SEAFDEC Training Department

Text/Reference Book

Southeast Asian Fisheries Development Center

TRB/No. 18 (Revised)

April 1981

PRACTICAL AND RAPID DIAGNOSIS AND FISH POPULATION

Takeyuki Doi

(Tokai Regional Fisheries Research Laboratory)

Tokyo, Japan

Training Department

Southeast Asian Fisheries Development Center

Issued by the Training Department, Southeast Asian Fisheries Development Center.
P.O. Box 4, Phrapradaeng, Samutprakarn, Thailand.

Text/Reference Book Series No. 18
(Revised edition)

April 1981

PRACTICAL AND RAPID DIAGNOSIS
OF
FISH POPULATION

Takeyuki Doi

Training Department
Southeast Asian Fisheries Development Center

CONTENTS

Practical and Rapid Diagnosis

of

Fish Population

Takeyuki Doi

(Tokai Regional Fisheries Research Laboratory)

Tokyo, Japan

1. Introduction

Problems of population analysis have been increasing year by year. This is attributed, on the one hand, to the programs of exploitation of fish resources having been carried out so successfully that the use of data processing is now required and, on the other, to the management of fisheries in the world having become very important not only to conserve marine resources at the optimum level, but also to deal scientifically with international fisheries problems. Moreover, stock assessment should be carried out as soon as possible. The scientific echo sounder is one technique for quick assessment. However, whereas by the use of the scientific echo sounder, population size is estimated quickly, diagnosis of population status cannot be obtained.

To meet such a demand for a technique giving quick results, I have developed a mathematical method capable of being used practically and rapidly when faced with actual problems of population studies. This method is a kind of unified field theory which is neither fragmental nor partially independent but applicable generally and systematically and synthetically to any situation of any species in any ground and at any time. By this method, therefore, the present data, whether perfect or not, are fully taken into a unified field based upon biology without mechanical calculations and fragmental formula.

This concept is an essential element in my newly developed unified field theory.

2. Flow of jobs

The general network of population studies is shown in Fig. 1 by arrow diagram from the view point of population studies as a whole. Actually situations such as those shown in Fig. 1 are very rare. Usually we do not have sufficient satisfactory data at our disposal. However, in spite of bad conditions, we must often make a diagnosis at the request of the Government, fishermen or any other organizations. Then rapid methods should be developed. I devised a rapid diagnosis method, called DOIRAP, which is suitable for use with a computer. The arrow diagram of DOIRAP is shown Fig. 2.

The basic data are divided into three categories as follows:

1. Growth law;
2. Life cycle;
3. Catch statistics

These categories can be found in the left side of Fig. 2. I shall now explain how we can obtain a diagnosis, which is the aim of this method, from the basic data, following the arrow diagram step by step.

If by only one theory we can solve every problem, we call it a unified field theory. In this theory, individual theories for individual occasions or situations are not taken into account. The DOIRAP system can be applied to any stationary situation of population problems. This is the first thing to emphasize. The second important consideration is the relationship between each job shown by arrows in Fig. 2. Considering growth as an example, this is analysed and then we obtain Growth law. In the usual method of analysis this is all that can be achieved. But in the DOIRAP system Growth is connected with vital factors such as metabolism, catabolism, biomass analysis, natural mortality, fishing mortality, population size, diagnosis 1, etc. No job is independent but all jobs are connected with each other. Absurd assumptions or results are rejected by such a connected network.

As mentioned above, in a process of analysis where we deal with combined data, jobs such as data collection, survey, research, etc., are not independent but are related with each other in a complicated pattern. If we treat a problem without any consideration for relations with other factors, biased results are obtained and not only relationships among jobs but also mutual influences upon other jobs cannot be understood.

Accordingly, in order to grasp relations between a research plan and jobs carried out in a process, the graphic representation of an arrow diagram can be used; that is a technique adopted in PERT (program evaluation and review technique). As the diagram is a means of checking the new plan, the total network of jobs is organically clarified, and charge share, necessity and importance of jobs may easily be understood.

Researches on marine resources are boundary sciences connected with two or more fields of research: they constitute interdisciplinary research. Interdisciplinary researches are becoming more necessary and urgent to develop sciences on boundary problems. Solutions of problems can be achieved more effectively and quickly by a group of persons who have a knowledge of different sciences rather than by persons who have the same knowledge. When heterogenous members with a different knowledge set up a team cooperatively, the work may be recognized as systematic, and splendid results may be expected. As research fields of fisheries are all interdisciplinary, valuable achievements cannot be expected within a single field of research. But "systematic" must not be taken to mean a style formation of organization; it denotes the smooth functioning of each unit within an organization.

In order to achieve outstanding results, competent researchers are required; there need not be many ordinary researchers but at least one of them must be an excellent researcher.

3. Growth

Growth law can be obtained from the sets of measurements of length L, weight and age t in individual fishes, as is represented by the following equations in the DOIRAP system.

$$\left.\begin{aligned} L &= a - be^{-kt} \\ &\quad \text{or } a\left[1 - e^{-k(t - t_o)}\right] \\ W &= AL^3 \end{aligned}\right\} \quad \text{...........(1)}$$

Constants a and k are calculated from the following difference equation

$$\left.\begin{aligned} L_{n+1} &= \beta_o + \beta_1 L_n \\ \beta_o &= a(1 - e^{-kt}) \\ \beta_1 &= e^{-kt} \end{aligned}\right\} \quad \text{..........(2)}$$

n : numerical value of t interval

The above equation is linear regressive. Therefore, we can calculate β_o and β_1. Then a and k are estimated from β_o and β_1, where b or t_o is to be decided from the initial conditions, as follows:

$$\left.\begin{aligned}
k &= -\frac{1}{t}\log_e \beta_1 \\
a &= \frac{\beta_0}{1-\beta_1} \\
b &= \frac{1}{n+1}\sum_{t=c}^{c+nt}\frac{a-\ell}{e^{-kt}} \\
t_0 &= \frac{1}{k}\log_e\frac{b}{a}
\end{aligned}\right\} \quad \ldots\ldots\ldots\ldots(3)$$

From a set of observed values of ℓ_i and W_i (i is number of individuals, $i = 1, 2, 3 \ldots\ldots n$, n is the sample size), parameter A can be estimated from the least square method as follows:

$$\left.\begin{aligned}
A &= \frac{\sum_{i=1}^{n} W_i L_i^3}{\sum_{i=1}^{n} L_i^6} \\
&\text{or} \\
\log_e A &= \frac{1}{n}\left[\sum_{i=1}^{n}\log_e W_i - 3\sum_{i=1}^{n}\log_e \ell_i\right]
\end{aligned}\right\} \quad \ldots.(4)$$

If we obtain the growth parameters a, k and A in Eqn. (1) from a set of observed values (age, W, L), the amount of metabolism and catabolism per day can be easily calculated.

According to the above equation not only coefficients of metabolism and catabolism but also amount of metabolism, catabolism, and growth per day are calculated as follows:

$$\left.\begin{aligned}
\alpha &= 3A^{\frac{1}{3}}ak \\
\beta &= 3k \\
\text{Amount of catabolism per day} &= \frac{\alpha}{365}W^{\frac{2}{3}} \\
\text{Amount of metabolism per day} &= \frac{\beta}{365}W
\end{aligned}\right\} \quad \ldots\ldots\ldots(5)$$

Growth of weight per day = difference between the above amounts supposing unit of age is one year. Equation (5) is derived from the fundamental growth law of body weight expressed in the differential equation as follows;

$$\frac{dW}{dt} = \alpha W^{2/3} - \beta W \qquad \text{...........(6)}$$

where

α : coefficient of catabolism

β : coefficient of metabolism

dW : increment of weight in time increment dt

Sakamoto (1977, 1978, 1979) carried out investigation on red sea breams in the Kii Strait, the eastern part of the Seto Inland Sea. I use this data as the basic input of the DOIRAP computer program.

Age and length data corresponding to arrow 1-2 in Fig. 2 is shown in Table 1, which are obtained from scale-reading. Length and weight of 318 individuals were also measured. From this input data, calculated results are as follows;

β_o = 128.95

β_1 = 0.7929

k = 0.2320/year

a = 622.7 mm

b = 616.7 mm

t_o = - 0.0421 year

A = 0.00002082 g/mm^3

α = 11.92/year

β = 0.5960/year

Then we can calculate length and weight in any month for any age, as well as amount of catabolism and metabolism which are shown in Table 2.

In the arrow diagram of Fig. 2, there are 6 phases in the computer program. The above calculations correspond to phase 1.

4. Life cycle

The second important input is the life cycle and reproduction.

Life cycle is the essential concept in studying biology. Basic components of life cycle are life span, maturity by age and fecundity by age.

The average number of years the species under consideration lives is considered to be its life span.

Rate of maturity varies according to age. Juveniles are of course immature and, as age advances, the rate of maturity increases. Over a given time all individuals mature. Maturity can be determined by shape, colour and size of eggs.

The number of eggs spawned by an individual also increases with age. A very large number of eggs is found in gonads, so a fraction sample in gonad has been examined.

The life cycle of red sea breams is represented in Fig. 3. The components of the life cycle connect with the following procedures of biomass analysis, estimate of natural mortality and life cycle analysis.

5. Biomass analysis and estimate of natural mortality

Biomass in each year or at a given age can be easily calculated under a given annual survival rate, because individual weight is already obtained in the previous section.

where W_x : individual weight at x-age

P_x : biomass at x-age

S : annual survival rate

N : number of population in given n-age after larval stage,

Px is represented as follows:

$$P_x = NS^{x-n} W_x \qquad \text{..........(7)}$$

Now N and S are unknown but the relative pattern can be calculated in various S, as described in Fig. 4 under the values of n = 1 and N = 10,000. This figure is also drawn by computer.

There are many methods of estimating natural mortality, although they are mostly rather difficult to apply. Here I show a little rough but simple and rapid method which is derived from the Biomass curve shown in Fig. 4.

Generally speaking, biomass increases at the young stage, has a dome at a given age and decreases to nearly zero at maximum age. In the example in Fig. 4, the curve of $S = 0.7$ seems to be adequate. The relationship between natural mortality and survival rate in the unexploited state is expressed as

$$S = e^{-M}$$

Then

$$S = -\log_e S = -\log_e 0.7 = 0.357$$

Now natural mortality of red sea breams is estimated as $M = 0.357$. In the arrow diagram of Fig. 2, we have finished the jobs $6 \rightarrow 7$, $7 - 8$, and $8 \rightarrow 10$ of phase 2.

The survival rate during egg to 1-age is much less than in the case of young or adult fishes. In the virgin or unexploited state the population maintains a given level. Therefore, the following equation is established:

$$S' \sum_{x=1}^{20} PS^{x-1} \; MTR(x) \; H(x) = 1 \qquad \ldots\ldots\ldots(8)$$

where,

S'	:	Survival rate during egg to 1-age
S	:	Annual survival rate of 1 age fish to 20-age.
P	:	Sex ratio
$MTR(x)$	:	Maturity proportion of x-age which is given in Fig. 3
$H(x)$	:	Fecundity of x-age which is also given in Fig. 3

S was already estimated to be 0.7 and P is assumed to be 0.5. Therefore unknown parameter S' in Eqn. (8) can be solved easily. We obtain;

$$S' = 0.419 \times 10^6$$

Then $$M' = -\log S' = 14.7$$

Where M' is natural mortality coefficient during egg to 1-age. M' cannot usually be observed by survey. However, by the theoretical process of life cycle we can estimate it deductively. This calculation is job 12 → 13 of phase 2 in the arrow diagram of Fig. 2.

