and  $\beta$ , respectively. This is the method I have used in my previous report (Sommani, 1987).

Note that the expression in equation (4) is the deterministic model. For the probabilistic one, the error term must be included in (4). The transformation given in equation (5) is based on the assumption that either the error term is additive to the exponent of the right hand side of equation (4), or it is multiplicative to the whole right hand side expression of that equation. Let  $F_{ci}$  be the calculated value of the fishing mortality coefficient at the length,  $L_i$ ,  $F_{ci}$  could be expressed as;



$$F_{ci} = ae^{-b/(L_i - L_o)} \quad (6)$$

where  $a$ ,  $b$  and  $L_o$  are the estimates of  $\alpha$ ,  $\beta$  and  $\gamma$ , respectively. For the method described above, these estimates are obtained by minimizing  $\sum (\ln F_i - \ln F_{ci})^2$  not  $\sum (F_i - F_{ci})^2$  as in the original model. In general, this usually results in the biases in the estimates, although in some cases the bias might be small. For example,  $a$  which is computed from  $e^{\ln \alpha}$  is the underestimate of  $\alpha$  (Ricker, 1975). Although Ricker recommended a method to improve this biased estimate, he still suggested avoiding the biases by employing the iterative method to minimize  $\sum (F_i - F_{ci})^2$  directly. For the detailed description of the method one may consult a textbook on nonlinear regression; for example Drapper and Smith (1981) and Ratkowsky (1983).

It is mentioned earlier that the variances of the coefficients of fishing mortality tend to increase with the sizes of the fish (see also Figures 2 and 3). Therefore, the following probabilistic model is assumed from equation (5);

$$F_i = \alpha e^{-\beta/(L_i - L_o)} + E(F_i) \cdot \epsilon_i \quad (7)$$

where  $E(F_i)$  is the expected value of  $F_i$  (= the right hand side expression of equation (5)) and  $\epsilon_i$  is the error term which is normally distributed with mean 0 and variance  $\sigma^2$ . In practice, however, since  $E(F_i)$  is not known it is replaced by its estimate,  $F_{ci}$ . Therefore, equation (7) is expressed as follows;

$$F_i = \alpha e^{-b/(L_i - L_o)} + F_{ci} \cdot \epsilon_i \quad (8)$$

The objective function is the error sum of squares, SSE, which would have to be minimized in order to obtain the estimates of the three parameters. According to equation (8), SSE is of the following form;

$$SSE = \sum W_i (F_i - F_{ci})^2 \quad (9)$$

where

$$W_i = F_{ci}^{-2} \quad (10)$$

The Gauss-Newton method (see Ratkowsky, 1983; Appendix 2.A) is used to estimate the three parameters by minimizing SSE. The least squares estimates are obtained by solving the following matrix equation;

$$\begin{bmatrix} AA & AB & AC \\ AB & BB & BC \\ AC & BC & CC \end{bmatrix} \begin{bmatrix} Da \\ Db \\ Dc \end{bmatrix} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \quad (11)$$

It should be noted that instead of solving  $a$ ,  $b$  and  $L_0$  directly, we must solve  $Da$ ,  $Db$  and  $Dc$  by using the following matrix equation;

$$\begin{bmatrix} Da \\ Db \\ Dc \end{bmatrix} = \begin{bmatrix} AA & AB & AC \\ AB & BB & BC \\ AC & BC & CC \end{bmatrix}^{-1} \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \quad (12)$$

These notations need some explanations. Firstly, let us consider  $Da$ ,  $Db$  and  $Dc$ . By letting  $k$  be the subscript denoting the  $k^{\text{th}}$  step of iteration, where  $k = 1, 2, 3, \dots, K$ . Now, we could define  $Da$ ,  $Db$  and  $Dc$  as follows:

$$Da_{k+1} = a_{k+1} - a_k \quad (13)$$

$$Db_{k+1} = b_{k+1} - b_k \quad (14)$$

$$Dc_{k+1} = L_{0,k+1} - L_{0,k} \quad (15)$$

and hence

$$a_{k+1} = a_k + Da_{k+1} \quad (16)$$

$$b_{k+1} = b_k + Db_{k+1} \quad (17)$$

$$L_{0,k+1} = L_{0,k} + Dc_{k+1} \quad (18)$$

The process of iteration is carrying until  $Da$ ,  $Db$  and  $Dc$  are less than some specific assigned values, for example I have used the figure of  $10^{-8}$ , the final estimates are obtained. But before this stage can be carried out, the elements of the matrices and vectors in equations (11) and (12) have to be computed first. Let  $J_{i1}$ ,  $J_{i2}$  and  $J_{i3}$  be the first partial derivatives of  $E(F_i)$  with respect to  $\alpha$ ,  $\beta$  and  $\gamma$ ; evaluate at  $a$ ,  $b$  and  $L_0$  (ignoring the subscript  $k$ ) respectively. These elements of the matrices and vectors can be calculated by the following equations;

$$D_1 = \sum W_i (F_i - F_{ci}) J_{i1} \quad (19)$$

$$D_2 = \sum W_i (F_i - F_{ci}) J_{i2} \quad (20)$$

$$D_3 = \sum W_i (F_i - F_{ci}) J_{i3} \quad (21)$$



$$AA = \sum W_i J_{i1}^2 \quad (22)$$

$$BB = \sum W_i J_{i2}^2 \quad (23)$$

$$CC = \sum W_i J_{i3}^2 \quad (24)$$

$$AB = \sum W_i J_{i1} J_{i2} \quad (25)$$

$$AC = \sum W_i J_{i1} J_{i3} \quad (26)$$

$$BC = \sum W_i J_{i2} J_{i3} \quad (27)$$

To compute the inverse matrix on the left-hand side of equation (11) or the matrix on the right-hand side of (12), the following symbols will be used. Let  $d$  be the determinant of the left-hand side matrix of (11) and  $aa$ ,  $bb$ ,  $cc$ ,  $ab$ ,  $ac$  and  $bc$  be the elements of the inverse matrix corresponding to their upper cases in equation (12). The calculation can be carried out as follows:

$$d = AA.BB.CC + 2.AB.BC.AC - BB.AC^2 - AA.BC^2 - CC.AB^2 \quad (28)$$

$$aa = (BB.CC - BC^2)/d \quad (29)$$

$$bb = (AA.CC - AC^2)/d \quad (30)$$

$$cc = (AA.BB - AB^2)/d \quad (31)$$

$$ab = (AC.BC - AB.CC)/d \quad (32)$$

$$ac = (AB.BC - BB.AC)/d \quad (33)$$

$$bc = (AB.AC - AA.BC)/d \quad (34)$$

According to the matrix equation (12),  $Da$ ,  $Db$  and  $Dc$  can be solved as follows:

$$Da = D_1.aa + D_2.ab + D_3.ac \quad (35)$$

$$Db = D_1.ab + D_2.bb + D_3.bc \quad (36)$$

$$Dc = D_1.ac + D_2.bc + D_3.cc \quad (37)$$

When the input parameters, say the results of equation (5), are used at the first step of iteration,  $F_{ci}$  is calculated from equation (6). Then the computations are carried out for equations (19) to (37). The next step is to calculate the new value of  $a$ ,  $b$  and  $L_0$  by substituting the results of (35), (36) and (37) into equations (16), (17) and (18). The new values are then employed to calculate the new  $F_{ci}$  from equation (6). With the new  $F_{ci}$ , equations (19) to (37) are calculated. This process is repeated until the requirements are met, i.e. when  $D_a$ ,  $D_b$  and  $D_c$  are less than the specific preassigned value. At this stage, the final estimates are obtained. Then, the residual or unexplained variance and the variance-covariance matrix of the estimates could be estimated by using the final estimates in the following equations,

$$s^2 = \frac{\sum W_i (F_i - F_{ci})^2}{n - 3} \quad (38)$$

and

$$\hat{V} = s^2 \begin{bmatrix} aa & ab & ac \\ ab & bb & bc \\ ac & bc & cc \end{bmatrix} \quad (39)$$

These are the asymptotic estimates. However,  $s^2$  does not, in general, lead to the unbiased estimated residual variance, but rather it is usually positively biased or overestimated. Hereafter, statements will be made with this fact in mind.

The sum of squares around the mean or total sum of squares (SST) and the variance around the mean or total variance ( $V(F)$ ) are calculated from the following equations;

$$SST = \sum W_i (F_i - \bar{F})^2 \quad (40)$$

$$\text{and} \quad V(F) = \frac{\sum W_i (F_i - \bar{F})^2}{n - 1} \quad (41)$$

where  $\bar{F}$  is the weighted arithmetic mean which is computed by

$$\bar{F} = \frac{\sum W_i F_i}{\sum W_i} \quad (42)$$

In order to determine roughly the magnitude of the variations of fishing mortality that could be explained by the variations in the fish lengths, the crude coefficient of determination ( $R^2$ ) should be estimated by the following expression:

$$R^2 = 1 - \frac{(n-1)s^2}{(n-3)V(F)} \quad (43)$$

The square root of this coefficient of determination is the correlation coefficient,  $R$ . Since  $s^2$  is positively biased, both  $R^2$  and  $R$  would be underestimated. However, they are still very useful indicators of the least magnitude of fishing mortality variations explained by the variations of the fish lengths in the model. For example,  $R^2$  will be the least fraction or the percentage of the variations of the fishing mortality coefficients which can be explained by the changes of the lengths.

