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THE BIOSTATISTICS OF FISHERY BIOLOGY: CALCULATION PROCEDURES OF FUNDAMENTAL BIOSTATISTICS

by

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Preface

This textbook was initially compiled as lecture notes on a part of the biostatistics for The Regional Training Course on Fish Population Dynamics held at SEAFDEC/TD in 1990; it was then revised. The textbook was compiled to demonstrate the basic principles of biostatistics and procedures for calculating the fundamentals of fishery biological work. The textbook is composed of two major parts, i.e., basic subjects from section 1 to section 4, and advanced subjects in section 5.

The basic subjects include tables and charts, the calculation procedures for the fundamental features of the groups, statistical tests of hypothesis, the correlation coefficient and a linear regression analysis. The advanced subjects include the procedures for the adoption of types using three growth equations, Logistic, Gompertz and von Bertalanffy, and shows the calculation procedure for each growth equation.

In this textbook, many examples and their solutions are given. However, these examples are hypothetical for easy understanding of statistical equations and the procedures of calculation. As most of the example results are calculated on a 10 digit calculator, slight numerical differences will be noticed when calculated on another digit calculator or when using a computer. A short section on biostatistics, focusing on fishery biological study is also included, however, I advice that for a more detailed study on biostatistics, further information should be obtained from other sources.

June 1991

Hiroyuki YANAGAWA

CONTENTS

1. Data Arrangement

This section shows some examples of statistical tables and charts, index numbers and the moving average method for data arrangement.

1.1 *Statistical Tables and Charts*

There are two major kinds of statistical tables defined by marks, attributes and variates. The attribute statistical table, Table 1, expresses income by sector, and shows the quality structure of the data. Table 2 shows variate statistical tables, which are expressed in body length by number of fish, and thereby show the quantity structure of the data.

Table 1. Income by sector. (US$)

Sector	1987	1988	1989
Marine capture fisheries	1,000	1,150	1,250
Freshwater fisheries	500	650	780
Aquaculture	300	450	520
Freshwater culture	100	120	150

Table 2. Body length frequency.

Body length in mm	Number of fish
10 - 20	35
20 - 30	98
30 - 40	140
40 - 50	108
50 - 60	72
60 - 70	25

From various statistical tables, many kinds of charts (diagrams) can be constructed i.e., bar graphs (column charts), pie charts, line charts etc. Four different interpretations of statistical charts obtained from Table 3 are shown in Figs. 1 to 4.

Table 3. Marine fishery production in ASEAN countries from 1981 to 1985.

(MT)

Country	1981	1982	1983	1984	1985
Brunei	3,455	3,462	4,964	5,165	5,480
Indonesia	1,404,276	1,490,300	1,682,019	1,712,804	1,821,725
Malaysia	686,446	627,001	686,463	600,473	574,354
Philippines	1,204,757	1,234,289	1,290,304	1,303,310	1,297,119
Singapore	15,620	18,830	19,099	25,042	22,761
Thailand	1,756,939	1,949,681	2,055,225	1,911,485	1,997,165

COLUMN CHART

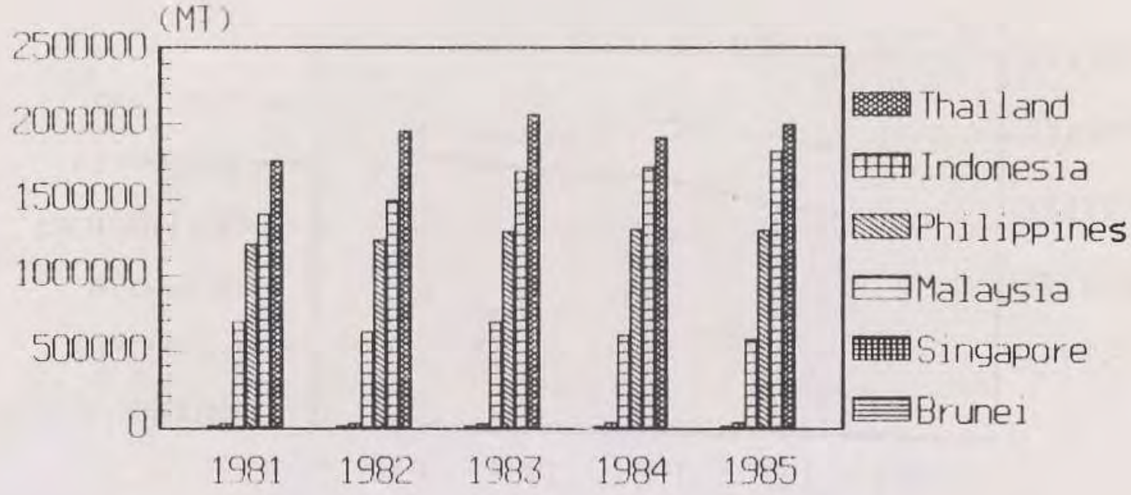

Fig. 1. Marine fishery production in ASEAN countries from 1981 to 1985.

STACKED COLUMN CHART

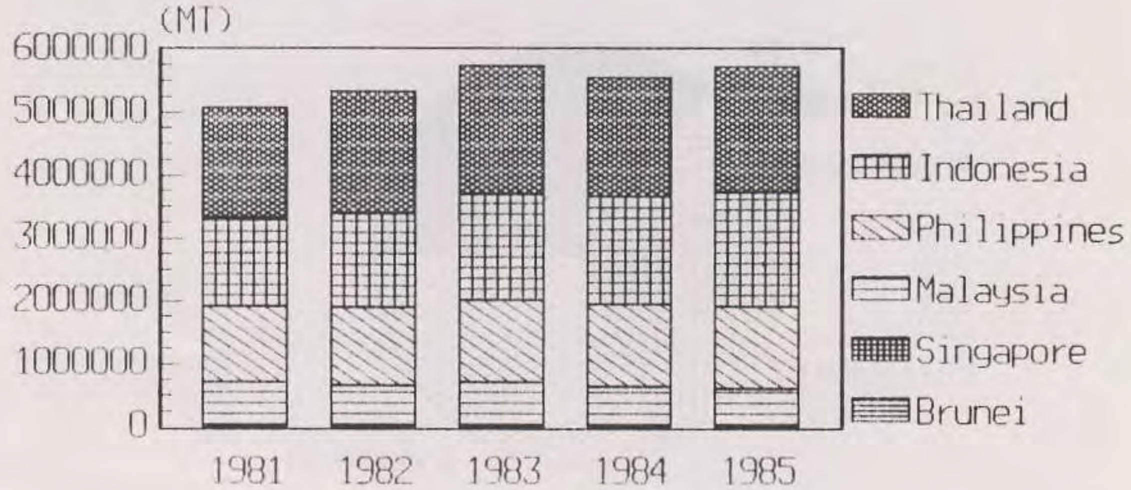

Fig. 2. Marine fishery production in ASEAN countries from 1981 to 1985.

LINE CHART

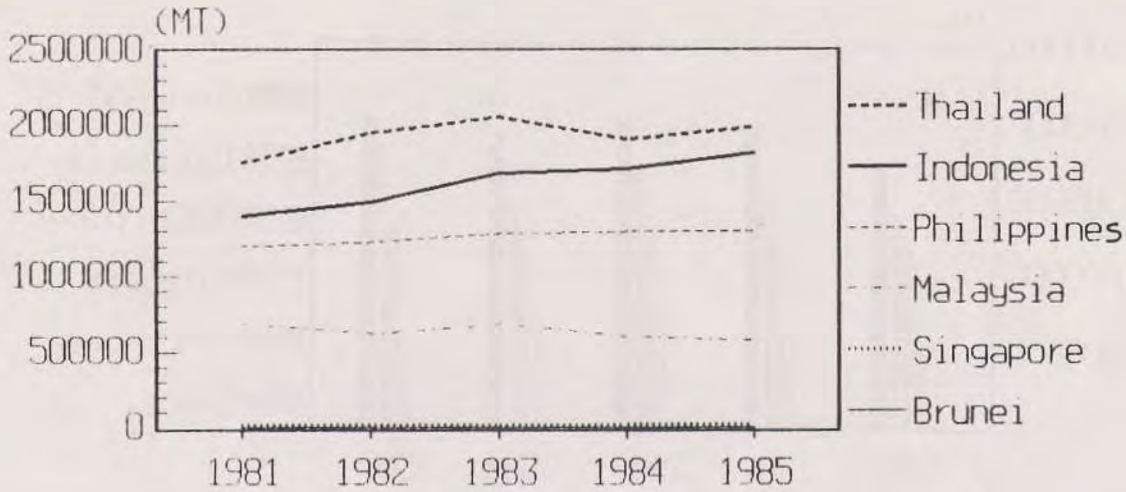

Fig. 3. Marine fishery production in ASEAN countries from 1981 to 1985.

PIE CHART

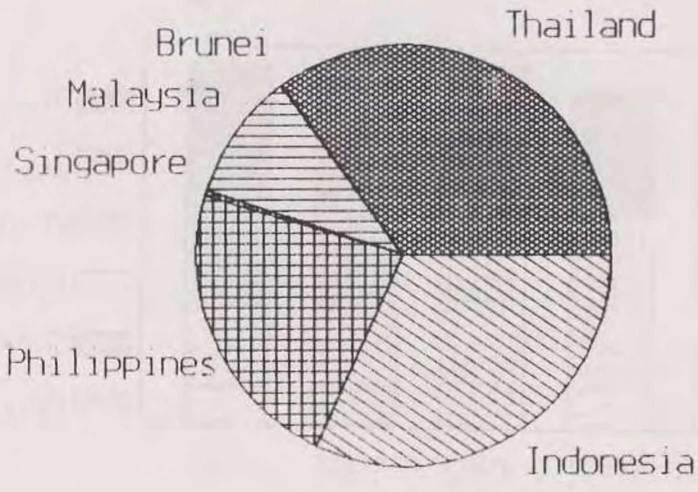

Fig. 4. Marine fishery production in ASEAN countries in 1985.

1.2 *Logarithmic Charts*

There are two kinds of logarithmic charts: the first is the whole logarithmic chart, which is described by logarithmic measurements on both vertical and horizontal axes; the second is a semi-logarithmic chart which describes logarithmic measurements on the vertical axis only.