6. Estimates of survival rate and availability obtained from catch statistics

It is natural that catch is important information in analysing fisheries problems. The most essential data for stock assessment are total catch and age composition of catch. In order to obtain age composition, many different studies must be carried out, such as aging character, reading age character, sampling methods from catch, etc. Therefore, sometimes we must use category composition instead of age composition, because categories are surveyed very easily in the fish market.

It is usual in estimating the survival rate S to use age compositions. Although there are several calculation methods, I shall present here the average age method (DOI, 1974).

Age of fish is not as wide as $0 - \infty$. Put a for the lowest age and $a + \Delta$ for the highest age, where Δ = (highest age) - (lowest age). The average age $\bar{x}$ of fishes, in a sample which is larger than 50 individuals, is regarded as normal distribution with mean, m, and standard deviation $\frac{\sigma}{\sqrt{n}}$. Here m and σ are:

$$M(a, S, \Delta) = \frac{xS^x}{S^x} = a + k\ (S, \Delta) \qquad \text{.........(9)}$$

and value of m and σ are indicated in Table 3, as changes of S and Δ. Accordingly in calculating S by average age method, the following steps are carried out successively;

1. To find a and Δ
2. Calculation of $\bar{x}$
3. To obtain k: $k = \bar{x} - a$
4. To search for S in Table

Corresponding to k and Δ under consideration, numerical interpolation or graphical interpolation may be adopted. Table 3 is built in DOIRAP program.

Fishing mortality F is defined by:

$$S = e^{-(M+F)} \qquad \text{........(10)}$$

that is:

$$F = -(\log_e S) - M \qquad \text{........(11)}$$

F is a instantaneous coefficient, which is a little difficult to grasp. Therefore, we usually use rate of exploitation E which is ratio of catch to total population.

$$E = \frac{F}{M+F}\left[1 - S\right] \qquad \text{........(12)}$$

In the sea, populations of the younger age are more abundant than those of older ones. But in the actual catches, the number of younger specimens is sometimes not greater than that of the older ones up to a given age (full recruited age). Introducing the availability Q_x (x is age), which is defined by

$$\frac{x\text{-age Catchable proportion}}{x\text{-age population in the sea}} = Q_x$$

Q_x is less than 1 up to fully recruited age, and Q_x is equal to 1 in the age beyond fully recruited age. Q_x is determined by mesh size, fishing area, fishing season, etc., which are conditions relevant to human activity of fishing operations. Q_x can be calculated from age compositions.

If we know the age composition of catch $C(x)$, natural mortality M and fishing mortality F, availability Q_x is calculated as follows:

Assuming age $x = r$ is fully recruited age, in the schematic representation of Fig. 5, shaded parts are catchable phases. Therefore,

$$Q_x = 1 \text{ at } x \geqslant r$$

$$Q_x < 1 \text{ at } x = r - 1$$

The general formula for estimating Q_x is derived from the model of Fig. 5, as shown in the equation below.

$$\frac{C(x)}{Q_x} = C(x-1)\left[\frac{1-Q_{x-1}}{Q_{x-1}}\,e^{-M} + e^{-(M+F)}\right] \quad\ldots\ldots(13)$$

If we put $x = r$ in Eqn. (11), Q_{r-1} can be easily calculated. And then Q_x can be successively calculated back to age x less than r.

At the fishing ground of red sea breams under consideration, age determination of fishes of older ages is not so easy. The reliable age composition as a input is shown in Table 4, in unit of catch per day per boat. The total catch per year in recent years (1974-1977) averaged 126 tons. These data are input data correšponding to the arrow 4-5 in the diagram of Fig. 2.

Calculated results of survival rate are as follows:

Survival rate $S = 0.274$
(at the present level)

By Eqns. (10, (11), and (12) coefficient of fishing mortality F and rate of exploitation are estimated.

$$F = 0.939$$

$$E = 0.526$$

Full recruited age is 2 age. Therefore,

$$Q_x = 1 \quad \text{at} \quad x \geqslant 2$$

And then by Eqn. (13), Q_1 is calculated:

$$Q_1 = 0.676$$

Thus the jobs 5-14, 5-16 and 14-15 of phase 3 are finished.

7. Estimate of population size

The last job to remain in phase 3 of the DOIRAP arrow diagram is arrow 15-17, which is to estimate population size. The rate of exploitation E has already been estimated and total catch Y is known as input data. Therefore, the catchable population size P_c is easily calculated as follows:

$$P_c = \frac{Y}{E} = \frac{126}{0.526} = 239 \text{ tons}$$

Number of population by age is necessary in the following diagnosis analysis. Calculation is carried out by the following procedures. Denoting N as the number of 1-age fishes in the sea, the number of fishes by age in the sea and weight of fishes by age in the catchable phase are represented as below.

Age x	Availability Q	Individual weight $W(x)$	Number in the sea	Weight in catchable phase
1	$Q_1 = 0.6763$	$W(1)$	N	$NQW(1)$
2	1	$W(2)$	$N\left[Q_1S + (1-Q_1)S_o\right]$	$NW(2)\left[Q_1S + (1-Q_1)S_o\right]$
3	1	$W(3)$	$N\left[Q_1S + (1-Q_1)S_o\right]S$	$NW(3)\left[Q_1S + (1-Q_1)S_o\right]S$
4	1	$W(4)$	$N\left[Q_1S + (1-Q_1)S_o\right]S^2$	$NW(4)\left[Q_1S + (1-Q_1)S_o\right]S^2$
5	1	$W(5)$	$N\left[Q_1S + (1-Q_1)S_o\right]S^3$	$NW(5)\left[Q_1S + (1-Q_1)S_o\right]S^3$
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.

Where $W(x)$ are known in Table 2 and both survival rate of virgin stock S_o (= 0.7) and the present survival rate S (= 0.224) are already estimated. The population size estimated above is 239 tons, which is the total right of the hand column of the above representation. In this equation the unknown parameter is only N. Therefore, N is simply obtained the following values:

$$N = 87 \times 10^4 \quad \text{individuals}$$

Then we can calculate the above representation of both number in the sea and weight in the catchable phase. Calculated results are shown in Table 5.

8. Diagnosis 1 - size limitation or legal size

It is not rational to catch too small or juvenile fishes, because it decreases not only the future abundance but also destroys the normal reproduction mechanism.

Size limit can be found from the biomass analysis. The biomass curve as shown in Fig.4 has a dome at a certain age. We choose the adequate curve from several trials of various values of S. On the chosen or adopted curve, we know the age at which the biomass has a dome. Up to this age fishing should be prohibited, because the biomass steadily increases year by year. Above this age, biomass decrease gradually year by year even if there is no catch. Therefore, fish beyond this age can be caught.

In the example of red sea bream shown in Fig.4, the adequate curve adopted is the curve of $S = 0.7$ which has a dome in age 4. Therefore, it is recommendable not to take fishes below this age. The size limit or legal size at age 4 is about 38 cm as shown in Table 1 on the growth law. In the actual fisheries a lot of fishes smaller than 38 cm are caught. This is one reason of overfishing which is explained in the following section.

9. Diagnosis 2 - Is the present status of population underexploitation or overfishing?

The criterion to judge underexploitation or overfishing is the reproduction, that is the total number of eggs spawned by adults. If this amount becomes less than half of that of the virgin stock, we can judge that the present status is overfishing. The reason why such a decision may be adopted is the common concept of the maximum sustainable yield on which usually an abundance of adults is about a half of adults in virgin stock. This method is a very practical one under the lack of reproduction mechanism.

Actual calculation can be carried out from weight, natural mortality, fishing mortality, availability, fecundity, maturity percentage which are the important biological parameters of life cycle. Table 6 shows the form of calculation, illustrated by the red sea breams.

The number of fishes at the age of 1 year can be estimated as explained in the previous section (see Table 5). Then, by using the estimated values of natural mortality, fishing mortality, availability, weight, fecundity, naturity, etc., the numbers of each age-group in the sea N, catchable population number Nc, catchable biomass Pc, expected catch C, expected yield Y, total number of adults A, total number of eggs spawned EGG are all reconstructed. In the above calculation, the point to be careful is the following procedure:

$$N_{x+1} = Q_x N_x e^{-M} + (1 - Q_x) N_x e^{-(M+F)}$$

The same calculation can be carried out as for the virgin stock in which fishing mortality $F = 0$. The number of age 1 fishes in the sea can be adopted the same values in the present status, although it is a little doubtful theoretically.

After both calculations we can compare the present status with the virgin stock. Then the rate of decrease can be calculated for not only population size but also reproduction.

Table 6 indicates that the red sea bream population has suffered overfishing because rate of decrease of reproduction is about 4% which is much smaller than 50%. Therefore, expansion of this fishery presents a big problem. Regulation of fishing should be recommended. Phase 4 of the arrow diagram of Fig. 2 is thus completed.

10. Diagnosis 3 .... Optimum status of population under the present condition of fishing gear

We can judge the present status of population to be overfishing or underexploitation by Diagnosis 2. We must then proceed to estimate what level of exploitation is the optimum. For this purpose, the same calculation as shown in Table 6 is carried out with various values of F. Then after further calculations, total yield Y, rate of decrease of eggs spawned and efficiency of fishing $\frac{Y}{F}$ are plotted against F-values. Such a diagram is shown in Fig. 6. Optimum level of population status and fishing intensity should be decided first from the values of rate of decrease of eggs spawned which must be larger than 0.5, and secondly in connection with total catch Y and $\frac{Y}{F}$ which are economical or political elements rather than factors related to population dynamics.

The optimum value of fishing mortality F is 0.168 which is indicated at the 50% decrease rate of reproduction in Fig. 6. The present fishing mortality is estimated already as being 0.939, which is much larger than 0.168. Therefore, in order to maintain the optimum population level, fisheries should be reduced to 18% ($= \frac{0.168}{0.939}$) of the present magnitude.

Thus the Phase 5 calculation is completed.

11. Diagnosis 4 .... Optimum status of population in conditions of change in fishing gear

Assuming we can change fishing gear or net mesh size in future, what is the best fishery under which the optimum level of population can be maintained? This is the problem of Diagnosis 4 that is the calculation of phase 6 in the arrow diagram.

According to changes in fishing gear, values of availability Q change. Therefore many calculations for Diagnosis 4 must be carried out for various combinations of Q and F. Calculations themselves are quite the same as those of Diagnosis 3. Calculated results can be summarized as in Fig. 7 which indicated isopleth diagram of total yield Y and contour line of rate of decrease of reproduction (eggs spawned), which was presented by DOI (1974). In Fig. 7 abscissa is fishing mortality F and ordinate shows corresponding values of Q by age.

The ordinate should be read as shown in the following examples.

Example values of ordinate Q by age.