The t-test can also be conducted as described by Ratkowsky (1983) by dividing the estimate into its standard error. The standard error of the estimate is the positive square root of the diagonal elements of the estimated variance-covariance matrix. This t-value does not have statistical properties as in the linear case. Thus, it should be treated as an approximate test. The t-test of 'a' could be computed from the following expression:

$$t = \frac{a}{s_a} \quad (44)$$

The variability index is calculated by dividing the standard error of the estimate by the estimate, for example the variability of  $b$  is computed by,

$$V.I. = \frac{s_b}{b} \quad (45)$$

This variability index may be expressed as a fraction or percentage, as one prefers.

It should be noted here that some authors do not recommend either the F-test or the t-test due to the risk of rejecting the true hypothesis because the exact variance is not known, but rather they recommend the confidence region be drawn (see for example Drapper and Smith, 1981). This is rather tedious or even impossible when one is concerned with three or more parameters. In my view, if we interpret the results with the utmost caution, the risk is not more serious than for the other method.



## MATERIALS, INPUT DATA AND PARAMETERS

Seventeen relationships between the lengths and the fishing mortality coefficients of eight species of fishes from the Gulf of Thailand (see Figure 1 for the study areas) were estimated and analyzed. The pelagic fishes included *Stolephorus heterolobus* (Ruppell), *Rastrelliger neglectus* (van Kampen) and *Decapterus maruadsi*. Fourteen data sets of the demersal fishes included those of two genera; namely *Saurida* spp. and *Nemipterus* spp. Two species of *Saurida* were *S. elongata* and *S. undosquamis* (Richardson). While the three species of the other genus, namely *Nemipterus* spp. were *N. japonicus*, *N. mesoprion* (Bleeker) and *N. nematophorus* (Bleeker).

The sources of data and Bertalanffy's growth parameters are given in Table 1. In this table, the study areas and years are also provided. If the coded ages and the length data are available, the growth parameters are reestimated from equation (1) directly by iterative method and these estimates are given instead. The detailed methodology of the estimation will be reported elsewhere. But it should be mentioned here that the estimates given are computed by weighted nonlinear regression with the weighting factor equal to the squares of the reciprocal of the estimated length.

Most of the length composition data are published in the report given in Table 1. The unpublished data of the demersal fishes were kindly provided by Mr. T. Boonwanich, except for those of *S. undosquamis* (Richardson) in area III which were provided by Miss S. Intong from her paper (Intong, 1976). The data of *D. maruadsi* were made available by Mr. S. Chullasorn and Miss P. Nuchmorn reanalyzed them; some are used here. The size composition data of *S. heterolobus* (Ruppell) were reported by Debtaranon (1972).

By using Bertalanffy's growth parameters given in Table 1 and the length composition data, the fishing mortality coefficients were estimated by the method described by Sommani (1987). The unity M-K ratios are assumed except for *R. neglectus* (Van Kampen), the M-K ratio is assumed to be 0.5 because the unity M-K ratio results in the unreasonably low estimates of coefficients of fishing mortality. The input values of F are usually 0.5 except in some cases the lower figure, 0.25, is used when the natural mortality is very low or the higher figure is used when some estimated fishing mortality coefficients are greater than unity. In either case, the exploitation ratios, F/Z, must be within the range of 0.7 to 0.9 as suggested in my previous report (Sommani, 1987).

The estimations and analyses obtained in this report were computed by using a pocket computer with an 8 Kbyte ramcard, but actually four Kbytes are enough.



## RESULTS AND DISCUSSION

The results of the estimations and analyses are summarized in Table 2 for the pelagic fishes and Table 3 for the demersal fishes. Figure 2 displays the relationships between the lengths and the fishing mortality coefficients of the three species of pelagic fishes, while those of the demersal fishes are shown in Figures 3 to 6.

According to Table 2, the estimated residual or unexplained variances,  $s^2$ , of the pelagic fishes are low, ranging from 0.00656677 for *D. maruadsi* to 0.02949066 for *R. neglectus* (van Kampen), while that of *S. heterolobus* (Ruppell) lies between them and equals 0.02132984. The figures are very small when compared with the total variances,  $V(F)$ , which are in the same sequence as the species given previously, 0.48849096, 0.73798015 and 0.78954895.

The estimated variances of the three estimates,  $V(a)$ ,  $V(b)$  and  $V(L_0)$  which are the diagonal elements of the variance-covariance matrices are also given in Table 2. The estimated variances of  $b$  are the largest while those of  $a$  are the least. Although the estimated variances of  $L_0$  are quite high they are still very low as compared with  $V(b)$ 's. For example,  $V(a)$ ,  $V(b)$  and  $V(L_0)$  of *D. maruadsi* are equal to 0.00797584, 0.77813041 and 0.09828561 and those of *R. neglectus* (van Kampen) are 0.08485379, 3.93007095 and 0.16044488 respectively. The variance-covariance matrices show that  $a$  and  $b$  are positively correlated but  $a$  and  $L_0$  as well as  $b$  and  $L_0$  are negatively correlated.

The variability indices which are the measures of the variations of the estimate are rather small for all three species, indicating that the variations of these estimates are small. In other words, it might be stated that the three estimates are acceptably accurate. Among the three estimates, the variability indices of  $L_0$  are the least and less than 4% for all three species. However, for the other two estimates,  $a$  and  $b$ , the figures show some differences among species. The variability indices of  $a$  and  $b$  for *D. maruadsi* and *S. heterolobus* (Ruppell) are less than 25%. The highest values are those of  $b$  which equal 18.1452% for the latter species and 23.8916% for the first. The variability indices of  $a$  are slightly lower than those of  $b$ . They are equal to 12.7972% and 13.1413% for *D. maruadsi* and *S. heterolobus* (Ruppell), respectively. The variability indices of the same estimates for *R. neglectus* are larger than 25% but still less than 30%. They are equal to 29.6765% for  $a$  and 28.5130% for  $b$  indicating that in this case the variation of  $a$  is higher than that of  $b$ .

The t-tests of these parameters are given in Table 2. The results indicate that in all cases they are significantly different from 0 ( $P < 0.010$ ) and, in many cases, the probability is less than 0.001. The latter includes the tests of  $a$ ,  $b$  and  $L_0$  for *S. heterolobus* (Ruppell), of  $a$  and  $L_0$  for *D. maruadsi* and of  $L_0$  for *R. neglectus* (van Kampen). These results suggest that the parameters are well determined even though there are correlations among these parameters.

The coefficients of determination range from 96.7304% for *R. neglectus* (van Kampen) to 99.0398% for *D. maruadsi*. The values are rather high for this type of data. The results suggest that 96.7304%, 97.6844% and 99.0398% of the variations in the fishing mortality coefficients of *R. neglectus* (van Kampen), *S. heterolobus* (Ruppell) and *D. maruadsi* could be explained by the variations in the lengths of the fishes. The correlation coefficients are all highly significant; the calculated correlation coefficients are 0.9835 ( $df = 9$ ), 0.9854 ( $df = 11$ ), and 0.9952 ( $df = 5$ ) for *R. neglectus* (van Kampen), *S. heterolobus* (Ruppell) and *D. maruadsi* respectively, while the criticals with the same degree of freedom are 0.735, 0.684 and 0.874. These are much lower than the calculated values given above.

It should be noted that the high variations in the estimates of *R. neglectus* (van Kampen) are caused by only one pair of data ( $L = 16.0$ ,  $F = 0.26618$ ) which is indicated by the arrow in Figure 2 (upper right). If this point is excluded, the final estimates  $a$ ,  $b$  and  $L_0$  are 1.0679, 7.9224 and 11.6053. The estimated variance,  $s^2$ , is 0.01174032 and the total variance,  $V(F)$ , is 0.70521456. The coefficient of determination increases about 2% and equals 98.6682%. The final estimated variance-covariance matrix becomes

$$\hat{V} = \begin{pmatrix} 0.05109005 & 0.34040272 & -0.06390176 \\ 0.34040272 & 2.38684873 & -0.46159542 \\ -0.06390176 & -0.46159542 & -0.09207516 \end{pmatrix}$$

and hence the variability indices of  $a$ ,  $b$  and  $L_0$  are 0.21165193, 0.19501026 and 0.02614663 respectively. Both estimated variance and variability indices are much less than figures given in Table 2. The new calculated t-values of  $a$ ,  $b$  and  $L_0$  are equal to 4.725 ( $df = 8$ ,  $P < 0.005$ ), 5.128 ( $P < 0.001$ ) and 38.246 ( $P < 0.001$ ) respectively. The results of the new estimation and analysis are better than those given in Table 2. However, since the results in Table 2 are acceptable for the coefficient of determination, the variability indices and the t-tests are at least highly significant. The author will let them be as in Table 2. The new results are discussed only to point out how the high variations of the coefficients of fishing mortality at or around the middle part of the lengths' range could affect the results of the estimations.