Figs. 5 and 6 show the relationship between the body length and body weight of certain fish species on an ordinary measurement chart and a logarithmic chart. This length-weight relationship is expressed as a parabola in Fig 5. However, this curved line can be translated into a straight line using logarithmic calculations (Fig. 6). This is the preferred method for obtaining a detailed examination from the point of view of fishery biology.

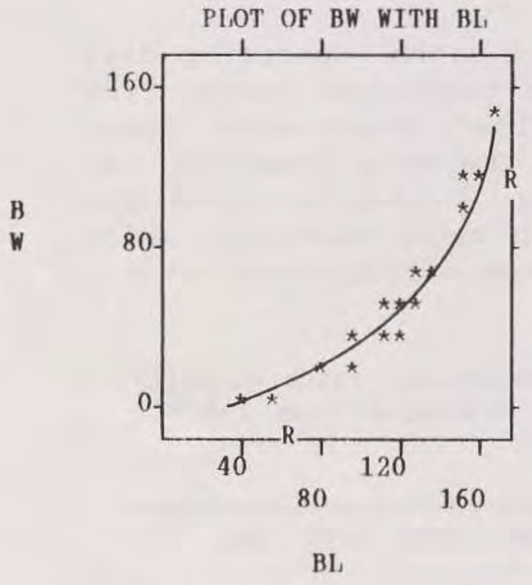

Fig. 5. Relationship between body length and weight in ordinary measurements.

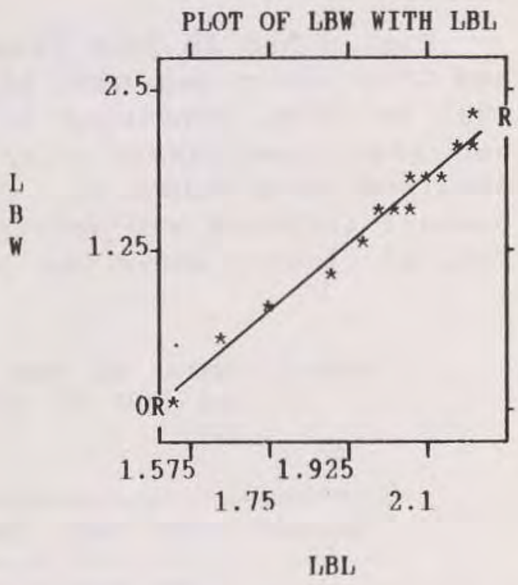

Fig. 6. Relationship between body length and weight in logarithmic measurements.

Figs. 7 and 8 show trends of total catch and index numbers of the catch both by ordinary measurement chart and a semi-logarithmic chart. The data applied was obtained from Table 4. In Fig.7, the trends shown of catch quantity by metric ton and catch quantity by index number vary; the degree of increase in catch quantity by weight appears to be higher than that by index number. However, this phenomenon never occurs in the semi-logarithmic chart (Fig. 8). Therefore, the semi-logarithmic chart is more convenient for a comparison of different unit data.

Table 4. Total catch and index numbers of catch from 1982 to 1989.

Year	1982	1983	1984	1985	1986	1987	1988	1989
Catch (MT)	345	232	447	568	950	960	1,060	1,240
Index (%)	100.0	67.2	129.6	164.6	275.4	278.3	307.2	359.4

Fig. 9 and 10 show trends of catch, operating days and CPUE (catch per unit effort) over eight years, from 1982 to 1989, expressed by ordinary measurement chart and semi-logarithmic chart. The data applied was obtained from Table 5. Fig. 9 clearly shows the overall increase and decrease in catch quantity, while Fig. 10 clearly shows the increase and decrease rates.

Table 5. Number of days of operation, catch quantity and CPUE of certain fisheries from 1982 to 1989.

Variable	1982	1983	1984	1985	1986	1987	1988	1989
No. of days	1,160	1,170	1,260	1,220	1,180	1,480	1,240	1,670
Catch (MT)	170	230	460	1,300	2,400	1,800	2,300	3,100
MT / day	0.15	0.20	0.37	1.07	2.03	1.22	1.85	1.86

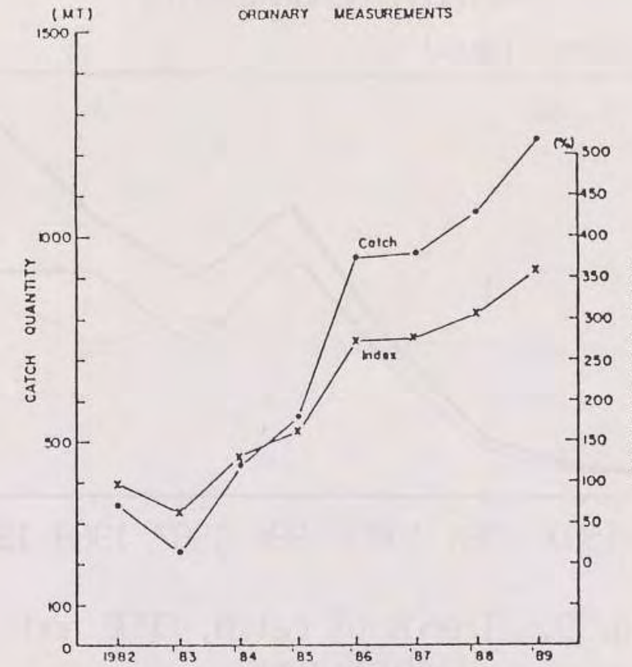

Fig. 7. Trends of catch and aggregate index number in ordinary measurements.

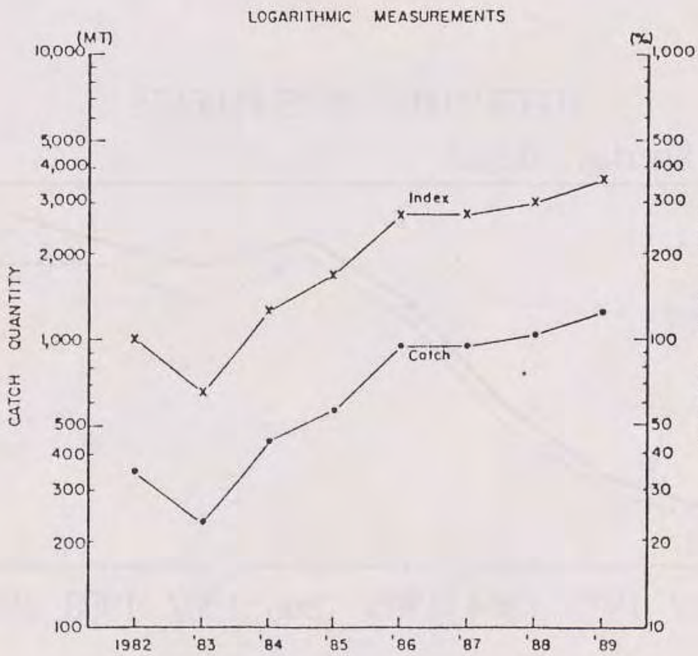

Fig. 8. Trends of catch and aggregate index number in logarithmic measurements.

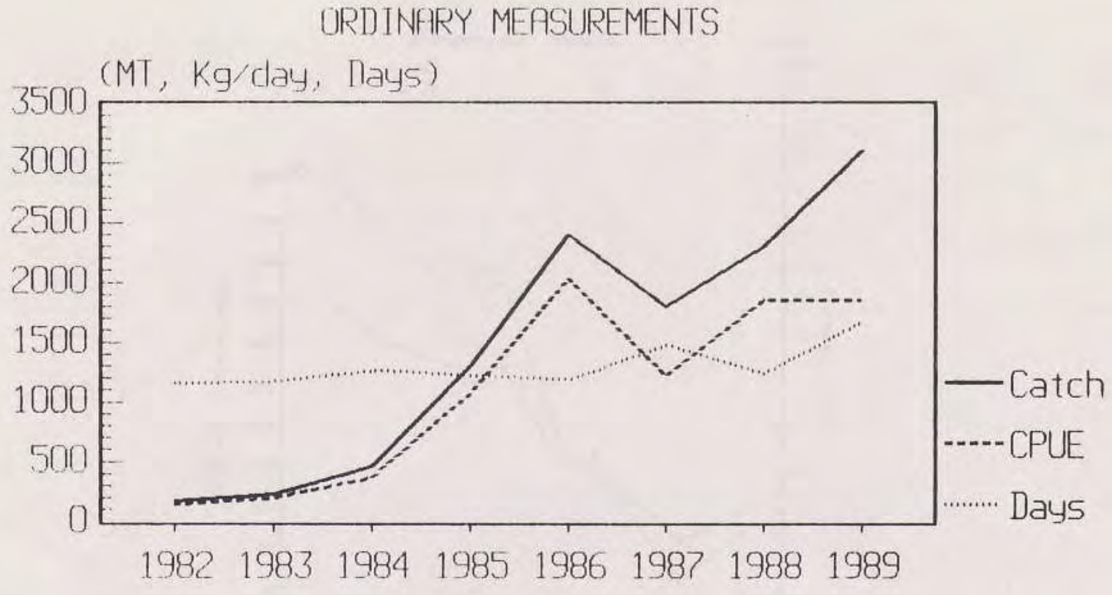

Fig. 9. Trends of catch, CPUE and fishing days.

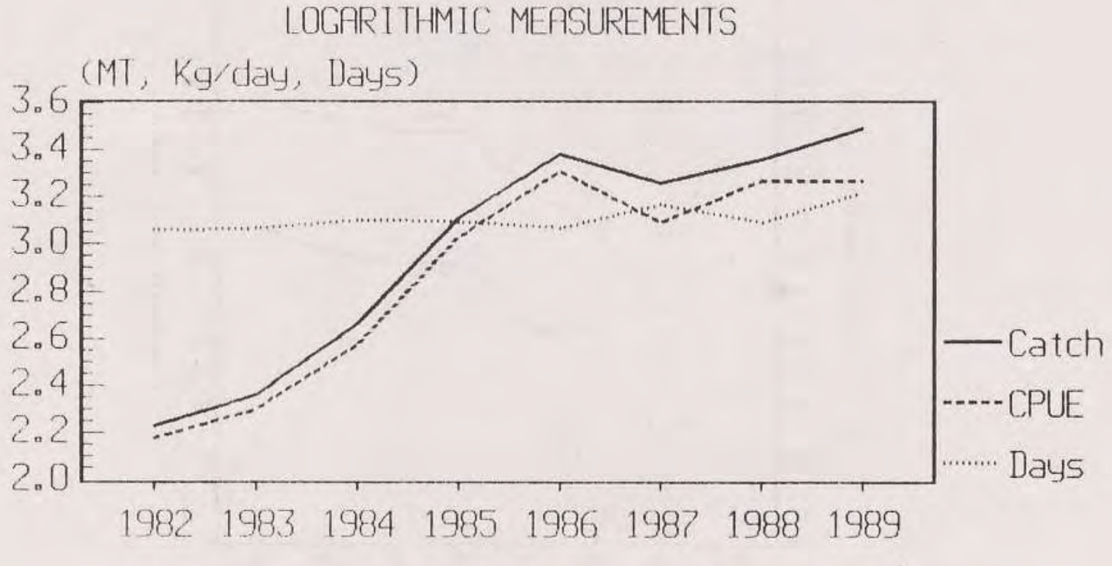

Fig. 10. Trends of catch, CPUE and fishing days.