1.4 $$\begin{cases} Q_2 = Q_3 = Q_4 = \dots = 1 \\ Q_1 = 1 - 0.4 = 0.6 \end{cases}$$

2.3 $$\begin{cases} Q_3 = Q_4 = Q_5 = \dots = 1 \\ Q_2 = 1 - 0.3 = 0.7 \\ Q_1 = 0 \end{cases}$$

3.7 $$\begin{cases} Q_4 = Q_5 = Q_6 = \dots = 1 \\ Q_3 = 1 - 0.7 = 0.3 \\ Q_2 = Q_1 = 0 \end{cases}$$

4.2 $$\begin{cases} Q_5 = Q_6 = Q_7 = \dots = 1 \\ Q_4 = 1 - 0.2 = 0.8 \\ Q_3 = Q_2 = Q_1 = 0 \end{cases}$$

$$5.0 \quad \begin{cases} Q_5 = Q_6 = Q_7 = \ldots = 1 \\ Q_4 = Q_3 = Q_2 = Q_1 = 0 \end{cases}$$

In Fig. 7, mark X represents the present status which is obtained in Diagnosis 2. Mark O shows a better status which is obtained in Diagnosis 3, on which level the maximum sustainable yield is available under the use of present fishing gear and method. Mark Δ indicates what appears to be the best status, i.e. the level of the most effectual maximum sustainable yield obtained if the present fishing gear or method is changed. However, as in the case of optimal status of population under the present conditions, this future status is not the problem of population dynamics but rather an economical or political problem. In any case, such a status should lie on the line of 50% decrease of reproduction.

12. Summary

1) A unified field theory, DOIRAP program for practical and rapid diagnosis method is presented in the form of an arrow diagram in Fig. 2.

2) Essential input data are as follows:

   i) data for age, length and weight of individual fishes measured or observed

   ii) Life cycle of the species under consideration including fecundity by age or by size of fish body

   iii) catch statistics and age composition or category composition

3) Aims of the survey and research are:

   i) Diagnosis 1 - size limitation

   ii) Diagnosis 2 - determining whether the present status of population is under-exploitation or overfishing.

   iii) Diagnosis 3 - optimum status of population under the present conditions of fishing gear.

   iv) Diagnosis 4 - optimum status of population in conditions of change in fishing gear or method.

4) Jobs shown in Fig. 2 are achieved by following the arrows of the diagram.

   The main analyses are: biomass analysis, life cycle analysis and age composition analysis. Through these analyses, population parameters such as natural mortality, survival rate, fishing mortality, rate of exploitation and availability are estimated.

5) After estimating population parameters, population size by age both in number and in weight can be calculated.

6) The four kinds of diagnoses can be carried out by using the population size and population parameters. Calculations are based on population size in the sea, catchable population, catch, yield, number of adults, total number of eggs spawned, rate of decrease of population size, rate of decrease of the total number of eggs which is most important to judge the diagnosis.

7) After completing the diagnosis, suitable measures for regulation of fisheries can be recommended.

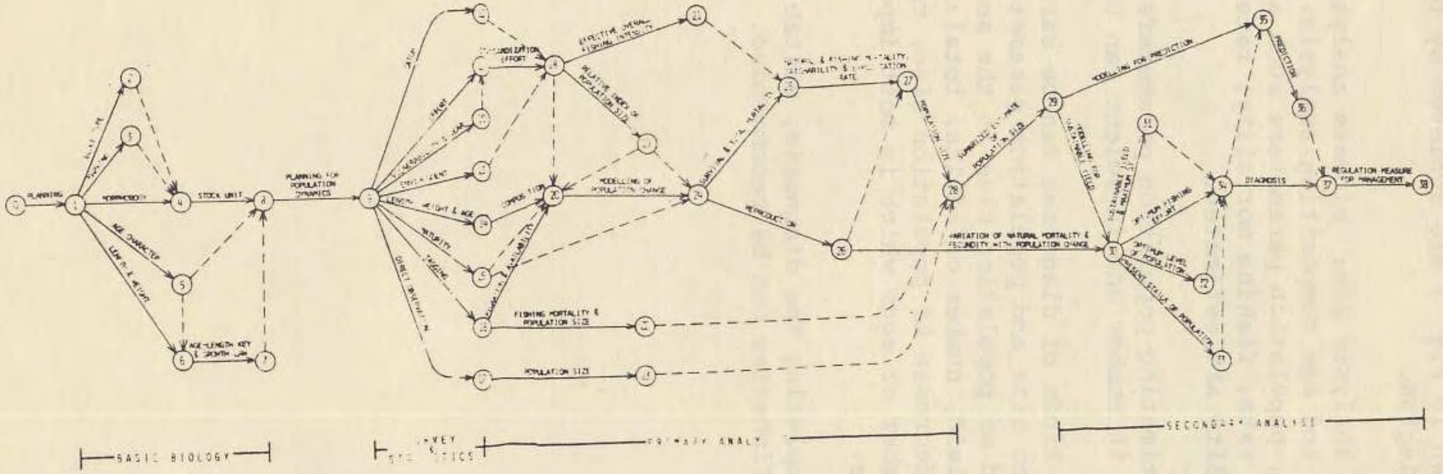
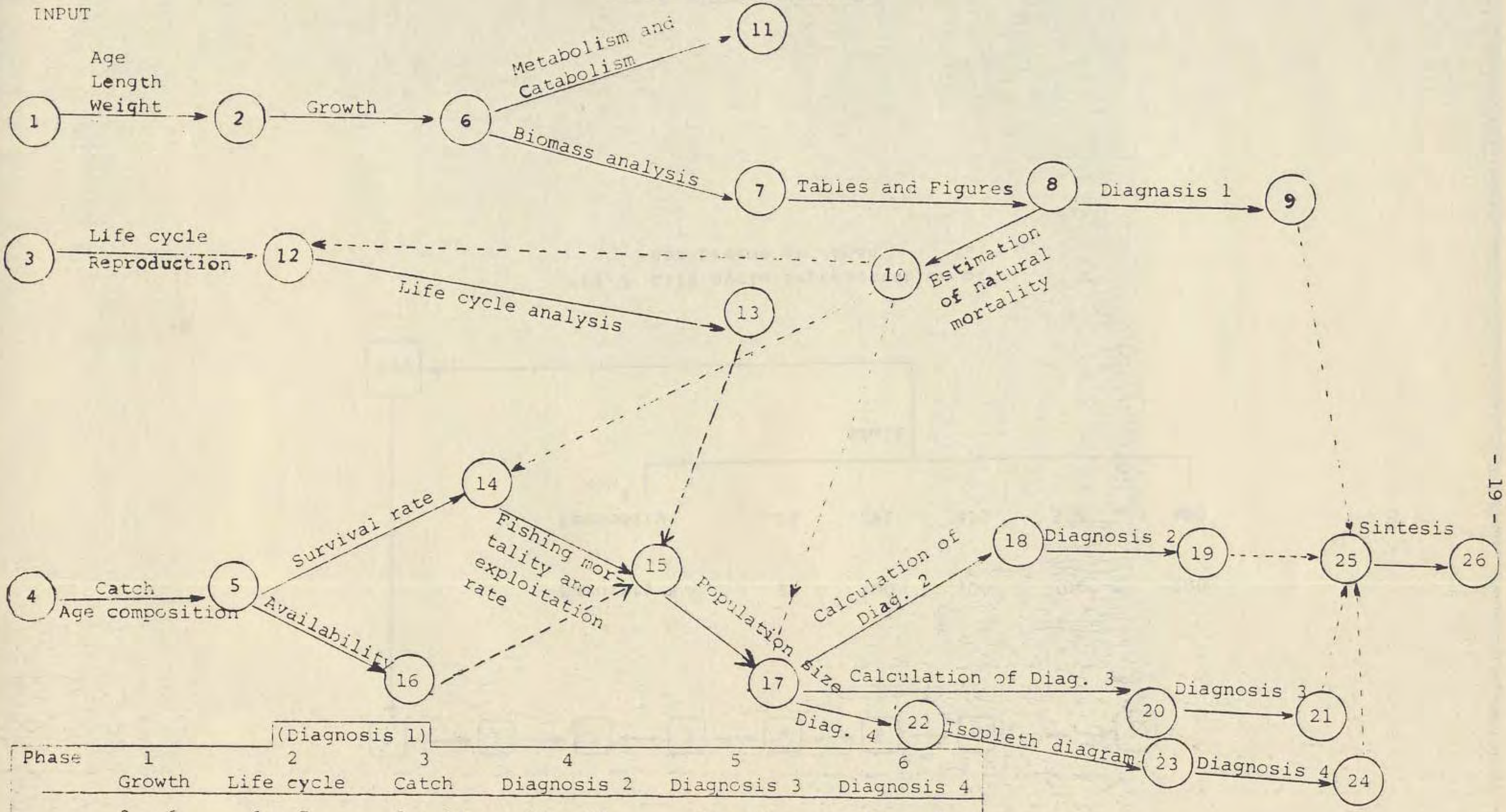


Fig. 1 General network of population studies shown by arrow diagram which indicates not only relations among each job but also process to approach to the aims reasonably

Doirap - Arrow diagram



Phase	1	2 (Diagnosis 1)	3	4	5	6
	Growth	Life cycle	Catch	Diagnosis 2	Diagnosis 3	Diagnosis 4
	2 - 6	6 - 7	5 - 14	17 - 18	17 - 20	17 - 22
	6 - 11	7 - 8	5 - 16	18 - 19	20 - 21	22 - 23
		8 - 9	14 - 15			23 - 24
		9 - 10	15 - 17			
		12 - 13				