The ultimate fishing mortality coefficients,  $F(L_{\infty})$ , are estimated to be 0.5515, 0.4172 and 0.5445 for *S. heterolobus* (Ruppell), *R. neglectus* (van Kampen) and *D. maruadsi* respectively. It would be valuable to examine these estimates together with the shapes of the fitted curves. In the results of *S. heterolobus* (Ruppell) and *D. maruadsi* the fishing mortality coefficients increase with the lengths rapidly for a while, then the rates of increase decline as the lengths approach the maximum length of the fish (Figure 2, left and lower right respectively). In other words, the rates at which the fishing mortality coefficients attain their ultimate values become smaller and smaller when the lengths of the fish approach the maximum lengths. On the other hand, for *R. neglectus* (van Kampen) the rises of the fishing mortality coefficients with the lengths are not similar to those of the previous two species, as the fish become larger the rates of increase do not decline as much as they do for the other two species. Instead, the rates of increase incline very slowly although the lengths of the fish are close to the ultimate length. According to Table 1, the ultimate length of this species is 19.94 cm and as displayed in Figure 2, upper right, even at a length of 18.0 to 19.0 cm, the fitted curve does not show any sign of the decline towards its ultimate value of 0.4172. Thus, in this species the fishing mortality changes abruptly to attain the ultimate fishing mortality coefficient. This abrupt or sudden change might be caused by the narrow range of the lengths or the interval between the length at which the fishing mortality is zero and the maximum length of the fish. In this case, this range is about 8 cm. from 11.8154 to 19.9407 cm.

The results of the analyses of the demersal species as given in Table 3 reveal that in thirteen out of fourteen data sets the estimated unexplained variances,  $s^2$ , are rather small ranging from 0.00056941 for the female *N. mesoprion* (Bleeker) in areas I-IX, 1978/1979 to 0.06395438 for the male *S. elongata* in areas V-VI in 1981, while the corresponding variances around the mean,  $V(F)$ , are 0.59716918 and 1.03964127. The unexplained variance of the female *N. japonicus* in areas V-VI during 1981 is exceptionally high and equal to 0.14952142 and the total variance is estimated to be 1.04346747. This result is not so surprising, since, as illustrated in Figure 5 (lower left), it can clearly be seen that the high estimated variance of the residuals is caused by the very high variations in the fishing mortality of the large fish. Note that although the unexplained variance is about 2.34 times the highest figure given above, the variances around the mean of the two cases are about the same.

As for the cases of the pelagic species, the estimated variance-covariance matrices show that  $a$  and  $b$  are positively correlated while both are negatively correlated with  $L_0$ . Since the residual variances are very high for the female *N. japonicus* then result in the very high estimated variances of  $a$ ,  $b$  and  $L_0$  which are

equal to 2.17065386, 14.10688363 and 0.32031990 respectively. The results of the analysis of the male *S. undosquamis* (Richardson) in areas I-IX during 1979 to 1980 also reveal that the estimated variances of  $b$  and  $L_0$  are quite high and equal to 8.74059310 and 0.58053492 while the estimated variance of  $a$  is only 0.02207138. Note that the estimated residual variance of this species is not as high as that of the former species. For the remaining twelve cases, the estimated variances of  $a$  are less than the magnitude of 0.10 except for the male *N. japonicus* in areas V-VI during 1981,  $V(a)$  is 0.20780402. The estimated variances of  $b$  in twelve cases are less than the magnitude of 4.0 and in nine data sets these variances are estimated to be lower than 2.0. In eleven of these twelve cases, the variances of  $L_0$  are estimated to be less than the figure of 0.10 except for the female *S. elongata* in areas I-IX during 1979 to 1980 in which  $V(L_0)$  is 0.12684931. In general, of the estimated variances of  $a$ ,  $b$  and  $L_0$  obtained from the same set of data,  $V(b)$  is the highest one, the next in value is  $V(L_0)$  and  $V(a)$  is the least. Except in the cases of the female *N. mesoprion* (Bleeker) in areas I-IX during 1978 to 1979 and of both the male and female *N. japonicus* in areas V-VI during 1981 in which  $V(a)$  is greater than  $V(L_0)$ . This exception occurs in only three of fourteen data sets.

The variability indices of the estimates are rather high for the female *N. japonicus* due to the very high variance of the residuals. In this case, the variability indices of  $a$ ,  $b$  and  $L_0$  are 44.008101%, 36.223264% and 9.865094% respectively. The male *S. undosquamis* (Richardson) in areas I-IX during 1979-1980 also has high variability indices for  $a$ ,  $b$  and  $L_0$  which are estimated to be 25.241422%, 39.702433% and 9.722861% respectively. For the remaining twelve cases, the variability indices of  $a$  are less than 20% and six of them are less than 15%. Eight variability indices for  $b$  are less than 20%, one index is less than 22%, two indices are lower than 35% and the remaining one for the female *S. elongata* is 35.086883% in areas V-VI in 1981. Of the other twelve cases, eleven of them reveal that the variability indices of  $L_0$  are less than 5%, and one, the male *N. mesoprion* (Bleeker) in areas V-VI in 1981 is 9.643863%, i.e. about the same magnitude as those of the exceptional two species given at the beginning of the paragraph. Thus, in general, it can be stated that the precision of the three estimates is very high. Among the three estimates,  $L_0$  is the most precise, then comes 'a' and finally  $b$  for which the precision is rather low when compared with the previous two estimates.

The results of the t-tests given in Table 3 suggest that since, for the female *N. japonicus*, the calculated t-value of 'a' is 2.272 ( $df = 8$ ,  $P < 0.100$ ), this parameter is not well determined. However, this calculated  $t$  is slightly less than the critical t-value of 2.306 ( $P < 0.050$ ) and the t-tests of  $b$  and  $L_0$  indicate that  $b$  is significant



( $P < 0.025$ ) and  $L_0$  is very highly significant ( $P < 0.001$ ). This suggests that the result of the t-test of 'a' is not so bad that the nonlinear model is not acceptable. In fact, if it is rejected we might possibly confront the risk of rejecting the true hypothesis. Further, the high t-values calculated for b and  $L_0$  suggest that the data-model does not fit badly. For the remaining thirteen data sets, nine of them reveal that all three estimates are very highly significant ( $P < 0.005$  or  $P < 0.001$ ). Three of the thirteen data sets indicate that the estimates for b are significant ( $P < 0.025$  or  $P < 0.050$ ) and that the other two estimates are very highly significant ( $P < 0.005$  or  $P < 0.001$ ). One case of the thirteen data sets reveals that b is significant ( $P < 0.050$ ), a is highly significant ( $P < 0.010$ ) and  $L_0$  is very highly significant ( $P < 0.001$ ). Since the t-test is not an exact test and the stated probability level is not exact, the exact conclusions can not be drawn at any stated probability level. However, these results suggest that the estimates of the parameters in the model are different from zero and all three parameters are equally important in the model so that none of them can be ignored.

The crude estimates of the coefficients of determination as given in Table 3 show that only one out of the fourteen coefficients of determination is less than 90% and equal to 88.5366% for the female *N. japonicus* in areas V-VI during 1981 while the remaining thirteen are very much higher than this level ranging from 95.0917% for the male *S. undosquamis* (Richardson) in areas I-IX during 1979 to 1980 to 99.9428% for the female *N. mesoprion* (Bleeker) in areas I-IX during 1978 to 1979. In all cases, the correlation coefficients which range from 0.9409 (df = 9) for the female *N. japonicus* to 0.9997 (df = 3) for the female *N. mesoprion* (Bleeker) are all highly significant ( $P < 0.010$ ). Therefore, even though  $s^2$  might be overestimated and is positively biased, it could be stated that in almost of the cases, more than 95% of the variations in fishing mortality coefficients can be explained by the changes in the fish lengths when the fishing mortality coefficient is expressed as the function of the fish lengths in the form of the derived model or the Johnson-Schumacher function.

Figure 3 illustrates the relationships between the lengths and the fishing mortality coefficients of *S. elongata* in areas I-IX during 1979 to 1980 (left) and in areas V-VI during 1981 (right). According to this Figure, the fitted curves rise rapidly for a while and then the rate of increase becomes smaller and smaller as the fish lengths increase. Finally, the fitted curves eventually approach the ultimate fishing mortality coefficients of 0.6267 and 0.5093 for the male and the female fish respectively for the former cases (Upper and lower left) and of 0.4368 and 0.3849 for the male and the female fish respectively for the latter (upper and lower right). For *S. undosquamis* (Richardson), in Figure 4 the results are shown for the stocks inhabiting area III during

1968 to 1972 (upper and lower left) and areas I-IX during 1979 to 1980 (upper and lower right). The shapes of the fitted curves are similar to those of *S. elongata*, that is, at first they increase rapidly but as the fish become larger and larger the fishing mortality coefficients eventually attain their ultimate fishing mortality coefficients of 0.8509 and 0.7693 for the male and the female fish respectively in the first cases and of 0.4324 and 0.5180 for the male and female fish respectively in the latter cases.

The results of the plots for both male and female *N. japonicus* in areas V-VI during 1981 show similar curves. Within the ranges of the lengths plotted in Figure 5 (left), the fitted curves do not approach the ultimate fishing mortality coefficients since the upper limits of the length ranges in that figure are far from the maximum lengths of 25.2240 cm and 27.1724 cm for the male and female respectively (Table 1). However, both would, as the fish become larger, finally attain the ultimate fishing mortality coefficients of 1.8360 and 2.0592 for the male and female fish respectively (Figure 5, upper and lower left).