1.3 *Index Numbers*

Aggregate index numbers and relative index numbers are demonstrated.

Example 1. The next table shows catch quantities of three major fish species caught in certain areas, from 1982 to 1989, and calculates the aggregate index number of the total catch.

Species	1982	1983	1984	1985	1986	1987	1988	1989
A	58	39	41	48	450	300	370	420
B	17	23	46	130	240	180	220	310
C	270	170	360	390	260	480	470	510
Total	345	232	447	568	950	960	1,060	1,240
Aggregate index no.	100.0	67.2	129.6	164.6	275.4	278.3	307.2	359.4

Solution:

1983; Aggregate index number = (232 / 345) * 100
= 67.246-----
= 67.2

1984; Aggregate index number = (447 / 345) * 100
= 129.565-----
= 129.6

|
|
|
1989.

Example 2. From the data obtained in Example 1, calculate the relative index number in this table.

Species	1982	1983	1984	1985	1986	1987	1988	1989
A	100.0	67.2	70.7	82.8	775.9	517.2	637.9	724.1
B	100.0	135.3	270.6	764.7	1411.8	1058.8	1294.1	1823.5
C	100.0	63.0	133.3	144.4	96.3	177.8	174.1	188.9
Relative index No	100.0	88.5	158.2	330.6	761.3	584.6	702.0	912.2

Solution:

1983; A = (39 / 58) * 100 = 67.241----- = 67.2

B = (23 / 17) * 100 = 135.294----- = 135.3

C = (170 / 270) * 100 = 62.962----- = 63.0

Relative index number (arithmetic mean)

= (67.2 + 135.3 + 63.0) / 3 = 88.5

¦

¦

¦

1989

1.4 *Moving Average*

The moving average method and an example chart are demonstrated.

Example 3. The following table shows a time series data of catches during a 20 year period. Calculate the three-years and five-years moving averages.

t	Catch	3-years moving average	5-years moving average
1	1,000		
2	1,500	1,433	
3	1,800	1,633	1,480
4	1,600	1,633	1,680
5	1,500	1,700	1,980
6	2,000	2,167	2,120
7	3,000	2,500	2,500
8	2,500	3,000	3,200
9	3,500	3,667	3,600
10	5,000	4,167	3,500
11	4,000	3,833	3,700
12	2,500	3,333	4,100
13	3,500	3,833	4,460
14	5,500	5,267	5,160
15	6,800	6,600	6,060
16	7,500	7,100	6,960
17	7,000	7,500	7,660
18	8,000	8,000	8,200
19	9,000	8,833	
20	9,500		

<u>*Solution*</u>:

3-years moving average

t = 2; (1,000 + 1,500 + 1,800) / 3 = 1,433.333----- = 1,433
t = 3; (1,500 + 1,800 + 1,600) / 3 = 1,633.333----- = 1,633
¦
t = 19

5-years moving average

```
t =  3;  (1,000 + 1,500 + 1,800 + 1,600 + 1,500) / 5 = 1,480
t =  4;  (1,500 + 1,800 + 1,600 + 1,500 + 2,000) / 5 = 1,680
     |
     |
     |
t = 18
```

Annual catch, three-years moving average and five-years moving average in time series from 1 to 20 are shown in Fig. 11.

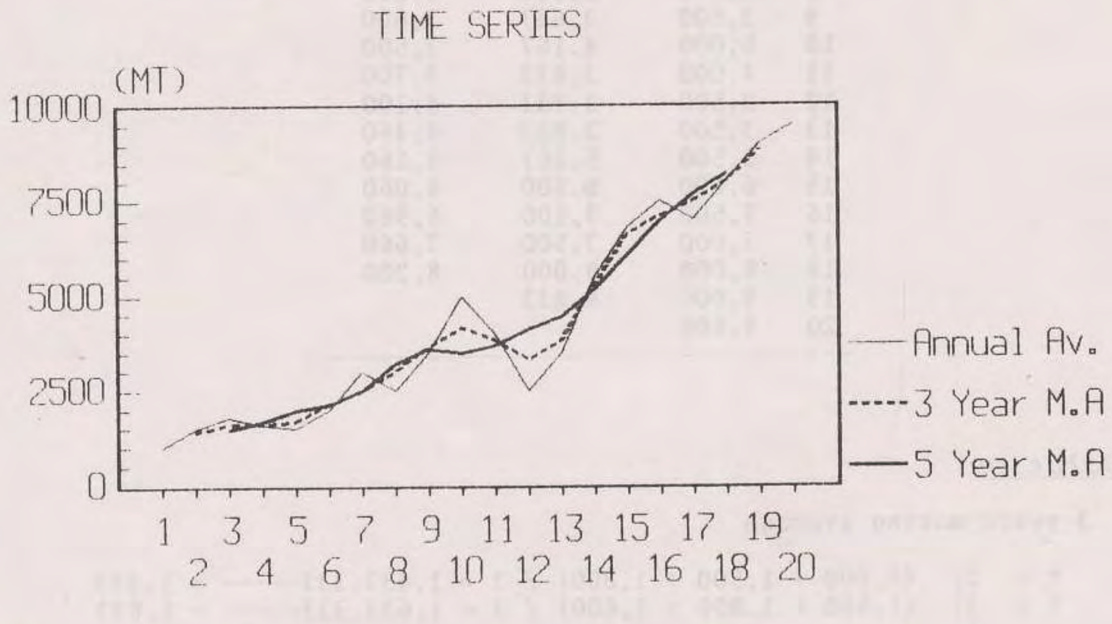

Fig. 11. Annual catch, 3 year moving average and 5 year moving average.

2. Features of Groups

This section shows the simple equations for calculating the features of the groups, i.e., mean, variance, standard deviation and confidence interval. Some examples of the methods used are shown.

2.1 *Mean*

Estimations of three types of mean are demonstrated.

Simple arithmetic mean (calculations based on the attribute statistical table).

$$\overline{X} = \frac{\sum_{i=1}^{N} X_i}{N} \qquad (1.1)$$

Simple arithmetic mean (calculations based on the variable statistical table).

$$\overline{X} = \frac{\sum_{i=1}^{N} f_i X_i}{\sum_{i=1}^{N} f_i} \qquad (1.2)$$

Weighted arithmetic mean.

$$\overline{X}_W = \frac{\Sigma_W X}{\Sigma_W} \qquad (1.3)$$

Example 4. The following table shows numbers of ovarian eggs from fish species A. Calculate the mean ovarian egg number.

Specimen	1	2	3	4	5
No. of eggs	1,800	1,650	1,960	2,050	1,760

Solution:

from (1.1): $$\bar{X} = \frac{1,800 + 1,650 + 1,960 + 2,050 + 1,760}{5}$$

$$= \frac{9,220}{5} = 1,844$$

Mean ovarian egg number is estimated as 1,844.

Example 5. The following table shows total length range of fish in species A. Calculate the mean total length.

Total length	No. of specimens
10 - 15	10
15 - 20	12
20 - 25	18
25 - 30	14
30 - 35	8

Solution:

Calculate items in the table as follows;

Total length	Intermediate of interval (X)	No. of specimens (f)	fX
10 - 15	12.5	10	125
15 - 20	17.5	12	210
20 - 25	22.5	18	405
25 - 30	27.5	14	385
30 - 35	32.5	8	260
Total	-	62	1,385

from (1.2): $\bar{X} = \frac{1,385}{62}$

$= 22.338---$

$= 22.3$

Mean total length of juveniles is estimated as 20.7 mm.

Example 6. **The following table shows total and mean numbers of cultured prawns harvested in fishing villages A, B, C and D. Calculate the weighted mean of cultured prawns harvested in all fishing villages.**

Fishing village	Total number of cultured prawns	Mean of cultured prawns
A	2,200	110
B	7,800	520
C	5,120	320
D	3,360	420

Solution:

from (1.3):

$$\bar{X}_W = \frac{(2,200/110)*110 + (7,800/520)*520 + (5,120/320)*320 + (3,360/420)*420}{(2,200/110) + (7,800/520) + (5,120/320) + (3,360/420)}$$

$$= \frac{18,480}{59} = 313.220\text{---}$$

$$= 313.2$$

The weighted mean of cultured prawns from all fishing villages is estimated as 313.2 prawns.

If calculated as a simple arithmetic mean it is as follows:

$$\bar{X} = \frac{110 + 520 + 320 + 420}{4}$$

$$= \frac{1,370}{4} = 342.5 \text{ (prawns)}$$

but, this is not a reasonable outcome because there is no consideration of case numbers.

2.2 *Variance and Standard Deviation*

Estimations of variance and standard deviation both in population and in sample are demonstrated.

Variance:
(Population)

$$\sigma^2 = \frac{\Sigma (X_i - \bar{X})^2}{N} \tag{1.4}$$

Standard deviation:
(Population)

$$\sigma = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{N}} \tag{1.5}$$

Variance:
(Sample)

$$S^2 = \frac{\Sigma (X_i - \bar{X})^2}{N-1} \tag{1.6}$$

Standard deviation:
(Sample)

$$S = \sqrt{\frac{\Sigma (X_i - \bar{X})^2}{N-1}} \tag{1.7}$$

Example 7. Calculate mean, variance and standard deviation from the following table. Variable A is a population.