Fig. 2 Arrow diagram for rapid diagnosis method

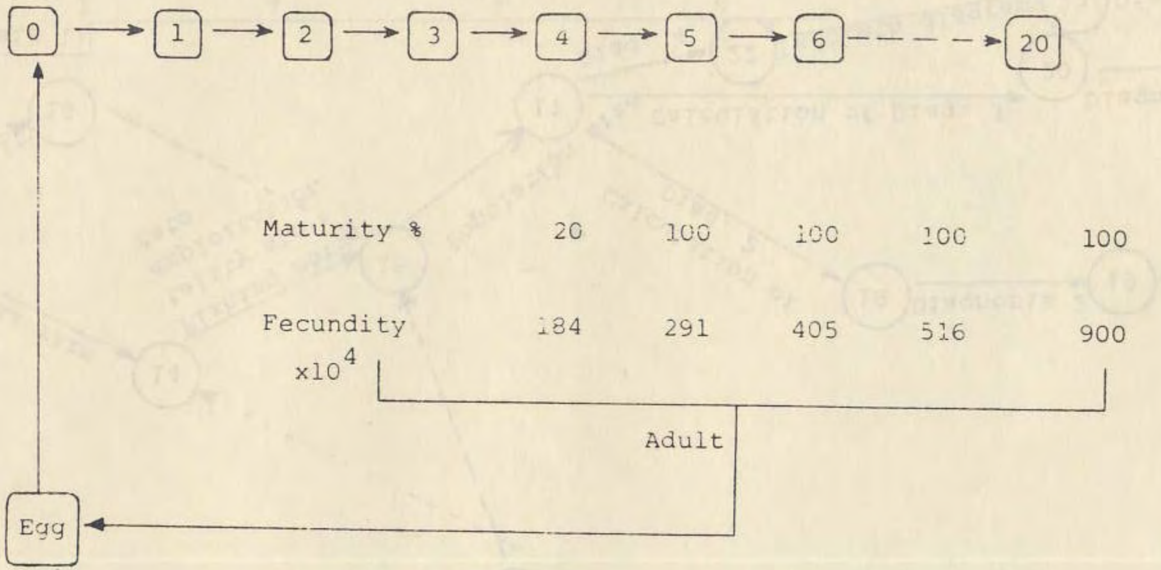
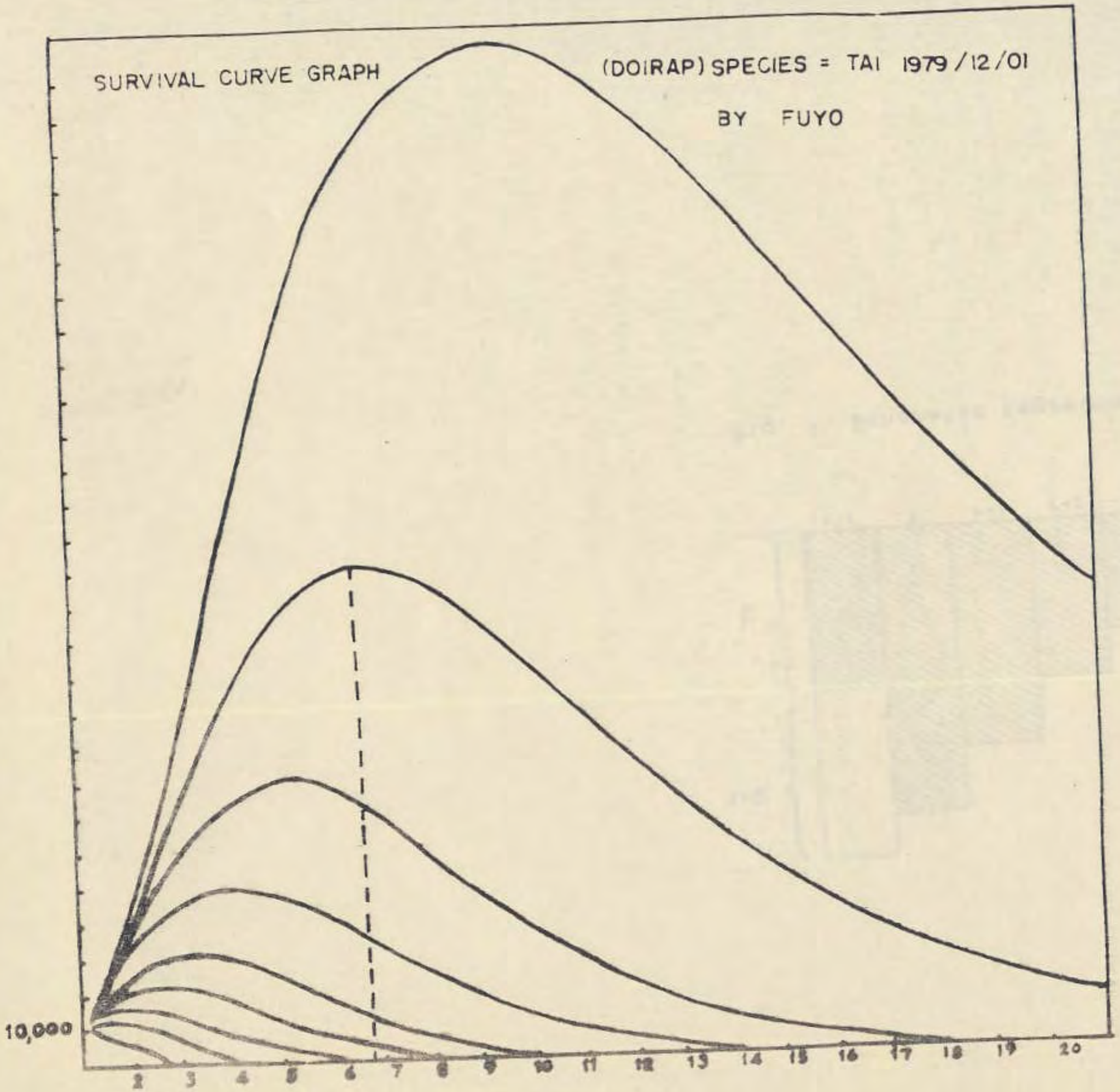


Fig. 3 Life cycle representation of sea-breams in Japan



	AGE	YIELD
S = 0.1	1.13	10103
S = 0.2	1.15	12283
S = 0.3	1.95	16302
S = 0.4	2.39	22647
S = 0.5	2.96	32756
S = 0.6	3.66	49662
S = 0.7	4.61	80129
S = 0.8	6.07	141693
S = 0.9	8.73	293659

Fig. 4 Biomass analysis of red sea breams.
The upper curve is S = 0.9 and the lowest one is S = 0.1