According to Figure 5, upper and lower right, the results of the plots for the male and female *N. mesoprion* (Bleeker) in areas I-VI during 1978 to 1979 indicate that the shapes of the fitted curves are different for each sex. For the male, the ascending limb of the fitted curve is not as steep as in the previous cases. The fitted regression line eventually approaches its ultimate fishing mortality coefficient of 0.2976. On the other hand, the fitted curve for the female fish increases steeply but as the fish grows the rate of increase does not become smaller as much as it should, so that the curve abruptly inclines to reach its ultimate fishing mortality coefficient of 1.1623 as in the case of *R. neglectus* (van Kampen). It is proposed that this is caused by the narrow range of lengths from  $L_0$  to  $L_\infty$  which in this case is equal to 9.8123 cm, from 6.8507 cm to 16.7620 cm.

As illustrated in Figure 6, the shapes of the fitted regression lines of the male and female *N. nematophorus* (Bleeker) in areas I-VI during 1978 to 1979 show that there are differences between the sexes, although the results of the plots look very similar. From  $L_0$  to the length of about 14.0 cm, the shapes of the fitted curves of both sexes are roughly the same. For the male, after this point, the fitted regression line will eventually attain its ultimate fishing mortality coefficient of 0.2636. The length interval between  $L_0$  and  $L_\infty$  is 12.7543 cm, from 6.2965 cm to 19.0499 cm. On the other hand, for the female beyond the length of 14.0 cm, the slope of the fitted curve does not decrease as rapidly as in the case of the male but rather it



suddenly attains its ultimate fishing mortality coefficient of 0.2144. The length range from  $L_0$  to  $L_\infty$  is 9.5561 cm, from 6.3050 cm to 15.8611 cm. Thus, the shape of the fitted curve of the female *N. nematophorus* (Bleeker) is the same as those of *R. neglectus* (van Kampen) and the female *N. mesoprion* (Bleeker) in areas I-VI during 1978 to 1979.

According to the shapes of the estimated regression line, two types of fitted curves are observed in this study.

The Type A curve is observed in the cases of *R. neglectus* (van Kampen), the female *N. mesoprion* (Bleeker) and the female *N. nematophorus* (Bleeker). In this type, the fitted curve increases rapidly at first but as the fish becomes larger the rate of increase does not decline as much as it should. Rather, the curve abruptly decreases to attain the ultimate fishing mortality coefficient. It is observed that, for these three species, the length interval between  $L_0$  to  $L_\infty$  is quite small, ranging from 8.12 cm for *R. neglectus* (van Kampen) to 9.81 cm for the female *N. mesoprion* (Bleeker).

The Type B curve, which is found in the cases of *Saurida* spp., *N. japonicus*, *D. maruadsi*, *S. heterolobus* (Ruppell) and other species and sexes not mentioned for the Type A curve above, is similar to other sigmoid curves. The curve rises rapidly at first but as the fish become larger the rate of rise will become smaller and smaller; and finally the curve will eventually approach the ultimate fishing mortality coefficient. These estimated regression curves are clearly illustrated in Figure 2 (left and lower right). Figure 3, Figure 4, Figure 5 (left and upper right) and Figure 6 (upper).

It is not known whether the Type A curve is a special characteristic of *R. neglectus* (van Kampen), the female *N. mesoprion* (Bleeker) and the female *N. nematophorus* (Bleeker) and their fisheries or if it is the result of a negative bias of the length interval between  $L_0$  and  $L_\infty$ . Since it occurs in only three of the seventeen data sets, it is likely that the latter be the case. The negative bias of the length interval between  $L_0$  and  $L_\infty$  might be caused by the fact that  $L_0$  is overestimated or  $L_\infty$  is underestimated or both. It is a must to prove which one of them is the cause of this bias or if it is caused by both.

## CONCLUSION

The model is derived to express the relationships between the fish lengths and the fishing mortality coefficients. The final result of the model derivation can be expressed in the deterministic form as;

$$F = \alpha e^{-\beta/(L - \gamma)}$$

where  $\alpha$ ,  $\beta$  and  $\gamma$  are the constants. The last parameter,  $\gamma$  is the length at which the fishing mortality is zero. This expression is of the same type as the Johnson-Schumacher function, with the minor exception that in our model the operation sign in the denominator of the exponent is the minus sign, instead of the positive sign as in the Johnson-Schumacher function. However, within the length interval from the length at which the fishing mortality is zero and the maximum length of the fish, the requirement of the Johnson-Schumacher function that the exponent expression must be negative is fulfilled. The ultimate fishing mortality coefficient with the highest possible fishing mortality coefficient of a given species and its fisheries is obtained at  $L_{\infty}$  and is always less than the actual asymptote which is attained as the fish length becomes infinite. The model is an asymmetric sigmoid curve.

The results of the analyses and the estimations of the parameters, using the Gauss-Newton method, reveal that in sixteen of the seventeen data sets or 94.12% the estimated unexplained or residual variances,  $s^2$ , are very small in relation to the variances around the mean,  $V(F)$  and are less than 0.07. The remaining and exceptional one case or 5.88%, the estimated unexplained variance, is rather high and equal to 0.14952142 and was obtained in the female *N. japonicus* in areas V-VI during 1981. In this latter case, the high residual variance is caused by very high variations in the fishing mortality of the large fish. Thus, in general, the unexplained variances are acceptably small even though these estimated variances are subjected to the possibility of positive biases. These small estimated unexplained variances in relation to the total variances result in the very high coefficients of determination which are, in the former sixteen cases, more than 95% while in the worst case or the latter this coefficient is about 88.54%. However, in all seventeen cases the correlation coefficients are highly significant ( $P < 0.010$ ). Although the stated probability level is not exact, but rather approximative, with the possibility that the coefficient of determination is negatively biased because  $s^2$  is positively biased, the results suggest that the derived model or the Johnson-Schumacher function can be used to express the relationships between the fish lengths and the fishing mortality coefficients very well. The results also reveal that with the use of this model, in 94.12% of the seventeen cases more than 95% of the variations in the fishing mortality coefficients can be explained by the changes in the fish lengths. Even in the worst case or 5.88% of the seventeen cases at least 88.50% of the variations in the fishing mortality coefficients can be explained by the changes in the fish lengths.





The results of the approximate t-tests and close examinations of the variability indices reveal that ten out of seventeen or 58.82% of the data sets indicate that all three estimates are very highly significant ( $P < 0.005$  or  $P < 0.001$ ). Two of the seventeen data sets or about 11.76% reveal that two estimates are highly significant ( $P < 0.010$ ) but the other estimates, usually  $L_0$  are very highly significant ( $P < 0.001$ ). The results obtained from four of the seventeen cases or about 23.53% indicate that the estimates,  $b$ , are significant ( $P < 0.050$  or  $P < 0.025$ ), while the estimates,  $a$ , are highly significant ( $P < 0.010$ ) or very highly significant ( $P < 0.001$ ) and the estimates,  $L_0$ , are very highly significant ( $P < 0.001$ ). In one last case or about 5.88% the estimate,  $a$  is not significant ( $P < 0.100$ ),  $b$  is significant ( $P < 0.025$ ) and  $L_0$  is very highly significant ( $P < 0.001$ ). Thus, the calculated t-values of 50 out of 51 ( $3 \times 17$ ) estimates or 98.04% are high or very high while only one or 1.96% is low and hence the corresponding variability indices are low or very low with one exception for the estimate,  $a$ , for the female *N. japonicus* whose index is high at about 44.00%. The results lead to the conclusion that in most cases the three parameters are well or very well determined in the model and are greater than zero. All three parameters in the model are equally important to the model so that none of them can be ignored. The variations of the estimates of these parameters are low and result in the high precisions of the estimates. But the degree of precision differs among the three estimates. The estimate,  $L_0$  is the most precise in the model, and this is indicated by its variability indices which are all less than 10% and 82.35% of them are less than 5%. The next most precise estimate is 'a'; about 82.35% of the seventeen indices are less than 20% and 47.06% are less than 15%, about 11.77% of the variability indices are between 25% and 30% and the last and exceptionally high index or about 5.88% of the total is about 44.00%. The precision of  $b$  is rather low when compared with the other two estimates; only 52.94% of its variability indices are less than 20%, about 29.41% of them range from 23% to 35% and the remaining 17.65% are between 35% and 40%.

The two types of fitted curves are observed according to their shapes. The Type A curve which is found in the cases of *R. neglectus* (van Kampen), the female *N. mesoprion* (Bleeker) and the female *N. nematophorus* (Bleeker) rises rapidly at first but as the fish become larger and larger the rate of increase does not decrease as much as it should, as in the other sigmoid curves, but rather the curve abruptly declines to attain the ultimate fishing mortality coefficient. It is proposed that this is caused by the negative bias of the length interval between  $L_0$  and  $L_\infty$ . This negative bias might be the result of the positive bias of  $L_0$  or the negative bias of  $L_\infty$  or both.