Variable	a	b	c	d
I	20	25	18	21

Solution:

from (1.1): $$\bar{X} = \frac{20 + 25 + 18 + 21}{4} = \frac{84}{4} = 21$$

from (1.4): Variance (Population)

$$= \frac{(20-21)^2+(25-21)^2+(18-21)^2+(21-21)^2}{4}$$

$$= \frac{1 + 16 + 9 + 0}{4} = \frac{26}{4} = 6.5$$

from (1.5): Standard deviation (Population)

$$= \sqrt{6.5} = 2.549\text{-----} = 2.5$$

Example 8. The following table shows the number of cultured prawns obtained from random sampling at A and B fishing villages. Calculate the mean, variance and standard deviation.

Fishing village	Farm 1	Farm 2	Farm 3	Farm 4	Farm 5
A	1,880	2,200	1,900	1,650	2,150
B	1,620	1,850	1,920	1,720	1,950

Solution:

Fishing village A

-Mean-

from (1.1): $$\bar{X} = \frac{1{,}880 + 2{,}200 + 1{,}900 + 1{,}650 + 2{,}150}{5}$$

$$= \frac{9{,}780}{5} = 1{,}956$$

-Variance-

from (1.6): $$s^2 = \frac{(1{,}880-1{,}956)^2+(2{,}200-1{,}956)^2+(1{,}900-1{,}956)^2+(1{,}650-1{,}956)^2+(2{,}150-1{,}956)^2}{5-1}$$

$$= \frac{5{,}776 + 59{,}536 + 3{,}136 + 93{,}636 + 37{,}636}{4}$$

$$= \frac{199{,}720}{4} = 49{,}930$$

-Standard deviation-

from (1.7):

$$S = \sqrt{49,930} = 223.450\text{-----} = 223.5$$

Fishing village B

-Mean-

from (1.1):
$$\bar{X} = \frac{1,660 + 1,850 + 1,920 + 1,720 + 1,950}{5}$$

$$= \frac{9,060}{5} = 1,812$$

-Variance-

from (1.6):
$$S^2 = \frac{(1,620-1,812)^2+(1,850-1,812)^2+(1,920-1,812)^2+(1,720-1,812)^2+(1,950-1,812)^2}{5 - 1}$$

$$= \frac{36,864 + 1,444 + 11,664 + 8,464 + 19,044}{4}$$

$$= \frac{77,480}{4} = 19,370$$

-Standard deviation-

from (1.7):

$$S = \sqrt{17,370} = 139.176\text{-----} = 139.2$$

2.3 *Confidence Interval*

An estimation of confidence interval by t distribution is demonstrated.

$$\bar{X} - t_{\alpha} \frac{s}{\sqrt{n}} \leq \mu \leq \bar{X} + t_{\alpha} \frac{s}{\sqrt{n}} \qquad (1.8)$$

Example 9. Estimate to a 95% confidence interval the population mean using t distribution.

The sample mean $\bar{X} = 41.9756$, standard deviation $s = 1.0837$ and $N = 41$ ($df = 41 - 1 = 40$) are given.

<u>*Solution*</u>:

$t_{0.05}(40) = 2.021$ --- obtained from the following table.

[t distribution table]

$df \setminus \alpha$	.2	.1	.05	.02	.01
29	1.311	1.699	2.045	2.462	2.756
30	1.310	1.697	2.042	2.457	2.750
40	1.303	1.684	<u>*2.021*</u>	2.423	2.704
60	1.296	1.671	2.000	2.390	2.660
120	1.289	1.658	1.980	2.358	2.617

from (1.8):

$$41.9756 - 2.021 \left(\frac{1.0837}{\sqrt{41}} \right) \leq \mu \leq 41.9756 + 2.021 \left(\frac{1.0837}{\sqrt{41}} \right)$$

$$41.9756 - 0.3420 \leq \mu \leq 41.9756 + 0.3420$$

$$41.6336 \leq \mu \leq 42.3176$$

$$41.6336 \leq 41.9756 \leq 42.3176$$

Example 10. Estimate to a 95% confidence interval the mean at fishing village A from the results of Example 8, using *t* distribution.

Solution:

Fishing village A

$\alpha = 0.05$, $df = 5 - 1 = 4$
$t_{0.05}(4) = 2.776$ --- obtained from the following table.

[*t* distribution table]

$df \setminus \alpha$	.2	.1	.05	.02	.01
4	1.533	2.132	***2.776***	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	2.365	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355

$$1{,}956 - 2.776 \left(\frac{223.5}{\sqrt{5}} \right) \le \mu \le 1{,}956 + 2.776 \left(\frac{223.5}{\sqrt{5}} \right)$$

$$1{,}956 - 277.5 \le \mu \le 1{,}956 + 277.5$$

$$1{,}678.5 \le \mu \le 2{,}233.5$$

$$1{,}678.5 \le 1{,}956.0 \le 2{,}233.5$$

3. Difference between Groups

This section shows a hypothetical test between groups. Statistical hypothesis will use *t*-test for mean, *F*-test for variance and Chi square test for frequency in this section.

3.1 *t-test*

The procedure for *t*-test and examples of the test are demonstrated.

The procedure for testing the difference between two population means, using the *t* distribution, is as follows (Equal variance; refer 3.2: *F*-test):

(1) $Ho: \mu_1 = \mu_2$ (there is no difference between the means of two populations.)

(2) Calculate t_o value.

$$t_o = \frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1 + n_2 - 2} * \frac{n_1+n_2}{n_1 n_2}}} \qquad (1.9)$$

where

$$s_1^2 = \frac{\sum_{i=1}^{n_1} (X_i - \bar{X}_1)^2}{n_1 - 1}$$

$$s_2^2 = \frac{\sum_{j=1}^{n_2} (X_j - \bar{X}_2)^2}{n_2 - 1}$$

(3) Calculate degree of freedom

$$df = n_1 + n_2 - 2$$

(4) Decide the significance level

(5) Refer the critical region (t_α value) from t distribution table.

(6) Compare t_o value with t_α value.

$$\text{Decision} \left\{ \begin{array}{l} t_o \geq t_\alpha \text{ --- Reject hypothesis} \\ t_o < t_\alpha \text{ --- Accept hypothesis} \end{array} \right.$$

Example 11. The next table shows the crop of cultured prawns from each farm at A and B fishing villages. Test the hypothesis that the two crops are equal using *t*-test at a 0.05 significance level. (Equal variance - refer to Example 13 - *F*-test)

Fishing village	Farm 1	Farm 2	Farm 3	Farm 4	Farm 5
A (X_1)	836	1,092	998	889	951
B (X_2)	860	980	1,010	888	-

Solution:

Ho: Population mean A (μ_1) = Population mean B (μ_2)

Fishing village A

$$\text{Mean} \quad \bar{X}_1 = 4{,}766 \;/\; 5 = 953.2$$

$$\begin{aligned}\text{Variance} \quad S_1{}^2 &= (836-953.2)^2/4+(1{,}092-953.2)^2/4 \\ &\quad +(998-953.2)^2/4+(889-953.2)^2/4 \\ &\quad +(951-953.2)^2/4 \\ &= 3{,}433.96+4{,}816.36+501.76+1{,}030.41+1.21 \\ &= 9{,}783.7\end{aligned}$$

$$\text{Standard deviation} \quad S_1 = \sqrt{9{,}783.7} = 98.912\text{-----}$$

$$= 98.9$$

Fishing village B

$$\text{Mean} \quad \bar{X}_2 = 3{,}738 \;/\; 4 = 934.5$$

$$\begin{aligned}\text{Variance} \quad S_2{}^2 &= (860-934.5)^2/3+(980-934.5)^2/3 \\ &\quad +(1{,}010-934.5)^2/3+(888-934.5)^2/3 \\ &= 1{,}850.1+690.1+1{,}900.1+720.8 \\ &= 5{,}161.1\end{aligned}$$

$$\text{Standard deviation} \quad S_2 = \sqrt{5{,}161.1} = 71.840\text{-----}$$

$$= 71.8$$

$$t_0 = \frac{|\,953.2 - 934.5\,|}{\sqrt{\dfrac{(5-1)\,9{,}783.7 + (4-1)\,5{,}161.1}{5+4-2} * \dfrac{5+4}{5 * 4}}}$$

$$= \frac{18.7}{\sqrt{\dfrac{39{,}134.8 + 15{,}483.3}{7} * \dfrac{9}{20}}}$$

$$= \frac{18.7}{\sqrt{7{,}802.6 * 0.45}} = \frac{18.7}{59.3}$$

$= 0.31534$-----

$= 0.315$

$df = 5 + 4 - 2 = 7$

$t_{0.05}(7) = 2.365$ --- obtained from the following table.

[t distribution table]

$df \setminus a$	.2	.1	.05	.02	.01
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	***2.365***	2.998	3.499
8	1.397	1.860	2.306	2.896	3.355

Accept hypothesis ; there is no significant difference between two means at the 5% level.

Example 12. The following table shows the number of vertebrae of clupeoid fish captured in different localities. Test the hypothesis that the vertebral number of the two groups is equal using a 0.05 significance level. (Unequal variance - refer to Example 14 - F-test)

* Unequal variance: Calculate a t_o value and a degree of freedom by using the following formulae.

$$t_o = \frac{|\bar{X}_1 - \bar{X}_2|}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} \tag{1.10}$$

$$df = \left[\frac{c^2}{n_1-1} + \frac{(1-c)^2}{n_2-1} \right]^{-1} \tag{1.11}$$

where

$$c = \frac{s_1^2 / n_1}{s_1^2 / n_1 + s_2^2 / n_2} \tag{1.12}$$

Locality	a	b	c	d	e	f	g	h
A (X_1)	66	69	70	72	72	68	67	-
B (X_2)	63	64	63	65	62	63	63	62

Solution:

$Ho: \mu_1 = \mu_2$

Locality A

Mean $\bar{X}_1 = 69.14$

Variance $S_1^2 = 5.48$

Standard deviation $S_1 = 2.34$

Locality B

Mean $\bar{X}_2 = 63.13$

Variance $S_2^2 = 0.98$

Standard deviation $S_2 = 0.99$

from (1.10): $$t_o = \frac{|69.14 - 63.13|}{\sqrt{\frac{5.48}{7} + \frac{0.98}{8}}}$$

$$= \frac{6.01}{0.95} = 6.3263\text{-----} = 6.33$$

from (1.11): $$c = \frac{5.48 / 7}{(5.48 / 7) + (0.98 / 8)}$$

$$= \frac{0.78}{0.78 + 0.12} = 0.8666\text{-----} = 0.87$$

$$df = \left[\frac{(0.87)^2}{7 - 1} + \frac{(1 - 0.87)^2}{8 - 1} \right]^{-1}$$

$$= (0.1262 + 0.0024)^{-1}$$
$$= (0.1286)^{-1}$$
$$= 7.77604\text{-----} = 7.8$$

$t_{0.05}(8) = 2.306$ --- obtained from

$t_{0.05}(7) = 2.365$ --- the following table.