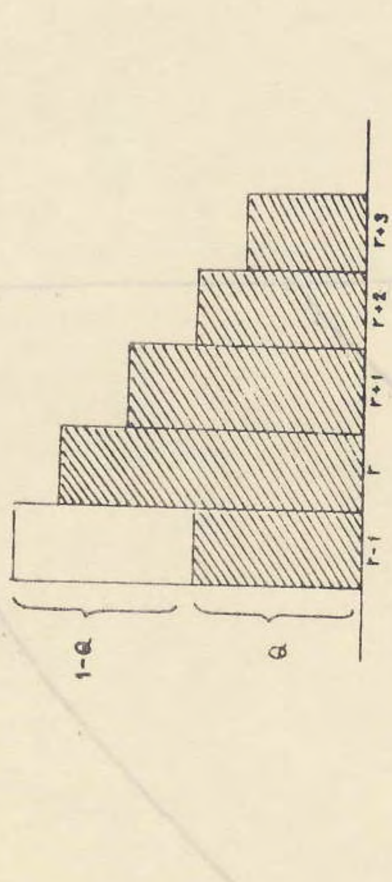


Fig. 5 Schematic representation of Q

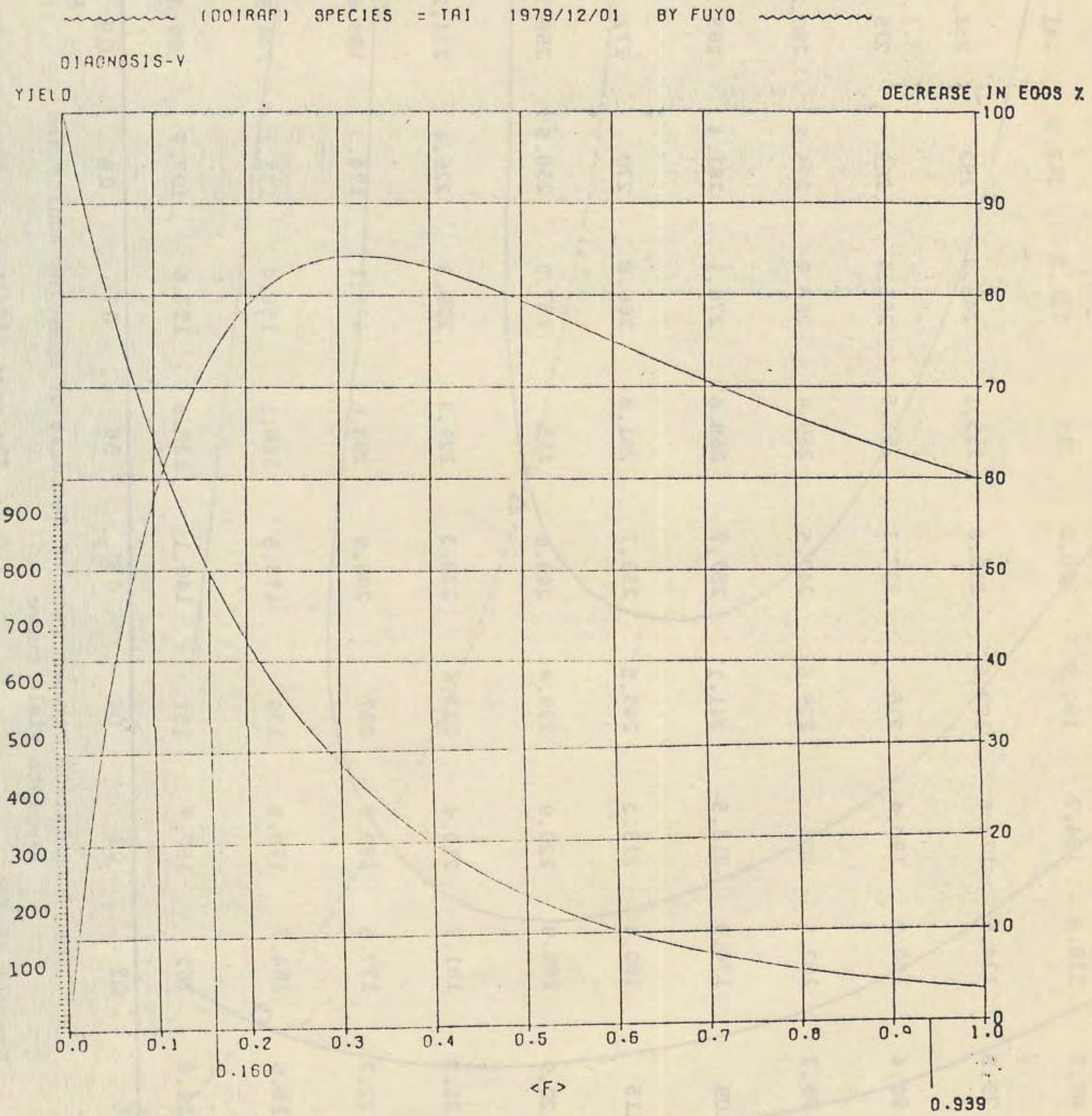


Fig. 6 Diagram for diagnosis 3

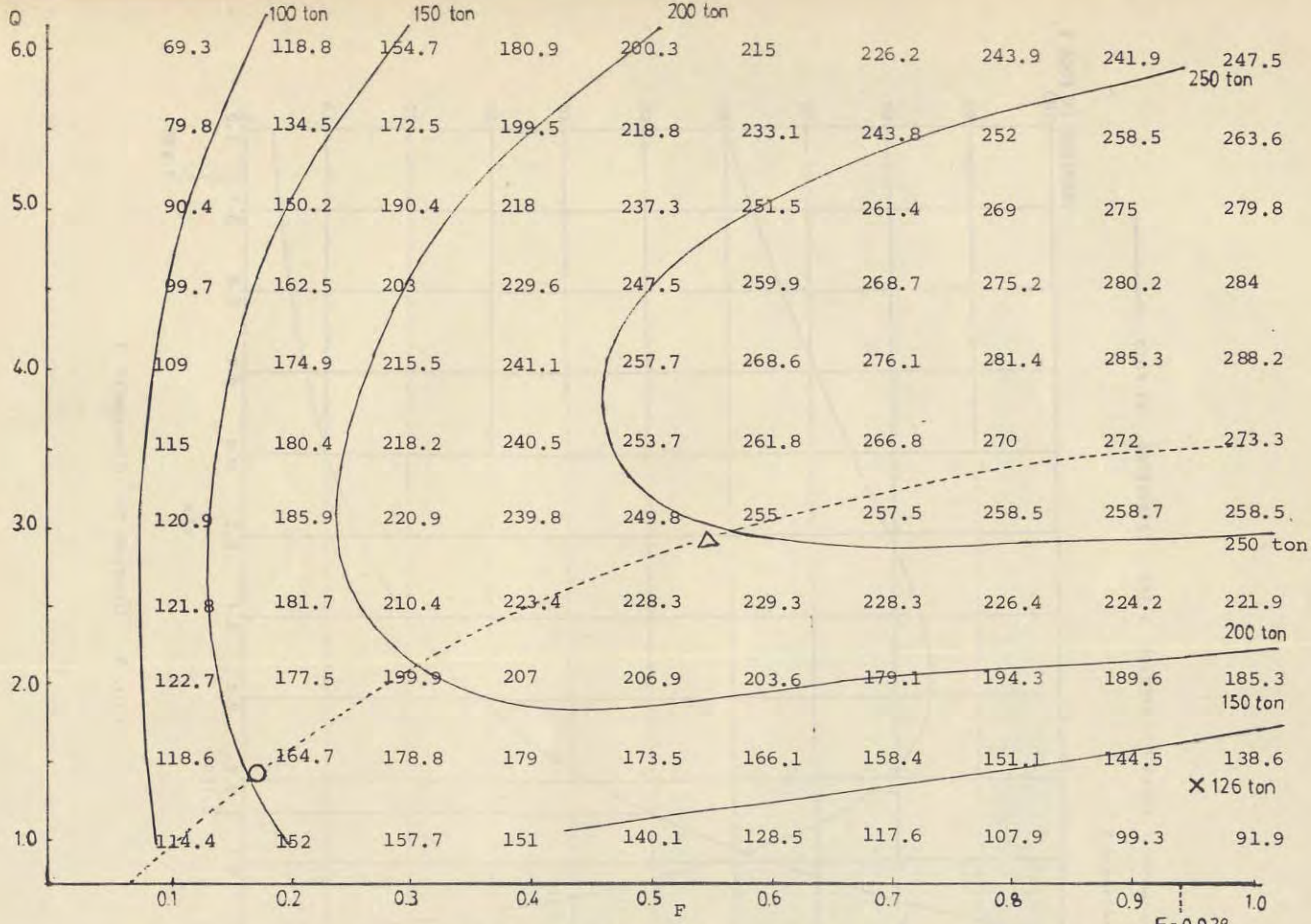


Fig. 7 Relation between rate of decrease of spawning adults and isopleth yield curve

× : Present status

O : Better status when fishing effort maintains the present level

△ : The most effective maximum sustainable yield can be provided that is the best status.

Table 1. Age and length

Age	Length mm.
1	137.7
2	240.3
3	320.4
4	381.3
5	432.5
6	457.5
7	490.7
8	531.8
9	549.7

Table 2. Anabolism and catabolism

Age (May)	Weight g	Amount of anabolism g/day	Amount of catabolism g/day	Growth per day g/day
1	50	0.445	0.0953	0.35
2	270	1.373	0.515	0.86
3	653	2.474	0.245	1.23
4	1133	3.573	2.150	1.41
5	1649	4.588	3.144	1.44
6	2154	5.483	4.107	1.38
7	2621	6.249	4.998	1.25
8	3036	6.893	5.789	1.10
9	3434	7.483	6.548	0.93

$a = 622.7$

$K = 0.2320$

$A = 0.00002082$

$b = 616.7$

$\alpha = 3A^{\frac{1}{3}}aK = 11.999$

$\beta = 3K = 0.696$

Table 3. Average age and survival rate

--- Table of K (S, Δ) ---

S \ Δ	1	2	3	4	5	6	7	8	9	10	∞
0.05	0.048	0.052	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053	0.053
0.10	0.091	0.108	0.107	0.111	0.111	0.111	0.111	0.111	0.111	0.111	0.111
0.15	0.130	0.166	0.174	0.176	0.176	0.176	0.176	0.176	0.176	0.176	0.176
0.20	0.167	0.226	0.244	0.248	0.250	0.250	0.250	0.250	0.250	0.250	0.250
0.25	0.200	0.286	0.318	0.328	0.332	0.333	0.333	0.333	0.333	0.333	0.333
0.30	0.231	0.345	0.396	0.416	0.424	0.427	0.428	0.428	0.429	0.429	0.429
0.35	0.259	0.404	0.478	0.512	0.527	0.534	0.537	0.538	0.538	0.538	0.538
0.40	0.286	0.462	0.562	0.615	0.642	0.655	0.661	0.664	0.666	0.666	0.667
0.45	0.310	0.517	0.647	0.724	0.768	0.792	0.805	0.811	0.815	0.816	0.818
0.50	0.333	0.571	0.733	0.839	0.905	0.945	0.969	0.982	0.990	0.995	1.000
0.55	0.355	0.623	0.819	0.957	1.050	1.114	1.155	1.181	1.197	1.208	1.222
0.60	0.375	0.673	0.904	1.078	1.206	1.298	1.363	1.408	1.439	1.460	1.500
0.65	0.394	0.721	0.988	1.201	1.368	1.496	1.594	1.667	1.721	1.760	1.857
0.70	0.412	0.767	1.069	1.323	1.534	1.705	1.844	1.955	2.043	2.111	2.333
0.75	0.429	0.811	1.149	1.444	1.701	1.922	2.110	2.269	2.403	2.515	3.000
0.80	0.444	0.852	1.225	1.563	1.868	2.142	2.387	2.605	2.797	2.966	4.000
0.85	0.459	0.892	1.298	1.679	2.034	2.364	2.670	2.954	3.215	3.456	5.667
0.90	0.474	0.930	1.369	1.790	2.195	2.582	2.953	3.308	3.647	3.969	9.000
0.95	0.487	0.966	1.436	1.898	2.351	2.795	3.232	3.659	4.079	4.490	14.000

Note: 1. K = average age - lowest age

2. Δ = highest age - lowest age

3. Steps to use this table

(1) Calculation of average age from age composition

(2) To obtain K value by Note 1

(3) To obtain Δ by Note 2

(4) To find K in the column of Δ in Table 2 and read S by use of interpolation.

Table 4. Age composition

Age	Catch per day per boat	Remarks
1	6.332	
2	3.855	fully recruited age
3	0.878	
4	0.361	

Table 5. Population size by age

Age X	Weight Wg	In the Sea		Catchable	
		Population number	Biomass	Population number	Biomass
1	50	87	44	59	30
2	270	36	97	36	97
3	653	10	65	10	65
4	1133	3	34	3	34
5	1649	1	16	1	16

Table 6. Example of calculation for diagnosis 2

$M = 0.357 \quad So = e^{-M} = 0.7$

$F = 0.936 \quad S = e^{-(M+F)} = 0.274$

$E = \frac{F}{M+F}(1-S) = 0.526$

					Present Status & population							Virgin stock				
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
Age X	Weight W_g	Fecundity H 104	Maturity MTR	Availability Q	N 104	NC 104	PC ton	C 104	Y ton	A 104	EGG 104	N	NC	PC	A	EGG
1	50	-	0	0.6763	87	59	29	31	15	0	0	87	59	29	0	0
2	270	-	0	1	36	36	97	19	51	0	0	61	61	164	0	0
3	653	184	0.2	1	10	10	65	5	34	2	181	43	43	280	9	787
4	1133	291	1	1	3	3	30	1.4	16	3	391	30	30	339	30	4346
5	1649	405	1	1	1	1	11	0.4	6	1	149	21	21	345	21	4232
6	2154	515	1	1	0	0	4	0.1	2	0	52	15	15	323	15	3772
7	2621	617	1	1	0	0	2	0	0.8	0	17	10	10	262	10	3160
8	3036	706	1	1	0	0	0.4	0	0.2	0	5	7	7	212	7	2532
9	3434	782	1	1	0	0	0.2	0	0.1	0	2	5	5	171	5	1964
Total					137	108		57	126	6	797	290	262		96	20793
Present/Virgin (rate of decrease in %)					47					5.9	3.8	100			100	100

Note 6 13 : Number of population in the sea

7 14 : Catchable number of population

8 15 : Catchable biomass

9 : Catch in number

10 : Yield

11 16 : Number of adults

12 17 : Number of eggs spawned

APPENDIX

Ordinary methods

The following items are interesting and useful topics concerning with ordinary methods. Arrangement order is not important. We can read any item in any order. Somewhere practical excercises are added.

1. Basic guide to theoretical statistics
2. Relative index of abundance
3. Theory of population change
4. Tagging and marking experiment
5. Reproduction mechanism
6. Sustainable yield and the maximum sustainable yield
7. Fisheries management
8. Prediction and forecasting
9. Ecosystem and carrying capacity (This item is omitted here)

I. Basic guide to theoretical statistics

Theoretical statistics is necessary for research planning and data processing. As it is very wide and deep in studying theoretically, I cannot describe it in short. Here only important parts, which are often used in population analysis, are introductively summarized. Details should be referred to specialistic literatures of statistics.

1. Fields of theoretical statistics

What are kinds of statistics? There are lots of fields and branches as follows:

Theory of probability;
Distribution function;
Sampling distribution;
Theory of sampling;
Theory of testing hypothesis;
Theory of inference;
Parameter estimate;
Design of experiment;

Quality control;
Variance analysis;
Multivariate statistical analysis;
Time series;
Simulation;
Monte Carlo method;
Operations research;
Information theory.

Some of the above items are explained briefly in the following sections.

2. Sampling distribution

2-1 Normal distribution

When type of distribution of random variable **x** is a normal distribution with mean m and standard deviation σ, denoted by $N(m, \sigma)$, probability density $f(x)$ is:

$$f(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\frac{(x-m)^2}{2\sigma^2}}$$

Namely,

$$\text{Prob.}\ \{t_1 \leq x < t_2\} = \int_{t_1}^{t_2} f(x)\,dx$$

If there are many random variable x_i $(i = 1, 2, ---i, ---n)$, with normal distribution $N(m_i, \sigma_i)$ respectively, distribution of newly compound variable L, defined by;

$$L = \sum_{i=1}^{n} \ell_i x_i$$

ℓ_i : constant

is normal distribution with

$$\text{mean } m_L = \sum_{i=1}^{n} \ell_i m_i$$

$$\text{standard deviation } \sigma L = \sqrt{\Sigma \ell_i^2 \sigma_i^2}$$

Putting

$$m_1 = m_2 = ---- = m_n = m$$

$$\sigma_1 = \sigma_2 = ---- = \sigma_n = \sigma$$

$$\ell_1 = \ell_2 = ---- = \ell_n = \frac{1}{n}$$

we obtain the well-known formulae as follows:

$$L = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$m_L = m$$

$$\sigma_L = \frac{\sigma}{\sqrt{n}}$$

2-2 Sampling distribution from normal distribution

The samples x_i (i = 1, 2, ------ n) from N(m, σ) can show the sample mean $\overline{x}$ and unbiased estimate of standard deviation s:

$$\overline{x} = \frac{1}{n} \Sigma x_i$$

$$s = \sqrt{\frac{\Sigma (x - \overline{x})^2}{n - 1}}$$

And then calculated values t;

$$t = \frac{(\overline{x} - m) \sqrt{n}}{s}$$

is t-distribution with degree of freedom (n-1). From this relationship we can estimate a confidencial interval of population mean m. And also, χ^2 calculated from the following equation,

$$\chi^2 = \Sigma \frac{(x_i - \overline{x})^2}{\sigma \quad 2}$$

is χ^2-distribution with degree of freedom (n-1).