The Type B curve, which is observed in cases of *Saurida* spp, *N. japonicus*, *S. heterolobus* (Ruppell), *D. maruadsi* and other species and sexes not mentioned for the Type A curve, has a similar shape to that in the other asymmetric sigmoid curve. That is it rises at first rapidly but as the fish grows up the rate of increase becomes smaller and smaller, finally the curve will eventually approach its ultimate fishing mortality coefficient.

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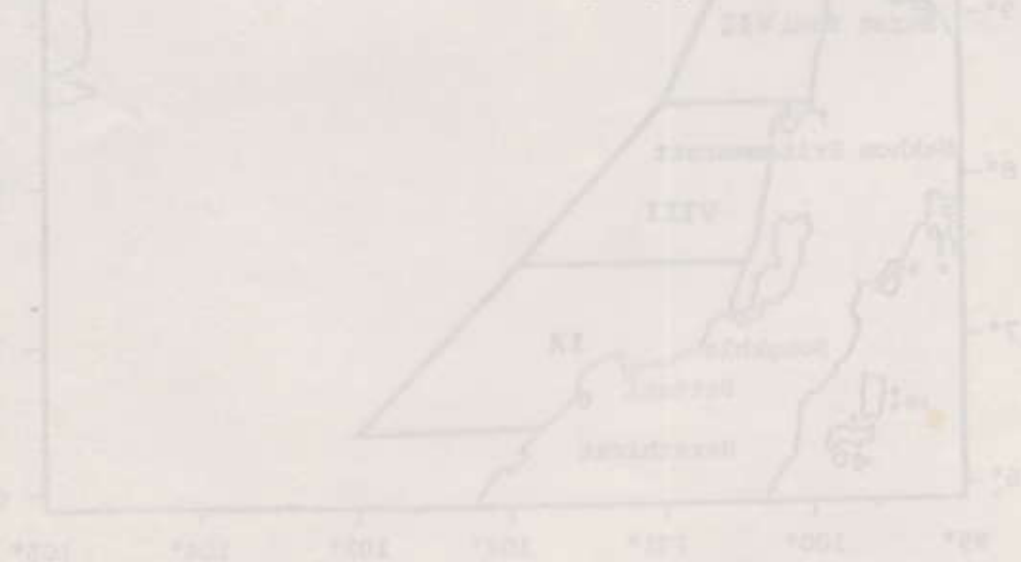


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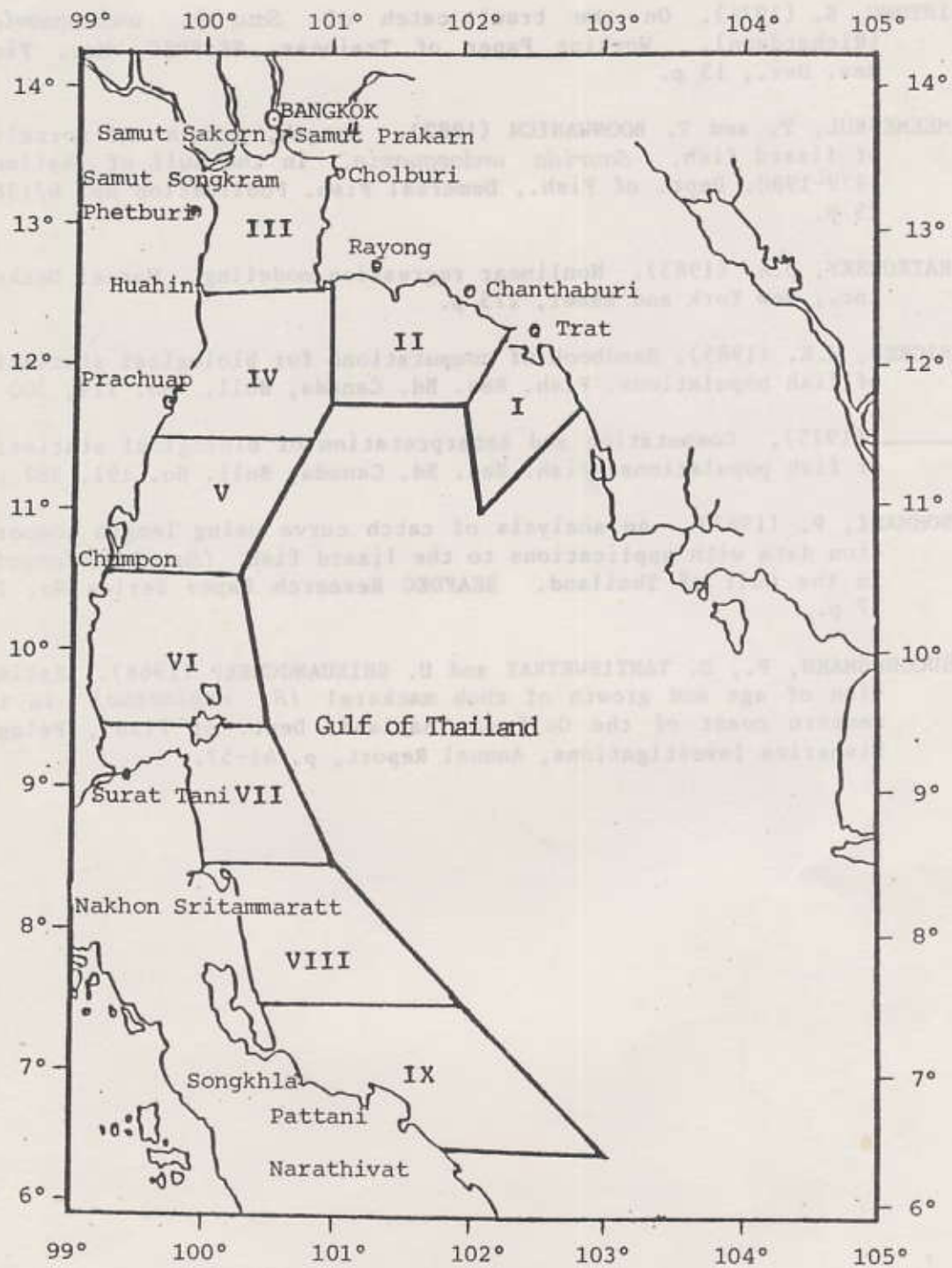


Figure 1. The Gulf of Thailand showing the study areas.

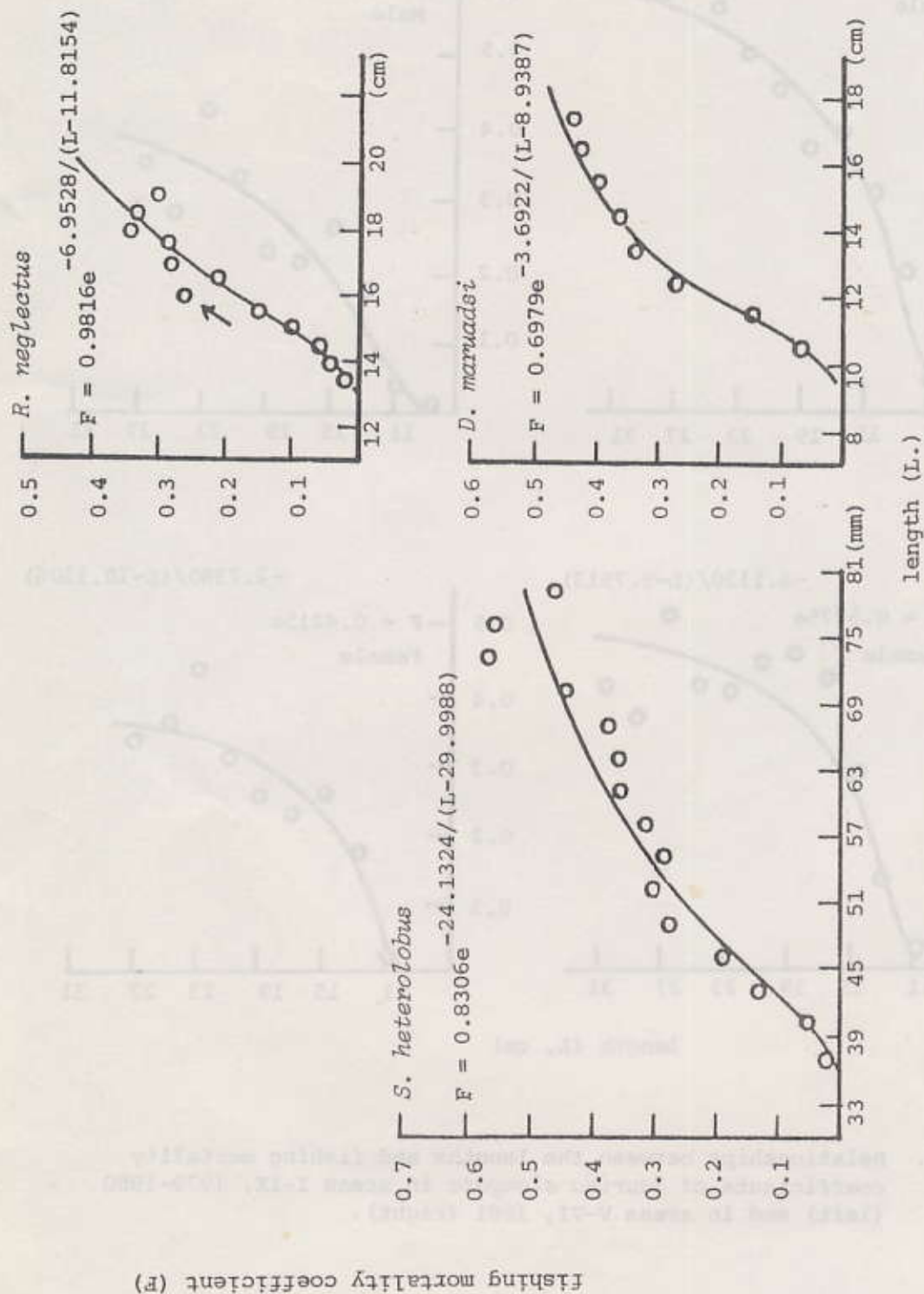


Figure 2. Relationships between the lengths and the fishing mortality coefficients of *Stolephorus heterolobus* in the East and Inner Gulf, 1970-1972 (left); of *Rastrelliger neglectus* in the West of the Gulf, 1962-1967 (upper right) and of *Decapterus maruadisi* in the Middle part of the Gulf, 1975-1979 (lower right).