[*t* distribution table]

df \ *α*	.2	.1	.05	.02	.01
4	1.533	2.132	2.776	3.747	4.604
5	1.476	2.015	2.571	3.365	4.032
6	1.440	1.943	2.447	3.143	3.707
7	1.415	1.895	***2.365***	2.998	3.499
8	1.397	1.860	***2.306***	2.896	3.355

Reject hypothesis; there is a significant difference between means of vertebral number at localities A and B at the 5% level.

3.2 *F-test*

The procedure for *F*-test and examples of the test are demonstrated.

The procedure for the test of equal variance is as follows:

(1) *Ho* : $\sigma_1^2 = \sigma_2^2$ (There is no difference between the variance of two populations.)

(2) Calculate F_o value.

$$F_o = \frac{s_1^2}{s_2^2} \qquad (s_1^2 > s_2^2) \tag{1.12}$$

where

$$s_1^2 = \frac{\sum_{i=1}^{n_1} (X_i - \bar{X}_1)^2}{n_1 - 1}$$

$$s_2^2 = \frac{\sum_{j=1}^{n_2} (X_j - \bar{X}_2)^2}{n_2 - 1}$$

(3) Calculate degree of freedom

$$df: \quad v_1 = n_1 - 1, \quad v_2 = n_2 - 1$$

(4) Decide the significance level

(5) Refer to the critical region (F_{v2}^{v1} (α) value) from *F* distribution table.

(6) Compare F_o value with F_{v2} (α) value.

$F_o \geq F_{v2}^{v1}$ (α) --- Reject hypothesis

Decision (

$F_o < F_{v2}^{v1}$ (α) --- Accept hypothesis

Example 13. Test the hypothesis that, between two groups, the variance of the crops of cultured prawns from Example 11 at *t*-test, is equal to a 0.05 significance level.

Solution:

$Ho: \sigma_1^2 = \sigma_2^2$

from the results of Example 11; $s_1^2 = 9,783.7$

$s_2^2 = 5,161.1$

$$F_o = \frac{s_1^2}{s_2^2} = \frac{9,783.7}{5,161.1} = 1.89566 \text{---} = 1.90$$

F_3^4 (0.05) = 9.12 --- obtained from the following table.

[F distribution table] $\alpha = .05$

$v_2 \setminus v_1$	2	3	4	5	6
3	9.55	9.28	*9.12*	9.01	8.94
4	6.94	6.59	6.39	6.26	6.16
5	5.79	5.41	5.19	5.05	4.95
6	5.14	4.76	4.53	4.39	4.28
7	4.74	4.35	4.12	3.97	3.87

Accept hypothesis; there is no significant difference of variance ratio between two fishing villages at 5% level. (Equal variance)

Example 14. Test the hypothesis that, between two groups, the variance of vertebral number from Example 12 at t-test, are equal, using a 0.05 significance level.

Solution:

$Ho: \sigma_1^2 = \sigma_2^2$

from the results of Example 12; $s_1^2 = 5.48$

$s_2^2 = 0.98$

$$F_0 = \frac{s_1^2}{s_2^2} = \frac{5.48}{0.98} = 5.59183\text{---} = 5.59$$

F_7^6 (0.05) = 3.87 --- obtained from the following table.

[F distribution table] α = .05

$v_2 \setminus v_1$	2	3	4	5	6
3	9.55	9.28	9.12	9.01	8.94
4	6.94	6.59	6.39	6.26	6.16
5	5.79	5.41	5.19	5.05	4.95
6	5.14	4.76	4.53	4.39	4.28
7	4.74	4.35	4.12	3.97	***3.87***

Reject hypothesis; there is a significant difference of variance ratio between two localities at 5% level. (Unequal variance)

3.3 *Chi Square Test*

Procedure for Chi square test and examples of the test are demonstrated.

Procedure for the test of adoption of observed frequency to expected frequency (multinominal probability) is as follows:

(1) $Ho: f_1 = mp_1,\ f_2 = mp_2,\ ---- f_k = mp_k$ (observed frequency is adopted with expected frequency)

(2) Calculate F_i (expected frequency) against f_i (observed frequency)

(3) Calculate X_o^2 value.

$$X_o^2 = \sum_{i=1}^{k} \frac{(f_i - F_i)^2}{F_i} \qquad (1.13)$$

(4) Calculate degree of freedom

$$n = k - 1$$

(5) Decide the significance level

(6) Refer the critical region (X_α^2 value) from X^2 distribution table.

(7) Compare X_o^2 value with X_α^2 value.

Decision ($X_o^2 \geq X_\alpha^2$ --- Reject hypothesis
$X_o^2 < X_\alpha^2$ --- Accept hypothesis

Example 15. The following table shows the sex ratio of species A by age. Test the hypothesis that the number of males and females are equal using X^2 test at an 0.05 significance level.

* *df = 1 or f_i < 5* : Calculate a X_o value by using the following formula (Yate's correction).

$$X_o^2 = \sum_{i=1}^{n} \frac{(|f_i - F_i| - 0.5)^2}{F_i} \qquad (1.14)$$

Age	Male	Female
2	462	484
3	9	25

Solution:

Age 2; *Ho; Male : Female = 1 : 1*

Calculate items in the table as follows;

	Observed frequency (f)	Expected frequency (F)	$\lvert f-F \rvert -0.5$	$(\lvert f-F \rvert -0.5)^2$	$(\lvert f-F \rvert -0.5)^2/F$
Male	462	473	10.5	110.25	0.233
Female	484	473	10.5	110.25	0.233
Total	946	946	-	-	0.466

$X_0{}^2 = 0.466$, $df = 2 - 1 = 1$

$X_{0.05}(1) = 3.841$ --- obtained from the following table.

[X^2 distribution table]

$df \backslash \alpha$	.2	.1	.05	.02	.01
1	1.642	2.706	***3.841***	5.412	6.635
2	3.219	4.605	5.991	7.824	9.210
3	4.642	6.251	7.815	9.837	11.345
4	5.989	7.779	9.488	11.668	13.277
5	7.289	9.236	11.070	13.388	15.086

Accept hypothesis; sex ratio is equal at age 2.

Age 3; *Ho; Male : Female = 1 : 1*

Calculate items in the table as follows;

	Observed frequency (f)	Expected frequency (F)	$\lvert f-F\rvert-0.5$	$(\lvert f-F\rvert-0.5)^2$	$(\lvert f-F\rvert-0.5)^2/F$
Male	9	17	7.5	56.25	3.309
Female	25	17	7.5	56.25	3.309
Total	34	34	-	-	6.618

$X_0{}^2 = 6.618$, $df = 2 - 1 = 1$

$X_{0.05}(1) = 3.841$

Reject hypothesis; sex ratio is not equal at age 3.

Example 16. The next table shows characteristic variations in hybrid species. Test hypothesis that these variations follow Mendel's laws.

Character	A	B	C	D	Total
Observed frequency	152	45	58	29	284

Solution:

$Ho;\ A : B : C : D = 9 : 3 : 3 : 1$

Calculate items in table as follows;

Character	f	p	F	$f-F$	$(f-F)^2/F$
A	152	9/16	159.75	-7.75	0.376
B	45	3/16	53.25	-8.25	1.278
C	58	3/16	53.25	4.75	0.424
D	29	1/16	17.75	11.25	7.130
Total	284	1	284.00	-	9.208

$X_0{}^2 = 9.208,\qquad df = 4 - 1 = 3$

$X_{0.05}(3) = 7.815$ --- obtained from the following table.

[X^2 distribution table]

$df \setminus a$	.2	.1	.05	.02	.01
1	1.642	2.706	3.841	5.412	6.635
2	3.219	4.605	5.991	7.824	9.210
3	4.642	6.251	*7.815*	9.837	11.345
4	5.989	7.779	9.488	11.668	13.277
5	7.289	9.236	11.070	13.388	15.086

Reject hypothesis; Mendel's law cannot be applied to these hybrid samples.

4. Relationship between Characters

This section shows examples of the correlation coefficient between characters, and linear regression analysis.

4.1 *Correlation Coefficient*

An estimation of the correlation coefficient is demonstrated.

The formula for estimation of simple (Pearson's) correlation coefficient is as follows:

$$r = \frac{\Sigma\, x_i y_i}{\sqrt{\Sigma x_i^2\ \Sigma y_i^2}} = \frac{\Sigma\, xy}{\sqrt{\Sigma x^2\ \Sigma y^2}} \qquad (1.15)$$

r values: $0 < r < +1$ ----- Positive correlation

$-1 < r < 0$ ----- Negative correlation

$r = 0$ ----- Zero correlation

(refer Fig. 12)

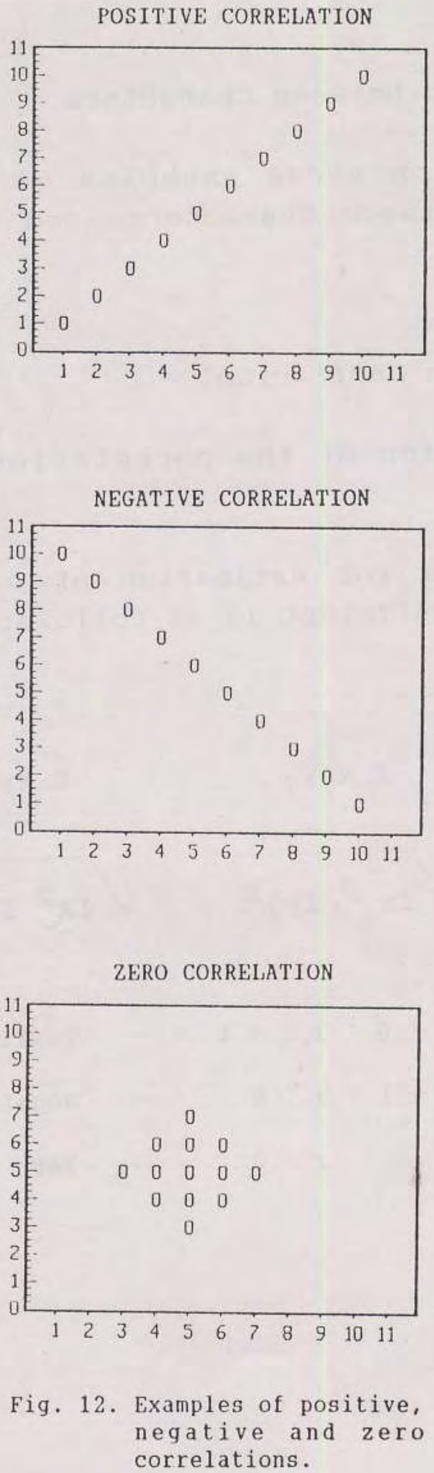

Fig. 12. Examples of positive, negative and zero correlations.