2-3 F-distribution

Presume there are two X^2 distributions. The ratio between them represents F-distribution: that is,

$$F = \frac{\dfrac{X_1^2}{n_1}}{\dfrac{X_2^2}{n_2}}$$

where n_1 and n_2 are the degrees of freedom in X_1^2 and X_2^2 respectively.

2-4 Exponential distribution

Probability density of exponential distribution is

$$\lambda e^{-\lambda x} \quad (0 \leq x < \infty)$$

The population mean and standard deviation are calculated accordingly as follows.

$$\text{mean} = \frac{1}{\lambda}$$

$$\text{standard deviation} = \frac{1}{\lambda}$$

Putting $\overline{x}$ as a sample mean from an exponential distribution,

$$2n\lambda\overline{x}$$

is X^2 distribution with degree of freedom 2n. Therefore the confidence interval of population parameter λ can be calculated on the level of significance α by the following relation.

$$X_o^2 \leq 2n\lambda\overline{x} \leq X_1^2$$

where,

$$\text{Prob.}\ \{X^2 \leq X_o^2\} = \frac{\alpha}{2}$$

$$\text{Prob.}\ \{X^2 > X_1^2\} = \frac{\alpha}{2}$$

3. Theory of sampling

3-1 Random sampling

It is primarily important to select samples at random. Random sampling can be carried out by dice, by random number, etc.

Randomness cannot be easily attained.

The following notations are adopted here:

N : Total number of a population

n : Sampled number

x_i : Attributed values of samples i ($i = 1, 2$ ------ n)

m : Mean value of population

σ : Standard deviation of population

V : Variance

We want to know the total sum X (for instance, catch), mean $\overline{x}$ (for instance mean length), ratio p (for instance, ratio of juveniles to - catch).

In simple sampling, samples are selected at random without any classification of population.

3-2-1 Estimate of total sum X

$$X = \frac{N}{n} \sum_{i=1}^{n} x_i$$

$$V(X) = N^2 \frac{N-n}{N-1} \cdot \frac{\sigma^2}{n}$$

3-2-2 Estimate of mean $\overline{x}$

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$V(x) = \frac{N-n}{n-1} \cdot \frac{\sigma^2}{n}$$

3-2-3 Estimate of ratio p

We put x = 1 for samples which have a definitive properties and x = o for samples which have no definitive properties. Then:

$$p = \frac{1}{n} \sum_{i=1}^{n} x_i$$

$$V(p) = \frac{N-n}{N-1} \frac{\sigma^2}{n}$$

Because m = p and $\sigma^2 = p(i - p)$, $\overline{V}(p)$ can be transformed into:

$$\overline{V}(p) = \frac{N-n}{N-1} \cdot \frac{p(1-p)}{n}$$

3-2-4 Decision of sample size n

How many samples should we select from a population in estimating with a given accuracy?

This is a principal standpoint of sampling theory.

Now we introduce coefficient of variance ε as a measure of accuracy. In considering total sum X, coefficient of variance is:

$$\frac{\sqrt{V(x)}}{X} = \frac{N\sigma}{X} \sqrt{\frac{N-n}{n(N-n)}} = \frac{\sigma}{m} \sqrt{\frac{N-n}{n(N-1)}}$$

We want to estimate X with accuracy of less than ε . Number of samples n can be decided from the following equation:

$$\frac{\sigma}{m} \sqrt{\frac{N-n}{n(N-1)}} \leq \varepsilon$$

Although the values of ε is ordinarily 1% - 5%, ε may be 10% - 20% in research field of fisheries resources.

As ε becomes small (high accuracy), n becomes large.

As for estimate of mean $\overline{x}$, equation of C.V. is the same describe above.

As for estimate of ratio p, following equation is adopted:

$$\sqrt{\frac{1-p}{p}}\sqrt{\frac{N-n}{n(N-1)}} \leq \varepsilon$$

or

$$\sqrt{\frac{1}{N}\left(\frac{1}{r}-1\right)\left(\frac{1}{p}-1\right)} \leq \varepsilon \qquad N-1 \fallingdotseq N \quad \frac{n}{N} = r \quad \text{(sampling ratio)}$$

Before sampling, we don't know exact values of m, σ, p. Therefore we assume approximate values to such parameters and then we can calculate sample number n with the given accuracy. Relation between p, ε, N and n are presented in Table 1 and 2.

3-2-5 Aimed precision and achieved precision

The accuracy I mentioned in the above is aimed accuracy, not achieved precision.

Achieved accuracy or confidence interval of estimate can be calculated from ordinary statistical technique described in section of this Appendix.

For example, in the case of estimate of ratio p, the following equation gives us the interval estimate with a given probability.

$$\Pr\left\{\alpha \leq \frac{r-np}{np(1-p)} \leq \beta\right\} = \frac{1}{\sqrt{2\pi}} \int_{\alpha}^{\beta} e^{-\frac{t^2}{2}}\, dt$$

3-3 If we can separate a population into several strata within which properties of unit are almost uniform, stratified sampling is better than simple sampling, because we can estimate population parameters more clearly from rather smaller samples.

i = number of stratum, i = 1, 2, ----L

j = number of samples in i-stratum

$$X = \sum_{i=1}^{L} \left(\frac{Ni}{ni} \sum_{j} x_{ij} \right)$$

$$V(X) = \sum_{i=1}^{L} N_i^2 \frac{Ni-ni}{Ni-1} \frac{\sigma_i^2}{Ni}$$

$$\bar{x} = \frac{1}{N} \sum_{i=1}^{L} \left(\frac{Ni}{ni} \sum_{j} x_{ij} \right)$$

$$V(\bar{x}) = \frac{1}{N^2} \sum_{i=1}^{L} N_i^2 \frac{Ni - ni}{Ni - 1} \frac{\sigma_i^2}{ni}$$

(p is same with $\bar{x}$ as described previously)

3-4 Cluster sampling

In cluster sampling, units are not uniform within a cluster but properties of distribution in each cluster are expected as uniform. Classifying a population into M clusters, m clusters are sampled from the total M clusters. And all units in sampled clusters are measured. The methods to estimate population parameters are as follows:

$$X = \frac{M}{m} \sum_{j}^{m} \sum_{k=1}^{Nj} x_{jk}$$

$$V(X) = M^2 \frac{M-m}{M-1} \frac{\sigma_e^2}{m}$$

$$\bar{x} = \frac{X}{N}$$

$$V(\bar{x}) = \frac{M^2}{N^2} \frac{M - m}{M - 1} \frac{\sigma_e^2}{m}$$

(p is same with $\bar{x}$)

where $\sigma_e^2 = \frac{1}{M} \sum_{j=1}^{M} (X_j - \overline{X})^2$

3-5 Ratio estimate

$$x_j = \begin{cases} 1 \text{ (has A-attribute)} \\ 0 \text{ (no A-attribute)} \end{cases} \qquad Y_j = \begin{cases} 1 \text{ (has B-attribute)} \\ 0 \text{ (no B-attribute)} \end{cases}$$

mean $\overline{x}$ mean $\overline{y}$

Standard deviation Standard deviation

$$\sigma_x = P_A(1 - P_A) \qquad \sigma_y = P_B(1 - P_B)$$

$$\frac{P_A}{P_B} = \frac{\Sigma X_i}{\Sigma Y_i}$$

$$V\left(\frac{P_A}{P_B}\right) \doteq \left(\frac{P_A}{P_B}\right)^2 \frac{N-n}{N-1} \frac{1}{n} \left[\frac{\sigma_x^2}{\overline{x}^2} + \frac{\sigma_y^2}{\overline{y}^2} - 2\frac{\mathrm{cov}(xy)}{\overline{x}\,\overline{y}}\right]$$

1) A and B are exclusive events

$$\mathrm{cov}(xy) = - P_A P_B$$

$$\therefore (\text{C.V. of } \frac{P_B}{P_A})^2 = \frac{N-n}{N-1} \frac{1}{n} \left[\frac{1}{P_A} + \frac{1}{P_B}\right]$$

2) A and B are inclusive events

$$\mathrm{cov}(xy) = P_A(1-P_B)$$

$$* (\text{C.V. of } \frac{P_B}{P_A})^2 = \frac{N-n}{N-1} \frac{1}{n} \left[\frac{1}{P_A} - \frac{1}{P_B}\right]$$

3-6 Application for tagging experiment (how many fishes with tags we may release into the sea)

In considering tagging problem we may take:

N : Number of fishes in the sea.

n : Catch in number

R : Number of tagged fishes released in the sea

r : Number of recaptured tagged fishes in catch n.

p : $\frac{R}{N}$ or $\frac{r}{n}$

As for accuracy of p-estimate I already mentioned above : namely

$$\varepsilon^2 = \frac{1-p}{p} \quad \frac{N-n}{n(N-1)}$$

In planning a tagging experiment, first, we put N and n in rough figures and we may calculate an probable values of p. And secondly we decide the number of R as R = pN.

Saving calculation, Tables 1 and 2 can be prepared. These tables are of course for any kind of estimate of p (for instance sex ratio, rate of juvenile, species composition, pregnancy rate, maturity rate, etc.)

When we know capture-recapture data, we can calculate abundance of fish resources N by the following steps:

$$p = \frac{r}{n}$$

$$N = \frac{R}{p}$$

4. Analysis of variance

Two-way layout in the analysis of variance is explained here. Two factors of B and V take various values in an experiment. Observed results are x_{ij}, represented as follows;

B_i	V_j 1	2	3	-------------- k	Total
1	x_{11}	x_{12}	x_{13}	x_{1k}	$T_{1.}$
2	x_{21}	x_{22}	x_{23}	x_{2k}	$T_{2.}$
3	x_{31}	x_{32}	x_{33}	x_{3k}	$T_{3.}$
-					
-					
n	x_{n1}	x_{n2}	x_{n3}	x_{nk}	$T_{n.}$
Total	T.1	T.2	T.3	T.k	T

$$S_{BV} = \Sigma\,\Sigma\; x_{ij}^{\,2} - \frac{T^2}{N} \qquad N = mk$$

$$S_B = \frac{1}{K}\; \Sigma\; T_i^{\,2} - \frac{T^2}{N}$$

$$S_V = \frac{1}{m}\; \Sigma\; T_j^{\,2} - \frac{T^2}{N}$$

$$S_{BxV} = S_{BV} - S_B - S_V$$

Factor	Variation	Degree of freedom	Unbiased estimate of variance
B	S_B	m - 1	$S_B/(m - 1)$
V	S_V	k - 1	$S_V/(k - 1)$
BxV	S_{BxV}	(m - 1)(k - 1)	$S_{B\ V}/(m - 1)\ (k - 1)$
BV	S_{BV}	mk - 1	

Because the ratios of each unbiased estimates of variances against the denominator of unbiased estimate of variance of BxV are F-distribution, significances of each factor can be tested by the table of F-distribution. If V is not factor but repeating only, the following is adopted for testing instead of the above.

B	S_B	$m - 1$	$S_B/(m - 1)$
R(B)	S_V+S_{BxV}	$(k - 1)m$	$(S_V + S_{BxV})/(k - 1)m$

5. Fitting of linear regression and interval estimate

In order to obtain linear relation from a scatter diagram of two variables x and y, the following process of calculation is necessary.