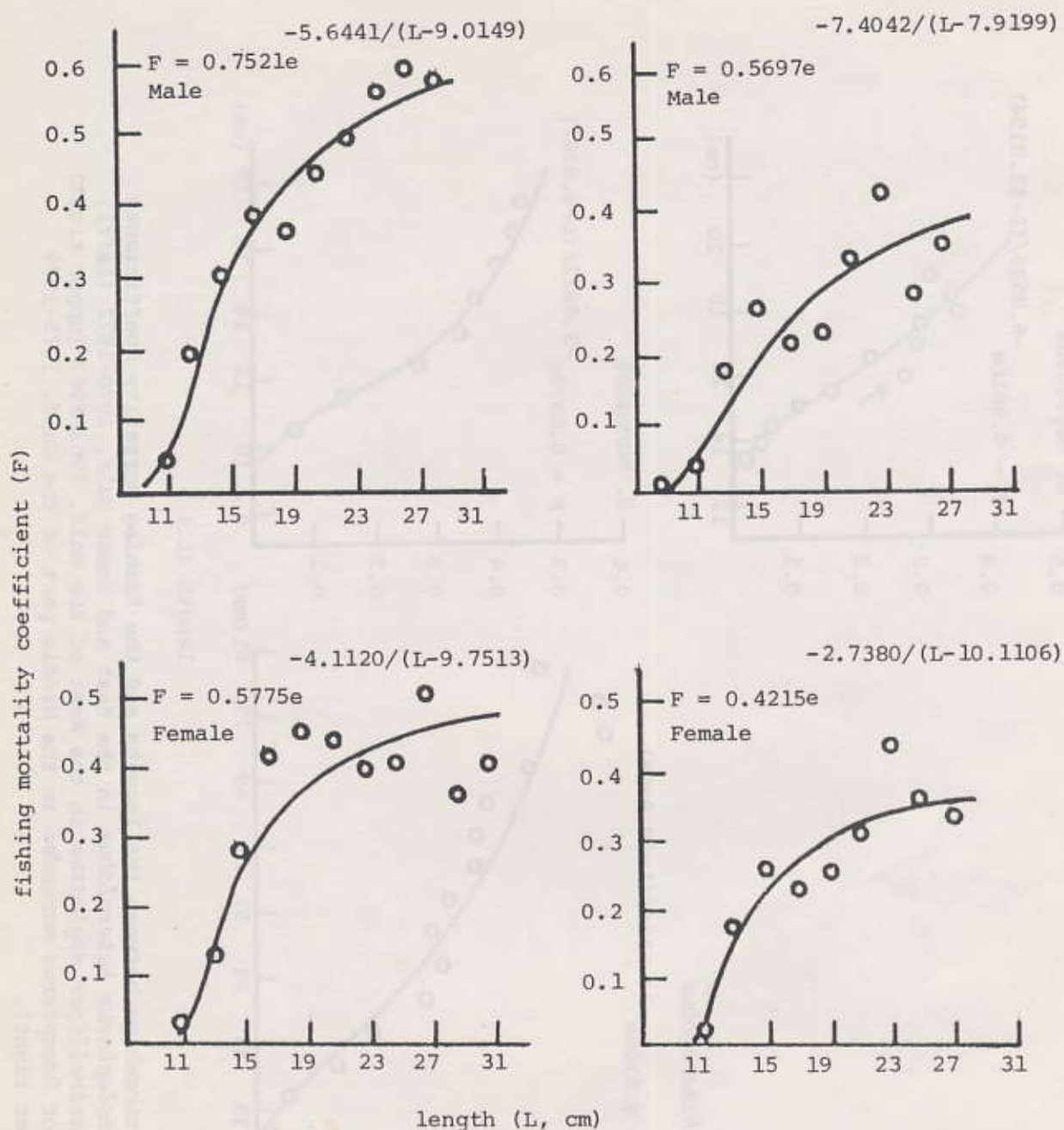


Figure 3. Relationships between the lengths and fishing mortality coefficients of *Saurida elongata* in areas I-IX, 1979-1980 (left) and in areas V-VI, 1981 (right).

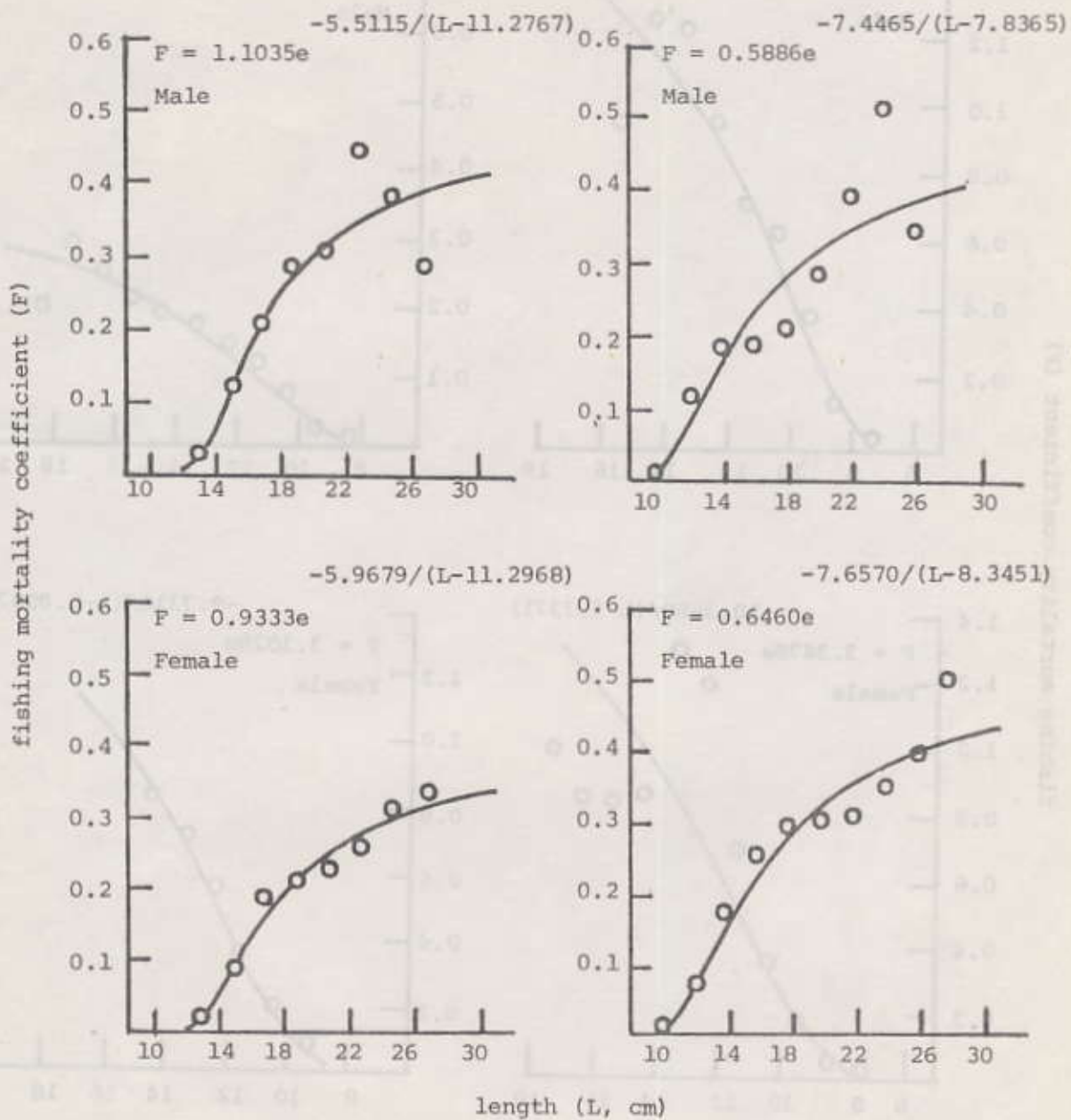


Figure 4. Relationships between the lengths and the fishing mortality coefficients of *Saurida undosquamis* in area III, 1968-1972 (left) and in areas I-IX, 1979-1980 (right).