Example 17. The next table shows data on body length and body weight of fish species A. Estimate the correlation coefficient between body length and body weight of this fish.

Body length in mm	Body weight in gram
150.0	108
116.3	39
93.8	21
128.2	62
135.7	70

Solution:

Calculate the table as follows:

	BL(X)	BW(Y)	$x=X-\bar{X}$	$y=Y-\bar{Y}$	x^2	y^2	xy
	150.0	108	25.2	48	635.04	2,304	1,209.6
	116.3	39	-8.5	-21	72.25	441	178.5
	93.8	21	-31.0	-39	961.00	1,521	1,209.0
	128.2	62	3.4	2	11.56	4	6.8
	135.7	70	10.9	10	118.81	100	109.0
Σ	624.0	300	-	-	1,798.66	4,370	2,712.9
Mean	124.8	60	-	-	-	-	-

from (1.15):

$$r = \frac{2,712.9}{\sqrt{(1,798.66) * (4,370)}}$$

$$= \frac{2,712.9}{7,860,144.2}$$

$$= \frac{2,712.9}{2,803.6}$$

$$= 0.96764\text{-----}$$

$$= 0.968$$

The correlation coefficient between body length and body weight is estimated as 0.968.

4.2 *Linear Regression*

An estimation of the linear regression equation is demonstrated.

The formula for estimation of *b* value (slope) in a linear regression line is as follows:

$$b = S_{xy} \,/\, S_{xx} \tag{1.16}$$

where

$$S_{xx} = \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$S_{xy} = \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

A linear regression line is expressed as follows:

$$y = \bar{y} + b\,(x - \bar{x}) \tag{1.17}$$

Example 18. The next table shows data on body length and body weight of fish species A (same data as Example 17). Estimate a linear regression equation for the relationship between body length and body weight of this fish.

Body length in mm	Body weight in gram
150.0	108
116.3	39
93.8	21
128.2	62
135.7	70

Solution:

Estimate b value (slope).

Calculate the following table.

	BL(X)	BW(Y)	X^2	XY
	150.0	108	22,500.00	16,200.0
	116.3	39	13,525.69	4,535.7
	93.8	21	8,798.44	1,969.8
	128.2	62	16,435.24	7,948.4
	135.7	70	18,414.49	9,499.0
Σ	624.0	300	79,673.86	40,152.9
Mean	124.8	60	-	-

$$\text{from (1.16): } b = \frac{\Sigma XY - (\Sigma X)(\Sigma Y) / N}{\Sigma X^2 - (\Sigma X)^2 / N}$$

$$= \frac{40{,}152.9 - (624)(300) / 5}{79{,}673.86 - (624)^2 / 5}$$

$$= \frac{40{,}152.9 - 37{,}440.0}{79{,}673.86 - 77{,}875.20}$$

$$= \frac{2{,}712.9}{1{,}798.66}$$

$$= 1.50828\text{-----}$$

$$= 1.508$$

The b value is estimated as 1.508.

Estimate the linear regression equation.

from (1.17):

$$y = \bar{y} + b\ (x - \bar{x})$$

$$y = 60 + 1.508\ (x - 124.8)$$

$$= 60 + 1.598\ x - 188.1984$$

$$= -\ 128.1984 + 1.508\ x$$

$$= -\ 128.198 + 1.508\ x$$

The linear regression equation adopted for body length and body weight is estimated as $y = -\ 128.198 + 1.508\ x$; the linear regression line is shown in Fig. 13.

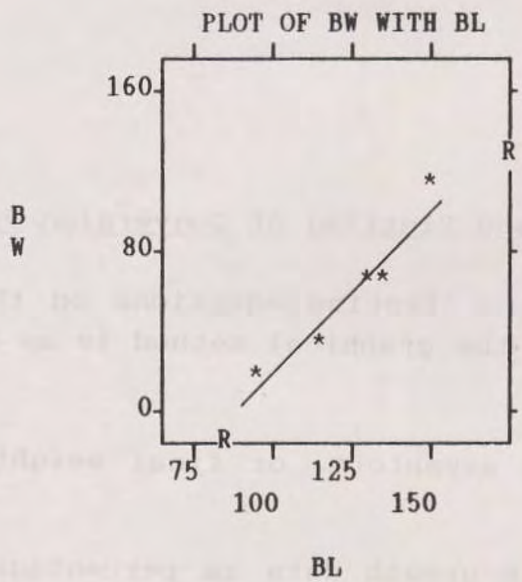

Fig. 13. Relationship between body length and body weight.

5. Growth Equations by Graphical Method

This section shows how to apply the data for suitable growth equations and how to calculate the growth equations which are adopted.

5.1 *Adoption of Types of Growth Equations*

Procedures for the adoption of growth equations, Logistic, Gompertz and von Bertalanffy, by graphical methods for growth in body weight as in Ricklefs (1967), are demonstrated. In the case of growth in body length, however, the two equations of Logistic and Gompertz are used. This is because, the cube equation used in Bertalanffy's equation is of the graphical method, thus this equation is unsuitable for growth in body length.

5.1.1 Calculation and Plotting of Conversion Factors

The procedure for testing equations on the growth curve according to the graphical method is as follows:

(1) Estimating the asymptote, or final weight, of the growth curve.

(2) Recalculate the growth data as percentages of the estimated asymptote.

(3) Plot the conversion factors which correspond to the percentage growth data as a function of time.

(4) If the resulting relationship follows a straight line through at least its lower half, the equation is appropriate; continue with step (5). If the relationship is curved through the whole range of the values the equation will not fit the growth data closely and step (3) must be repeated using a different equation.

(5) If the graph of the conversion factors either rises sharply or falls off in its upper portion, refine the estimate of the asymptote, recalculate the growth data as a percentage of the new asymptote, and graph the new conversion factors.

(6) Measure the slope of the resulting straight line which is directly proportional to the rate constant, K, of the growth equation.

Formulae of three types of conversion factor are as follows;

Logistic conversion factor:

$$C_W = \frac{1}{4} \log_e \left(\frac{W}{1-W}\right) \tag{1.18}$$

Gompertz conversion factor:

$$C_W = \frac{1}{e} \log_e \left(- \frac{1}{\log_e W}\right) \tag{1.19}$$

Bertalanffy conversion factor (adopted only for growth in body weight):

$$C_W = \frac{4}{9} \log_e \left(\frac{1}{3(1-W^{1/3})}\right) \tag{1.20}$$

Example 19. The following table shows the monthly age and weight of juvenile fish species A. Test which of the equations fits best from the three types of growth curves, Logistic, Gompertz or von Bertalanffy.

(gram)

Monthly age	Body weight
1	0.72
2	1.17
3	1.83
4	2.61
5	3.87
6	5.25
7	6.75
8	8.25
9	9.35
10	10.26
11	10.96
12	11.41

Solution:

(1) Estimate an asymptote.

12.0 grams.

(2) Calculate the percentage of asymptote at each month of age.

Monthly age 1; 0.72 / 12.0 = 0.060
2; 1.17 / 12.0 = 0.098
¦
¦
12

(3) Calculate the conversion factor for a Logistic curve.

From (1.18):

Monthly age 1; $C_W = (1/4) * \log_e (W/(1-W))$
$= (1/4) * \log_e (0.060/(1-0.060))$
$= 0.25 * \log_e (0.064)$
$= 0.25 * (-2.749)$
$= -0.687$

Monthly age 2; $C_W = (1/4) * \log_e (0.098/(1-0.098))$
$= 0.25 * \log_e (0.109)$
$= 0.25 * (-2.21)$
$= -0.554$

|
|
12

(4) Calculate the conversion factor on the Gompertz curve.

From (1.19):

Monthly age 1; $C_W = (1/e) * \log_e (-(1/\log_e W))$
$= (1/2.718) * \log_e(-(1/\log_e 0.060))$
$= 0.368 * \log_e (-(1/(-2.813))$
$= 0.368 * \log_e (-(-0.355))$
$= 0.368 * (-1.036)$
$= -0.381$

Monthly age 2; $C_W = (1/e) * \log_e (-(1/\log_e 0.098))$
$= 0.368 * \log_e (-(1/(-2.323))$
$= 0.368 * \log_e (0.430)$
$= -0.311$

|
12

(5) Calculate the conversion factor on the von Bertalanffy curve.

From (1.20):

Monthly age 1; $C_W = (4/9) * \log_e (1/(3(1-W^{1/3})))$
$= (4/9) * \log_e (1/(3(1-0.060^{1/3})))$
$= 0.444 * \log_e (1/(3(1-0.391)))$
$= 0.444 * \log_e (1/1.827)$
$= 0.444 * \log_e (0.547)$
$= 0.444 * (-0.603)$
$= -0.268$

Monthly age 2; $C_W = (4/9) * \log_e (1/(3(1-0.098^{1/3})))$
$= 0.444 * \log_e (1/(3(1-0.461)))$
$= 0.444 * \log_e (0.618)$
$= 0.444 * (-0.481)$
$= -0.214$

¦
12

Conversion factors were calculated on a 10 digit calculator in the following table.