Linear regression $\quad y = a + bx$

$$Sx = \sum_i (x_i - \overline{x})^2$$

$$Sy = \sum_i (y_i - \overline{y})^2$$

$$C = \sum_i (x_i - \overline{x})(y_i - \overline{y})$$

Then

$$b = C/Sx$$

$$a = \overline{y} - b\overline{x}$$

Coefficient of correlation $r = \sqrt{\dfrac{C}{Sx \cdot Sy}}$

The most likely estimate $\hat{y}$ is of course obtained by

$$\hat{y} = a + bx$$

And the interval estimate can be calculate by the following equations.

$$y = y \pm t(n-2, \alpha)\ \hat{\sigma} \sqrt{\frac{1}{n} + \frac{(x-\bar{x})^2}{Sx}}$$

where $\hat{\sigma} = (1 - r^2) \sqrt{\frac{Sy}{n - 2}}$

α : level of significance

t : t - distribution

In addition standard deviations of a and b are as follows:

$$\hat{\sigma}_a = \hat{\sigma} \sqrt{\frac{\Sigma\ x_i^2}{n\ Sx}}$$

$$\hat{\sigma}_b = \frac{\hat{\sigma}}{\sqrt{Sx}}$$

6. Time series

(1) C_t: observed values at time t.

(2) Self-correlation r_k

r_k is a coefficient of correlation between sets of C_{t-k} and C_t. r_o is of course equal to 1.

(3) Predictive equation

Putting C_t as value to be predicted at time t,

$$\hat{C}_t = \sum_{h=1}^{H} a_h C_{t-h}$$

where a_h must be obtained by solving the following equation.

$$\begin{pmatrix} r_o & r_1 & r_2 & \cdots & r_{H-1} \\ r_1 & r_o & r_1 & & r_{H-2} \\ r_2 & r_1 & r_o & & r_{H-3} \\ - & & & & \\ - & & & & \\ r_{H-1} & r_{H-2} & r_{H-3} & & r_o \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \\ \\ a_H \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \\ \\ r_H \end{pmatrix}$$

(4) Predicted values can be calculated from data in the past H period. However, H cannot be decided simply.

7. Multivariate statistical analysis (especially principal component analysis)

The general concept about principal component analysis in multivariate statistical analysis is described here briefly. A certain event is supposed to be expressed as p-dimensional vector, that is (x_1, x_2, ---x_p). The principal components of m-number ($m \leq p$) can be represented as follows:

$$X_1 = l_{11} x_1 + l_{12} x_2 + \text{-----------} + l_{1p} x_p$$

$$X_2 = l_{21} x_1 + l_{22} x_2 + \text{-----------} + l_{2p} x_p$$

$$X_3 = l_{31} x_1 + l_{32} x_2 + \text{-----------} + l_{3p} x_p$$

-

-

$$X_m = l_{m1} x_1 + l_{m2} x_2 + \text{-----------} + l_{mp} x_p$$

where l_{ij} are obtained by solving the following matrix.

$$(R - \lambda I)L = O$$

x are values normalized. R is a matrix of coefficient of correlation between (x_i, x_j). L is a matrix of l_{ij}. $\lambda_i (i = 1, 2 \text{-----} p)$ is given-values of matrix R, which can be calculated by solving the determinant $|R - \lambda I| = O$. I and O are identity matrix and zero matrix.

Loss of information in adopting m-principal component is

$$1 - \frac{1}{p} \sum_{i=1}^{m} \lambda_i$$

Therefore, we can discuss the problem under consideration from only a few principal components instead of lots of factors. It is enough if loss of information is less than 20%.

Calculations are advisable to use an electronic computer because actual calculations by hand are not so easy when p is larger than 4.

Table 1

Calculation of sample size n in estimating p (p = 0.1 - 1.0)

N: Population number **ε: coefficient of variance (accuracy)**

	P	100	1000	10000	100000	∞
ε = 0.01	0.1	100	989	9000	47371	90009
	0.2	100	976	8000	28571	40000
	0.3	100	959	7000	18921	23337
	0.4	99	938	6000	13044	15002
	0.5	99	909	5000	9091	10000
	0.6	99	870	4000	6250	6667
	0.7	98	811	3000	4110	4286
	0.8	96	714	2000	2000	2500
	0.9	92	526	1000	1099	1111
	1.0	0	0	0	0	0
ε = 0.05	0.1	97	783	2647	3475	3600
	0.2	94	615	1379	1575	1600
	0.3	90	483	854	925	933
	0.4	86	375	566	596	600
	0.5	80	286	385	398	400
	0.6	73	210	260	266	266
	0.7	63	146	169	171	171
	0.8	50	91	99	100	100
	0.9	31	43	44	44	44
	1.0	0	0	0	0	0
ε = 0.10	0.1	90	474	826	892	900
	0.2	80	286	385	398	400
	0.3	70	189	228	233	233
	0.4	60	130	148	150	150
	0.5	50	91	99	100	100
	0.6	40	63	66	67	67
	0.7	30	41	43	43	43
	0.8	20	24	25	25	25
	0.9	10	11	11	11	11
	1.0	0	0	0	0	0
ε = 0.20	0.1	69	184	220	225	225
	0.2	50	91	99	100	100
	0.3	37	55	58	58	58
	0.4	27	36	37	37	58
	0.5	20	24	25	25	25
	0.6	14	16	17	17	17
	0.7	10	11	11	11	11
	0.8	6	6	6	6	6
	0.9	3	3	3	3	3
	1.0	0	0	0	0	0

Table 2

Calculation of sample size n in estimating p (p = 0.01 ~ 0.10)

N: Population size ϵ : coefficient of variance (accuracy)

	p	100	1000	10000	100000	∞
$\epsilon = 0.01$	0.01	100	999	9900	90826	990000
	0.02	100	998	9800	83051	490000
	0.03	100	997	9700	76378	323333
	0.04	100	996	9600	70588	240000
	0.05	100	995	9500	65517	190000
	0.06	100	994	9400	61039	156667
	0.07	100	993	9300	57055	132857
	0.08	100	991	9200	53488	115000
	0.09	100	990	9100	50276	101111
	0.10	100	989	9000	47368	90000
$\epsilon = 0.05$	0.01	100	975	7984	28367	39600
	0.02	100	952	6622	16388	19600
	0.03	99	928	5640	11452	12933
	0.04	99	906	4898	8759	9600
	0.05	99	884	4318	7063	7600
	0.06	98	862	3853	5897	6267
	0.07	98	842	3470	5046	5314
	0.08	98	821	3151	4398	4600
	0.09	98	802	2880	3887	4044
	0.10	97	783	2647	3475	3600
$\epsilon = 0.10$	0.01	99	908	4975	9008	9900
	0.02	98	831	3289	4671	4900
	0.03	97	764	2443	3132	3233
	0.04	96	706	1936	2344	2400
	0.05	95	655	1597	1865	1900
	0.06	94	610	1355	1543	1567
	0.07	93	571	1173	1311	1329
	0.08	92	535	1031	1137	1150
	0.09	91	503	918	1001	1011
	0.10	90	474	826	892	900
$\epsilon = 0.20$	0.01	96	712	1984	2415	2475
	0.02	93	551	1091	1210	1225
	0.03	89	447	748	802	808
	0.04	86	375	566	596	600
	0.05	83	322	454	473	475
	0.06	80	281	377	390	392
	0.07	77	249	322	331	332
	0.08	74	223	280	287	288
	0.09	71	202	247	252	253
	0.10	69	184	220	225	225

2. Relative index of abundance

2-1 Standardization of fishing effort - by efficiency of fishing gear

If elements of

Efficiency of fishing gear;

Improvement of fishing boat and navigation apparatus crew; are quantified, corrections or standardization of fishing effort becomes easy. However, these elements change year-by-year. So, we can not standardize these with a slight trial.

Generally speaking, standardization will be carried out by comparison of catches of various kinds of gear operated in the same fishing ground in the same period.

Considering there are two kinds of gear (A and B), corresponding catch and effort are C_A, X_A and C_B, X_B.

Recognizing A as a standard, standardized total effort X is:

$$X = X_A + kX_B \quad \text{------------} \ (1)$$

$$k = \frac{C_B / X_B}{C_A / X_A} \quad \text{------------} \ (2)$$

where k is a correction coefficient.

Eqn. (2) is converted to the following;

$$X = \frac{C_A + C_B}{C_A / X_A}$$

This means we can obtain the standardized effort without estimation of k. When there are many kinds of fishing gear, the same way of calculation is derived, such as;

$$X = \frac{\Sigma \, Ci}{C_A / X_A} = \frac{\Sigma \, Ci}{C_A} X_A \quad \text{-----------} \ (3)$$

2-2 Standardization of fishing effort - by environmental conditions

The actual efficiency of operation is of course influenced by:

Calendar phenomena: Length of day time, tide;

Sea conditions: Wave, storm, current, tidal current;

Meteorological condition: Wind, fog storm, typhoon.

Therefore, correction should become necessary theoretically. However, such a procedure can be seldom adopted in the practical analysis because it is very difficult to obtain environmental factors during a long period based upon the same rule.

2-3 Effective overall fishing intensity and relative index of population size

Although the fishing effort is corrected and standardized, its influence upon abundance is not yet perfectly grasped. An operation in the fishing ground, where a fish school is dense, gives greater pressure on the abundance than an operation in an area where density is thin. Thus the concept of effective overall fishing intensity can be recognized, taking into account of distribution of fish density combined with distribution of fishing effort. The effective overall fishing effort is proportional to coefficient of fishing mortality.

Considering that fishing grounds are divided into subareas, we denote as follows:

i : No. of subareas.

A_i : Dimension of i-subarea.

X_i : Fishing efforts in i-subarea.

$\emptyset_i$: Fish density in i-subarea.

C_i : Catch in i-subarea.

We can calculate effective overall fishing effort according to calculation procedures described below.

Total area $A = \Sigma A_i$ ----------- (4)

Total population $N = \Sigma \emptyset_i A_i$ ----------- (5)

Mean density of fish $\emptyset = \frac{N}{A}$ ----------- (6)

and then

effective overall fishing effort

$$= \frac{1}{A} \frac{\Sigma \phi_i X_i}{\phi} \quad \text{------------ (7)}$$

$$\text{Effective fishing effort } \tilde{X} = \frac{\Sigma \phi_i X_i}{\phi} \quad \text{---------- (8)}$$

$$\text{Effectiveness of fishing effort } r = \frac{\tilde{X}}{X} \quad \text{---------- (9)}$$

where X is the total of an apparent raw effort, ΣX_i

If fish density is uniform in the whole area, r is equal 1. And if fishing efforts are distributed uniformly in the whole ground, r is equal 1, too. When operations are concentrated in dense area, r becomes larger than 1. On the contrary, many operations are distributed in the thin area, r becomes less than 1. Taking a total effort X is the same, the influence of effort upon fish population is stronger in case of $r > 1$ than in case of $r < 1$.

ϕ is usually unknown before analysis. Therefore, we adopt the general knowledge of:

$$\phi_i \propto \frac{C_i}{X_i} \quad \text{------------ (10)}$$

($\propto$ means proportional)

Substitute Eqn. (16) for ϕ_i value of the equations mentioned above, we obtain the following equations successively. (Here' (dash) means relative figures instead of absolute quantity).

$$\phi_i' = \frac{C_i}{X_i} \quad \text{------------ (11)}$$

$$N' = \Sigma A_i \frac{C_i}{X_i} \quad \text{------------ (12)}$$

$$\phi' = \frac{N'}{A} \quad \text{------------ (13)}$$

$$X = \frac{\Sigma C_i}{\phi'} \quad \text{------------ (14)}$$

$$f = \frac{1}{A} \frac{\Sigma C_i}{\phi'} = \frac{X}{A} \quad \text{--------- (15)}$$

According to Eqn. (11) - (15), we can calculate practically the effective overall fishing intensity (f), effective fishing effort (X) and relative index of population size (N').