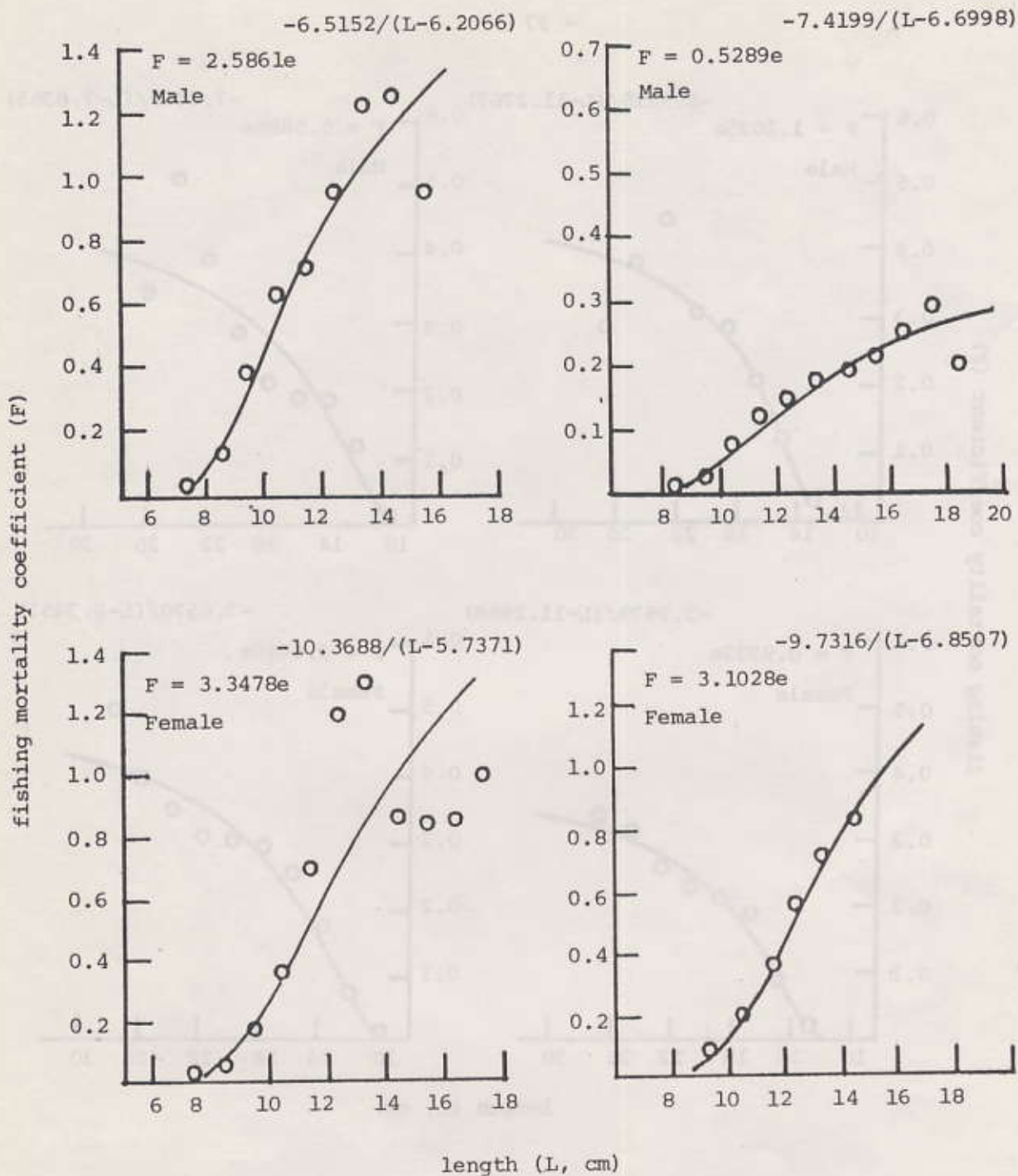


Figure 5. Relationships between the lengths and the fishing mortality coefficients of *Nemipterus japonicus* in areas V-VI, 1981 (left) and *N. mesoprion* in areas I-IX, 1978-1979 (right).

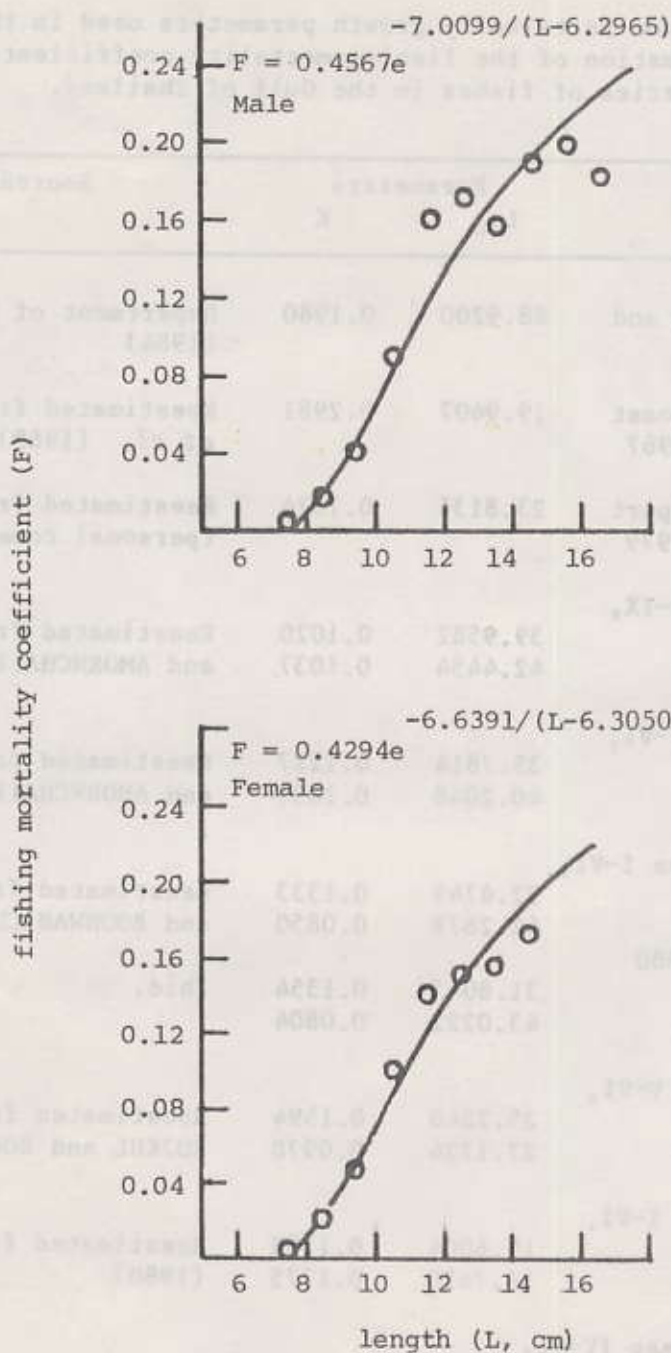


Figure 6 Relationships between the lengths and the fishing mortality coefficients of *Nemipterus nematophorus* in areas I-VI, 1978-1979.



Table 1. The von Bertalanffy growth parameters used in the estimation of the fishing mortality coefficients of 8 species of fishes in the Gulf of Thailand.

Species	Parameters		Sources*
	$L_{\infty}$	K	
<i>S. heterolobus</i> East and Inner Gulf	88.9200	0.1980	Department of Fisheries (1984)
<i>R. neglectus</i> West coast of the Gulf, 1962-1967	19.9407	0.2981	Reestimated from SUCONDHMARN <i>et al.</i> (1968)
<i>D. maruadsi</i> Middle part of the Gulf, 1975-1979	23.8134	0.1374	Reestimated from NUCHMORN (personal communication)
<i>S. elongata</i> areas I-IX, 1979-1980	male	39.9582	Reestimated from BOONWANICH and AMORNCHAIROJKUL (1982a)
	female	42.4454	
<i>S. elongata</i> areas V-VI, 1981	male	35.7814	Reestimated from BOONWANICH and AMORNCHAIROJKUL (1982b)
	female	40.2048	
<i>S. undosquamis</i> areas I-VI, 1979-1980	male	32.4749	Reestimated from MEEMESKUL and BOONWANICH (1982)
	female	42.2878	
	male	31.8043	ibid.
	female	43.0222	
<i>N. japonicus</i> areas V-VI, 1981	male	25.2240	Reestimated from AMORNCHAIROJKUL and BOONWANICH (1982)
	female	27.1724	
<i>N. mesoprion</i> areas I-VI, 1978-1979	male	19.6004	Reestimated from BOONWANICH (1980)
	female	16.7620	
<i>N. nematophorus</i> areas IV-VI, 1978-1979	male	19.0499	Reestimated from BOONWANICH (1981)
	female	15.8611	

\* Reestimated using the Gauss-Newton method.