Monthly Age	Weight in gr.	Percentage of asymptote (12.0 gr.)	Logistic conversion factor	Gompertz conversion factor	Bertalanffy conversion factor
1	0.72	0.060	-0.688	-0.381	-0.268
2	1.17	0.098	-0.555	-0.310	-0.214
3	1.83	0.153	-0.428	-0.232	-0.148
4	2.61	0.218	-0.319	-0.155	-0.079
5	3.87	0.323	-0.185	-0.045	+0.027
6	5.25	0.438	-0.062	+0.071	+0.145
7	6.75	0.563	+0.063	+0.204	+0.288
8	8.25	0.688	+0.198	+0.362	+0.465
9	9.35	0.779	+0.315	+0.510	+0.635
10	10.26	0.855	+0.444	+0.682	+0.835
11	10.96	0.913	+0.588	+0.882	+1.072
12	11.41	0.951	+0.741	+1.100	+1.333

(6) Plot the conversion factors (Fig. 14) from the previous table.

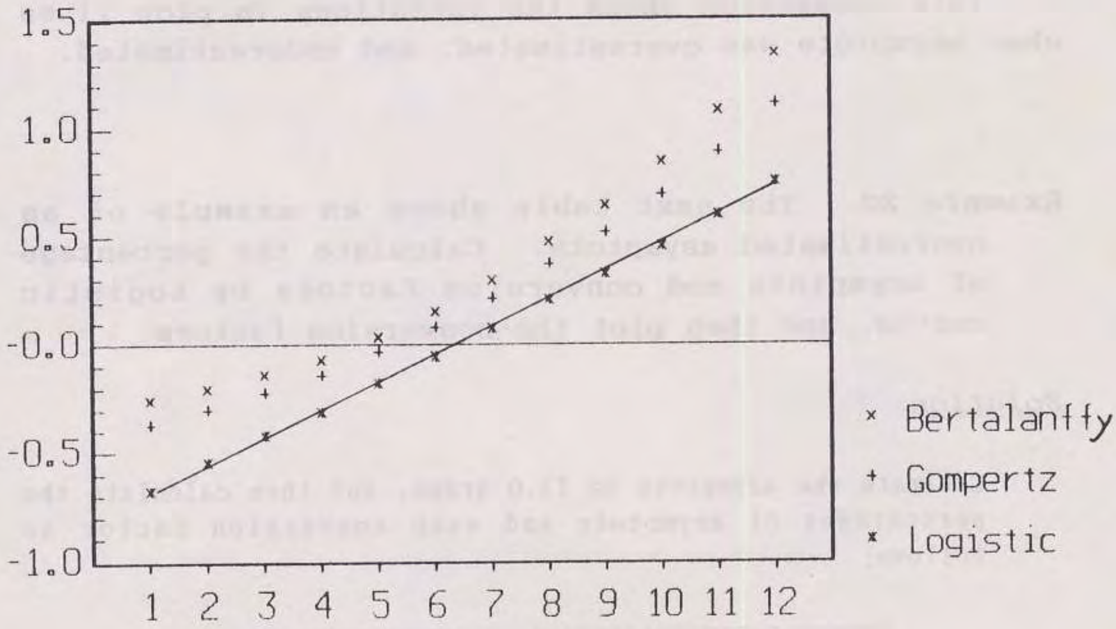

Fig. 14. Plots of conversion factors from three types of growth equations.

From Fig. 14, the resulting relationship of Logistic conversion factors follows a straight line, and therefore can be considered the most fitting growth equation for this example.

5.1.2 Estimation of Asymptote

The Logistic conversion factors plotted from the estimated asymptote (12.0 grams) followed a straight line not only on the lower half of the chart but also upper half (Example 19). Therefore, this estimation of asymptote can be considered as reasonable.

This subsection shows the variations in plot lines when asymptote was overestimated, and underestimated.

Example 20. The next table shows an example of an overestimated asymptote. Calculate the percentage of asymptote and conversion factors by Logistic curves, and then plot the conversion factors.

Solution:

Estimate the asymptote as 13.0 grams, and then calculate the percentages of asymptote and each conversion factor as follows;

Monthly Age	Weight in gr.	Percentage of asymptote (13.0 gr.)	Logistic conversion factor
1	0.72	0.055	-0.711
2	1.17	0.090	-0.578
3	1.83	0.141	-0.452
4	2.61	0.201	-0.345
5	3.87	0.298	-0.214
6	5.25	0.404	-0.097
7	6.75	0.519	+0.019
8	8.25	0.635	+0.138
9	9.35	0.719	+0.235
10	10.26	0.789	+0.330
11	10.96	0.843	+0.420
12	11.41	0.878	+0.493

The Logistic conversion factors obtained from the results are shown in Fig. 15 (overestimation).

Example 21. The next table shows an example of an underestimated asymptote. Calculate percentage of asymptote and conversion factors by Logistic curve, and then plot the conversion factors.

Solution:

Estimate the asymptote as 11.0 grams, and then calculate percentages of asymptote and each conversion factor as follows;

Monthly Age	Weight in gr.	Percentage of asymptote (11.0 gr.)	Logistic conversion factor
1	0.72	0.065	-0.667
2	1.17	0.106	-0.533
3	1.83	0.166	-0.404
4	2.61	0.237	-0.292
5	3.87	0.352	-0.153
6	5.25	0.477	-0.023
7	6.75	0.614	+0.116
8	8.25	0.750	+0.275
9	9.35	0.850	+0.434
10	10.26	0.933	+0.658
11	10.96	0.996	+1.379
12	11.41	-	-

The Logistic conversion factors obtained from the results are shown in Fig. 16 (underestimation).

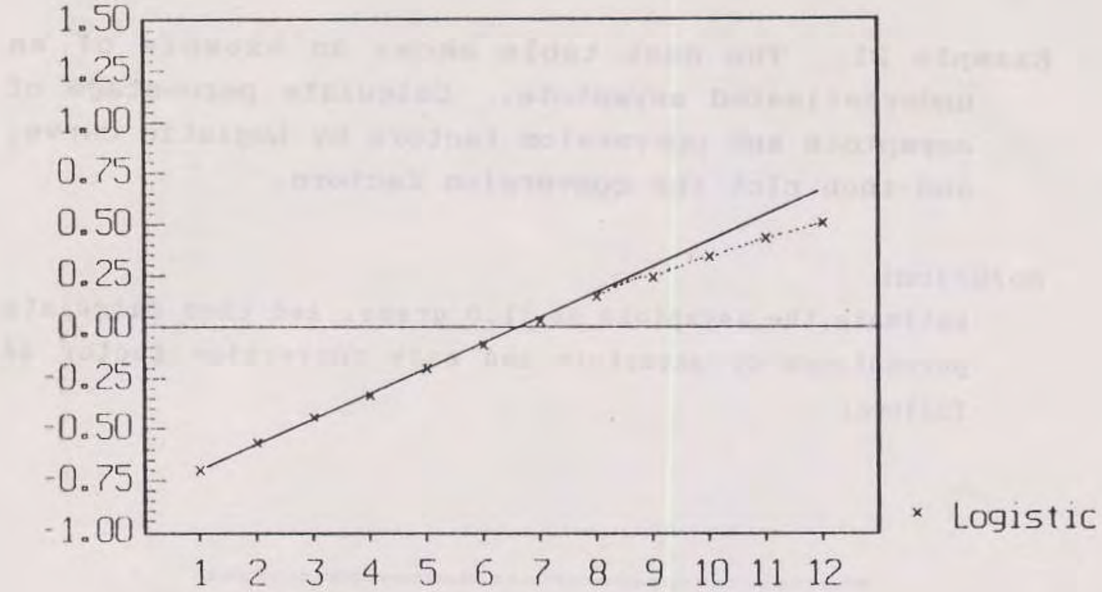

Fig. 15. Plots of Logistic conversion factors at overestimation.

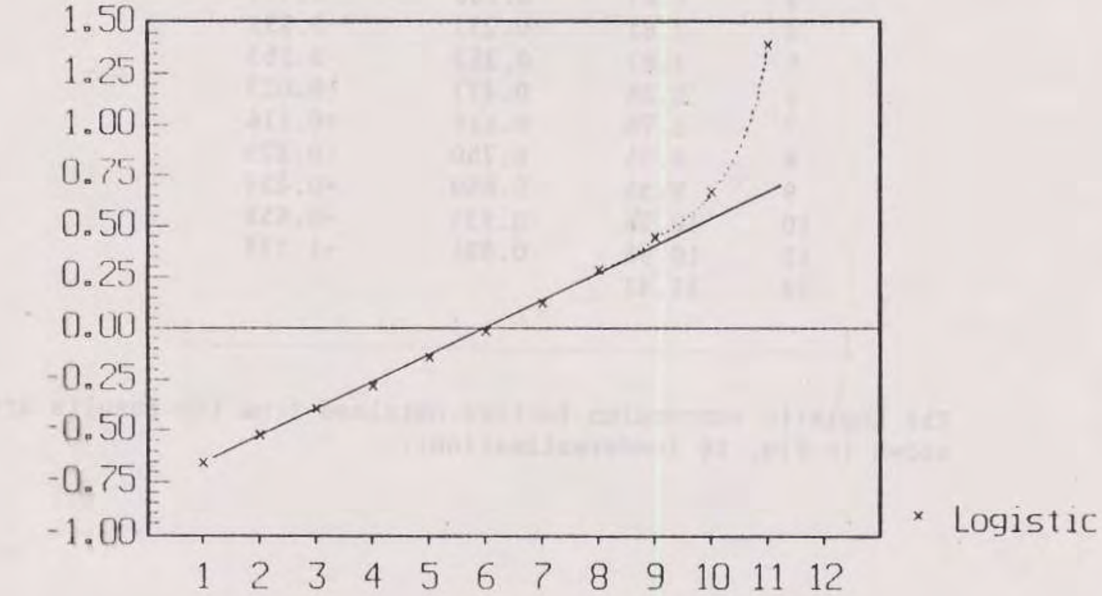

Fig. 16. Plots of Logistic conversion factors at underestimation.

5.2 *Fundamental Method for Calculating Growth Equations*

Calculation procedures for three types of growth equation as expressed in the previous section are demonstrated.

5.2.1 Logistic Growth Equation

The Logistic growth equation is expressed as follows:

$$W_t = \frac{W_\infty}{1 + e^{-k\,(t - t_o)}} \qquad (1.21)$$

where Wt is an estimated body weight at (monthly) age t, and L_∞ is an asymptote of body weight. W_t and W_∞ can be replaced by L_t and L_∞ in the case of the body length growth equation.