Table 2. Summarized results of the estimated relationships between the lengths and the fishing mortality coefficients of three species of pelagic fishes in the Gulf of Thailand.

	a	b	$L_0$	$R^2\%$	$s^2$	V(F)
<i>S. heterolobus</i> (1)	0.8306	24.1324	29.9988	97.6844	0.02132984	0.78954895
Variance-covariance matrix	a b $L_0$	0.01191374 0.44912749 19.17462661	-0.10336068 -4.63945177 1.19083270			
Variability index	0.13141342	0.18145244	0.03637660			
t-value	7.610	5.511	27.490			
t-critical (n=15)	4.318	4.318	4.318			
P <	0.001	0.001	0.001			
<i>R. neglectus</i> (2)	0.9816	6.9528	11.8154	96.7304	0.02949066	0.73798015
Variance-covariance matrix	a b $L_0$	0.08485397 0.56157170 3.93007095	-0.10791750 -0.78006897 0.16044488			
Variability index	0.29676449	0.28513032	0.03390128			
t-value	3.370	3.507	29.497			
t-critical (n=12)	3.250	3.250	4.781			
P <	0.010	0.010	0.010			
<i>D. maculatus</i> (3)	0.6979	3.6922	8.9397	99.0398	0.00656677	0.48849096
Variance-covariance matrix	a b $L_0$	0.00797584 0.07556800 0.77813041	-0.02538583 -0.27116498 0.09828561			
Variability index	0.12797205	0.23891572	0.03506907			
t-value	7.814	4.186	28.515			
t-critical (n=8)	6.859	4.032	6.859			
P <	0.001	0.010	0.001			

(1) East and Inner Gulf, 1970-1975.

(2) West of the Gulf, 1962-1967

(3) Middle part of the Gulf, 1975-1979.



Table 3. Summarized results of the estimated relationships between the lengths and the fishing mortality coefficients of five species of demersal fishes in the Gulf of Thailand.

	a	b	$L_0$	$R^2$	$s^2$	$V(F)$
<i>S. elongata</i> (1) male	0.7521	5.6441	9.0149	99.3063	0.00637967	0.71526516
Variance-covariance matrix	a b $L_0$	0.00294727 0.04219565 0.71673087	-0.01249960 -0.22275320 0.07292238			
Variability index	0.07218151	0.14999643	0.02995504			
t-value	13.854	6.667	33.383			
t-critical (n=10)	5.405	5.405	5.405			
P <	0.001	0.001	0.001			
<i>S. elongata</i> (1) female	0.5775	4.1120	9.7513	96.6622	0.03641469	0.87277756
Variance-covariance matrix	a b $L_0$	0.00552933 0.08165002 1.57481452	-0.02154770 -0.43627341 0.12684931			
Variability index	0.12875820	0.30518753	0.03652444			
t-value	7.767	3.277	27.379			
t-critical (n=11)	5.041	2.752	5.041			
P <	0.001	0.025	0.001			
<i>S. elongata</i> (2) male	0.5697	7.4042	7.9199	95.2154	0.06395483	1.03964127
Variance-covariance matrix	a b $L_0$	0.01046357 0.14044272 2.45857917	-0.01782756 -0.32641533 0.04517074			
Variability index	0.17955202	0.21177060	0.02683546			
t-value	5.569	4.722	37.264			
t-critical (n=10)	5.405	4.029	5.405			
P <	0.001	0.005	0.001			
<i>S. elongata</i> (2) female	0.4215	2.7380	10.1106	97.5429	0.02804852	0.85614823
Variance-covariance matrix	a b $L_0$	0.00279084 0.04442487 0.92287087	-0.01262614 -0.27306584 0.08352546			
Variability index	0.12532558	0.35086883	0.02858451			
t-value	7.979	2.850	34.984			
t-critical (n=9)	5.959	2.477	5.959			
P <	0.001	0.050	0.001			

(1) Areas I-IX, 1979-1980.

(2) Areas V-VI, 1981.

Table 3. Summarized results of the estimated relationships between the lengths and the fishing mortality coefficients of five species of demersal fishes in the Gulf of Thailand (continued).

	a	b	$L_0$	$R^2\%$	$s^2$	V(F)
<i>S. undosquamis</i> (3) male	1.1035	5.5115	11.2767	97.1934	0.03184004	0.81033969
Variance-covariance matrix	a	0.04638424	0.38275490	-0.09926772		
	b		3.66446000	-0.99098988		
	$L_0$		0.27953891			
Variability index	0.19516918	0.34732389	0.04688554			
t-value	5.124	2.879	21.329			
t-critical (n=8)	4.773	2.571	6.859			
P <	0.005	0.050	0.001			
<i>S. undosquamis</i> (3) female	0.9333	5.9679	11.2969	99.1524	0.00999829	0.84261542
Variance-covariance matrix	a	0.01029975	0.09980082	-0.02364456		
	b		1.12360833	-0.27758047		
	$L_0$		0.07152160			
Variability index	0.10873629	0.17761780	0.02367362			
t-value	9.197	5.630	42.241			
t-critical (n=8)	6.859	4.773	6.859			
P <	0.001	0.005	0.001			
<i>S. undosquamis</i> (4) male	0.5886	7.4465	7.8365	95.0917	0.05566330	0.85055103
Variance-covariance matrix	a	0.02207138	0.40948873	-0.09856245		
	b		8.74059310	-2.19956662		
	$L_0$		0.58053492			
Variability index	0.25241422	0.39702433	0.09722861			
t-value	3.962	2.519	10.285			
t-critical (n=9)	3.707	2.447	5.595			
P <	0.010	0.050	0.001			
<i>S. undosquamis</i> (4) female	0.6460	7.6570	8.3451	98.9496	0.01283180	0.95013301
Variance-covariance matrix	a	0.00369147	0.05517308	-0.01013727		
	b		1.00556117	-0.19384953		
	$L_0$		0.03930312			
Variability index	0.09404988	0.13096216	0.02375658			
t-value	10.633	7.636	42.094			
t-critical (n=10)	5.405	5.405	5.405			
P <	0.001	0.001	0.001			

(3) Inner Gulf, 1968-1972.

(4) Areas I-IX, 1979-1980.



Table 3. Summarized results of the estimated relationships between the lengths and the fishing mortality coefficients of five species of demersal fishes in the Gulf of Thailand (continued).

	a	b	$L_0$	$R^2\%$	$s^2$	V(F)
<i>N. japonicus</i> (5) male	2.5961	6.5152	6.2066	98.2584	0.02178568	0.93817903
Variance-covariance matrix	a	0.20780402	0.48939977	-0.07961077		
	b		1.29702992	-0.22024414		
	$L_0$		0.03918552			
Variability index	0.17559001	0.17480176	0.03189426			
t-value	5.695	5.721	31.354			
t-critical (n=9)	4.317	4.317	5.959			
P <	0.005	0.005	0.001			
<i>N. japonicus</i> (5) female	3.3478	10.3688	5.7371	88.5366	0.14952142	1.04346747
Variance-covariance matrix	a	2.17065386	5.24039250	-0.73638174		
	b		14.10688363	-2.07275694		
	$L_0$		0.32031990			
Variability index	0.44008101	0.36223264	0.09865094			
t-value	2.272	2.761	10.137			
t-critical (n=11)	1.860	2.752	5.041			
P <	0.100	0.025	0.001			
<i>N. mesoprion</i> (6) male	0.5289	17.4199	6.6998	98.0056	0.02100299	0.84246009
Variance-covariance matrix	a	0.00783805	0.12147149	-0.02432484		
	b		2.09394342	-0.43827880		
	$L_0$		0.09643863			
Variability index	0.16739663	0.19502166	0.04635141			
t-value	5.974	5.128	21.574			
t-critical (n=11)	5.041	5.041	5.041			
P <	0.001	0.001	0.001			
<i>N. mesoprion</i> (6) female	3.1028	9.7316	6.8507	99.9428	0.00056941	0.59716918
Variance-covariance matrix	a	0.08819411	0.24431006	-0.04752006		
	b		0.69604442	-0.13796927		
	$L_0$		0.02788627			
Variability index	0.09571352	0.08573026	0.02437599			
t-value	10.448	11.665	41.024			
t-critical (n=6)	7.453	7.453	12.941			
P <	0.005	0.005	0.001			

(5) Areas V-VI, 1981.

(6) Areas I-IX, 1978-1979.

Table 3. Summarized results of the estimated relationships between the lengths and the fishing mortality coefficients of five species of demersal fishes in the Gulf of Thailand (continued).

	a	b	$L_0$	$R^2\%$	$s^2$	V(F)
<i>N. nematophorus</i> (7)	0.4567	7.0099	6.2965	98.2329	0.03504855	0.98513043
male						
Variance-covariance matrix	a	0.00733710	0.09758138	-0.01397014		
	b		1.49828615	-0.22479522		
	$L_0$		0.03551900			
Variability index	0.18757487	0.17461739	0.02993156			
t-value	5.331	5.727	33.410			
t-critical (n=10)	4.029	5.405	5.405			
P <	0.005	0.001	0.001			
<i>N. nematophorus</i> (7)	0.4294	6.6391	6.3050	98.6010	0.01877094	0.95840852
female						
Variance-covariance matrix	a	0.00597980	0.07743514	-0.01130418		
	b		1.11953781	-0.17011744		
	$L_0$		0.02696834			
Variability index	0.18007076	0.15937075	0.02604601			
t-value	5.553	6.275	38.394			
t-critical (n=8)	4.773	4.773	6.859			
P <	0.005	0.005	0.001			

(7) Areas I-VI, 1978-1979.