Example 22. Estimate a Logistic growth equation using data obtained from Example 19.

Solution:

Estimate a regression equation adopted for the relationship between monthly age and the Logistic conversion factor.

X (Monthly age)	Y (Conversion factor)	X^2	XY
1	- 0.688	1	- 0.688
2	- 0.555	4	- 1.110
3	- 0.428	9	- 1.284
4	- 0.319	16	- 1.276
5	- 0.185	25	- 0.925
6	- 0.062	36	- 0.372
7	+ 0.063	49	+ 0.441
8	0.198	64	1.584
9	0.315	81	2.835
10	0.444	100	4.440
11	0.588	121	6.468
12	0.741	144	8.892
Σ 78	0.112	650	19.005
Mean 6.5	0.009	-	-

N = 12.

$$b = \frac{19.005 - (78)(0.112) / 12}{650 - (78)^2 / 12}$$

$$= \frac{19.005 - 0.728}{650 - 507}$$

$$= \frac{18.277}{143} = 0.12781\text{---}$$

$$= 0.128$$

$$\begin{aligned} y &= \bar{y} + b\ (x - \bar{x}) \\ &= 0.009 + 0.128\ (x - 6.5) \\ &= 0.009 + 0.128\ x - 0.832 \\ &= -\ 0.823 + 0.128\ x \end{aligned}$$

$$\begin{aligned} k &= 4 * b \\ &= 4 * 0.128 \\ &= 0.512 \end{aligned}$$

$$\begin{aligned} t_0 &= -\ (a\ /\ b) \\ &= -\ (-0.823\ /\ 0.128) \\ &= 6.42968\text{---} \\ &= 6.430 \end{aligned}$$

The Logistic growth equation is estimated as follows:

from (1.21):

$$W_t = \frac{12.0}{1 + e^{-0.512\ (t\ -\ 6.430)}}$$

The estimated body weight at each monthly age using the above growth equation is as follows:

Monthly age 1: $$W_t = \frac{12.0}{1 + e^{-0.512\ (1 - 6.430)}}$$

$$= \frac{12.0}{1 + e^{2.780}}$$

$$= \frac{12.0}{1 + 16.119} = 0.70097\text{---}$$

$$= 0.70$$

Body weight at monthly age 1 is estimated as 0.7 grams.

Monthly age 2: $$W_t = \frac{12.0}{1 + e^{-0.512\ (2 - 6.430)}}$$

$$= \frac{12.0}{1 + e^{2.268}}$$

$$= \frac{12.0}{1 + 9.660} = 1.12570\text{---}$$

$$= 1.13$$

Body weight at monthly age 2 is estimated as 1.13 grams.

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Monthly age	Estimated body weight in grams
1	0.70
2	1.13
3	1.77
4	2.68
5	3.90
6	5.34
7	6.87
8	8.29
9	9.46
10	10.34
11	10.95
12	11.34

Comparison between monthly body weight and estimated body weight calculated from the Logistic growth equation is shown in Fig. 17.

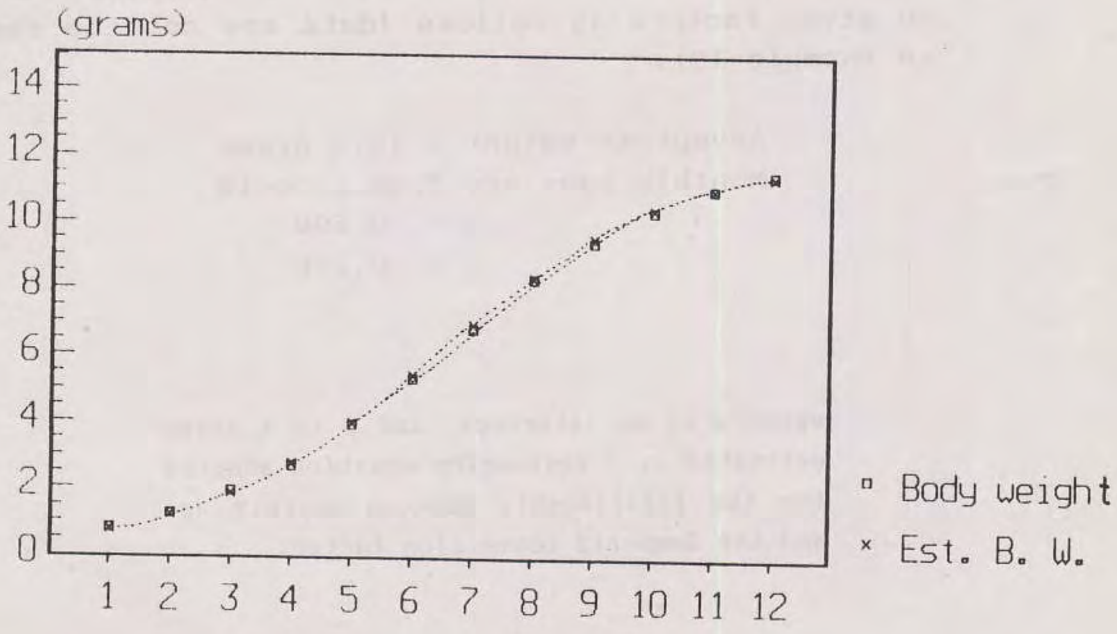

Fig. 17. Comparison between monthly body weight and estimated body weight.

5.2.2 Gompertz Growth Equation

The Gompertz growth equation is expressed as follows:

$$Wt = W_\infty \, e^{-e^{-k\,(t - t_o)}} \qquad (1.22)$$

where Wt is an estimated body weight at (monthly) age t, and L_∞ is an asymptote of body weight. W_t and W_∞ can be replaced by L_t and L_∞ in the case of the body length growth equation.

Example 23. Estimate a Gompertz growth equation based on given factors as follows (data are not the same as Example 19):

Asymptote weight = 12.0 grams
Monthly ages are from 1 to 12
$a = -0.690$
$b = 0.146$

where a is an intercept, and b is a slope estimated by a regression equation adopted for the relationship between monthly age and the Gompertz conversion factor.

Solution:

Estimate a k and a t_o values for the Gompertz growth equation as follows:

$$k = e * b$$
$$= 2.71828\text{---} * 0.146$$
$$= 0.39686\text{-----}$$
$$= 0.397$$

$$t_o = -\ (a\ /\ b)$$
$$= -\ (-0.690\ /\ 0.146)$$
$$= 4.72602\text{-----}$$
$$= 4.726$$

The Gompertz growth equation is estimated as follows:

from (1.22):

$$W_t = 12.0\ e^{-e^{-0.397\ (t\ -\ 4.726)}}$$

Estimated body weight at each monthly age using above growth equation is as follows:

Monthly age 1: $W_t = 12.0\ e^{-e^{-0.397\ (t\ -\ 4.726)}}$

$$= 12.0\ e^{-e^{-0.397\ (1\ -\ 4.726)}}$$

$$= 12.0\ e^{-e^{1.479}}$$

$$= 12.0\ e^{-4.390}$$
$$= 12.0 * 0.012 = 0.144$$
$$= 0.1$$

Monthly age 1 is estimated as 0.1 gram.

Monthly age 2: $W_t = 12.0\ e^{-e^{-0.397\ (2\ -\ 4.726)}}$

$$= 12.0\ e^{-e^{1.082}}$$

$$= 12.0\ e^{-2.951}$$

$$= 12.0 * 0.052 = 0.624$$

$$= 0.6$$

Monthly age 2 is estimated as 0.6 grams.

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Monthly age	Estimated body weight in grams
1	0.1
2	0.6
3	1.6
4	3.2
5	4.9
6	6.6
7	8.0
8	9.1
9	10.0
10	10.6
11	11.0
12	11.3

Monthly body weight and estimated body weight by Gompertz growth equation are shown in the above table.

5.2.3 Bertalanffy Growth Equation

The Bertalanffy growth equation for body weight is expressed as follows:

$$W_t = W_\infty (1 - e^{-k (t - t_o)})^3 \qquad (1.23)$$

The Bertalanffy growth equation for body length is expressed as follows:

$$L_t = L_\infty (1 - e^{-k (t - t_o)}) \qquad (1.24)$$

Example 24. Estimate a von Bertalanffy growth equation based on the given factors (data are not the same as Example 19):

Asymptote weight = 12.0 grams
Monthly ages are from 1 to 12
a = −0.659
b = 0.160

where a is an intercept, and b is a slope estimated by a regression equation adopted for the relationship between monthly age and the Bertalanffy conversion factor.

Solution:

Estimate k and a t_o values for the von Bertalanffy growth equation as follows:

$$k = \frac{9}{4} * b$$

$$= (9 / 4) * 0.160$$
$$= 0.360$$

$$t_o = - (a / b)$$
$$= - (-0.659 / 0.160)$$
$$= 4.11875$$
$$= 4.119$$

The von Bertalanffy growth equation is estimated as follows:

from (1.23):

$$W_t = 12.0 (1 - e^{-0.360 (t - 4.119)})^3$$

Estimated body weight at each monthly age using the above growth equation is as follows:

Monthly age 5: $W_t = 12.0\ (1 - e^{-0.360\ (5\ -\ 4.119)})^3$

$= 12.0\ (1 - e^{-0.317})^3$

$= 12.0\ (1 - 0.728)^3$

$= 12.0\ (0.272)^3$

$= 12.0 * 0.020 = 0.240$

$= 0.2$

⋮

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Monthly age	Estimated body weight in grams
1	-
2	-
3	-
4	-
5	0.2
6	1.4
7	3.2
8	5.1
9	6.8
10	8.2
11	9.2
12	10.0

Monthly body weight and estimated body weight by von Bertalanffy growth equation are shown in the above table.

Example 25. Estimate body length at each month of age based on given factors as follows:

Asymptote length = 105 mm

k = 0.360

t_o = 0.411

Solution:

For body length, from (1.24):

$$L_t = L_\infty \left(1 - e^{-k(t - t_o)}\right)$$

$$L_t = 105 \left(1 - e^{-0.360(t - 0.411)}\right)$$

Estimated body length based on the above von Bertalanffy growth equation is shown in the following table.

Monthly age	Estimated body length in mm
1	20.1
2	45.7
3	63.7
4	76.2
5	84.9
6	91.0
7	95.2
8	98.2
9	100.2
10	101.7
11	102.7
12	103.4

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