2-4 Transformation of relative index of abundance

We can easily estimate a relative index of abundance. If we know the converting ratio into the absolute number in the past, hereafter the absolute abundance can be calculated conveniently by use of the rate.

2-5 Analysis only by index of population density

The following equation is the fundamental one.

$$\frac{dC}{dt} = q\ f\ N \text{ ------------------(16)}$$

Namely, this means that catch per unit time is proportional to an abundance of population if no change of f. If we use fishing effort X instead of fishing intensity f, the following equation is obtained:

$$\frac{dC}{dt} = q\frac{X}{A}N = qX\frac{N}{A} = q\ X\ \emptyset \text{ ----------- (17)}$$

That is; catch per unit time is proportional to density if no change of X. Putting $\frac{dC}{dt}$ as C_t because it is a catch in a unit time, Eqns. (16) and (17) are transformed into:

$$\frac{Ct}{f} = qN \qquad \text{--------------- (18)}$$

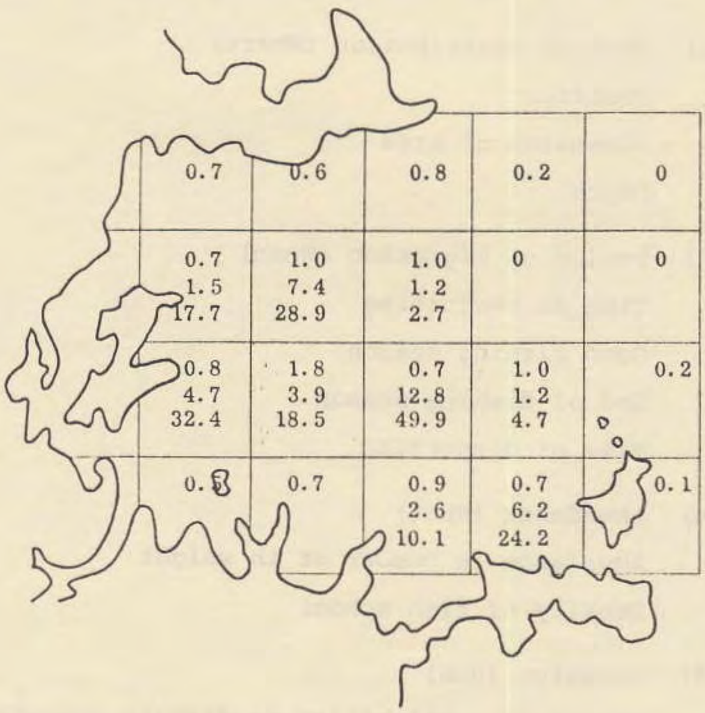
$$\frac{Ct}{X} = q\emptyset \qquad \text{--------------- (19)}$$

Namely, catch in unit time per fishing intensity or per fishing effort is proportional to abundance or density of population. Be careful we should use f for abundance and X, for density. Assuming there are no natural mortality, dispersion and recruitment, the population is decreased only by fisheries. Accordingly Eqn. (18) is modified as follows:

$$\frac{C_t}{f} = q\left[No - \sum_{t=0}^{t} C_t\right] = q\,No - q\sum_{t=0}^{t} C_t \text{ ------------ } (20)$$

(Practical excercise)

Catch statistics of KURUMA-prawn in Saeki Bay are shown below by area. Calculate the index of density, relative index of abundance, effective fishing effort, effective overall fishing intensity and effectiveness of fishing effort. --- in the middle of July, 1972 ---



Upper: Dimension of Area in km^2

Middle: Length of trawl operation in km

Lower: Number of prawns caught

3. Theory of population change

3-1 Representation of status of population

Representation of status of population can be described below.

(1) Quality of population (Who)

Kinds of species

Stock unit or local stock

(Difference of spawning grounds and behaviour)

Size of fish body

Growth stage and age

(2) Area of distribution (Where)

Position

Dimension of area

Depth

(3) Period of migration (When)

Time in recruiting

Good fishing season

End of fishing season

Time of dispersion

(4) Abundance (What)

Abundance in number or in weight

Density of fish school

(5) Occasion (How)

Migratory availability to fishing grounds

Spatial distribution of population density

3-2 Variation factors (Why)

A status of fish population shown in the above section varies because of various factors from the standpoint of temporal meaning as well as of spatial meaning. Causes are mainly classified into inner actions and outer pressures as follows:

- Variation factors
  - inner action (self-preservation)
    - reproduction
      - adult
      - eggs spawned
    - natural mortality
  - outer pressure
    - natural environment
      - physical factors (temperature, salinity, sea conditions)
      - Chemical factors
      - Biological factors (plankton, prey, competition between fishes)
    - artificial environment
      - fishing
      - release of juveniles
      - artificial change of fishing grounds (water pollution, reclamation work, artificial bank, draining)

The inner actions - self preservation - are proper natures which fish population have through natural selections of survival of the fittest. On the contrary, the outer pressures consist of two parts: environment of oceanographical conditions and human actions to utilize not only fish population but also sea properties.

3-3 Modeling

3-3-1 Importance of modeling

As described above, variation factors influence status of a population temporally and spatially. In order to solve a variation mechanism practically, modeling becomes necessary. If we make slight of modeling the obtained solutions would not be useful practically. Models should be set up after we understand physiology and ecology of fish and consider environmental factors concerned.

3-3-2 Classification of model

In treating a problem mathematically, the problem is transformed to formulation according to establishment of a model. And then we solve a mathematical formula analytically. It is usually a basic procedure. Classification of models is as follows, represented by two dichotomies.

Method	System of problem	
	Deterministic	Probabilistic
Deterministic		
Probabilistic		

The most models adopted in fisheries study are deterministic-deterministic ones. In the practical analysis, it is impossible to take account of all factors and conditions in establishing mathematical models. Even if we can establish formula, we cannot always solve them analytically. In such a case simulation of probabilistic model is one method to solve a problem. However, in the field of fisheries research, there are very few probabilistic simulations.

3-3-3 Cross action in discussing modeling

It is impossible to establish a model which represents or maps perfectly the actual fluctuation of population. Therefore, the following questions occur frequently. Some researchers who like the perfectness of 100% say "the model does not represent the actual status of population by simplifying". On the contrary researchers who like practicalness say "if we introduce many factors and conditions in modeling, we will not be able to solve. Accordingly any useful actions cannot be taken in diagnosis, management and prediction. It's like waiting for pigs to fly."

Only god knows the perfect reliability. As population dynamics is a applied science, we should set up a model by an adequate simplification.

3-3-4 Feedback

It is not so easy to establish a model mapping the aim even if we can simplify. Theoretical solutions obtained from the established model must be compared with the actual status of population. If we can find differences between them, we have to examine carefully the differences from various scientific points of view. And then we feedback such information in order to develop a more reliable model. In other words "plan-do-see" cycle is feedback. Such a cycle makes a progress of research. Fig. 1 shows an example of feedback indicated by flow chart.

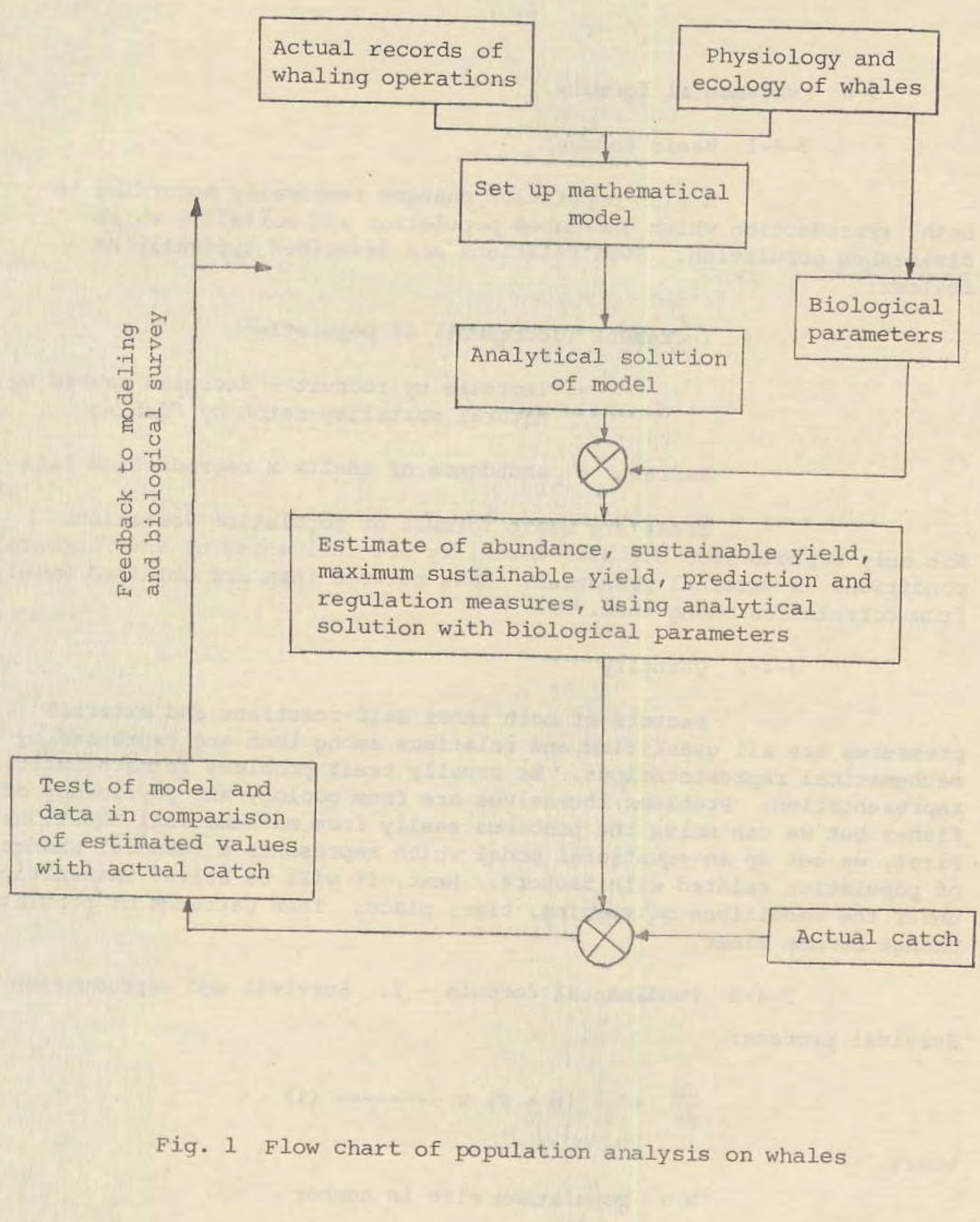


Fig. 1 Flow chart of population analysis on whales

3-4 Fundamental formula

3-4-1 Basic concept

A fish population changes temporally according to both, reproduction which increases population and mortality which diminishes population. Such relations are described typically as follows:

Increment (decrement) of population

= increase by recruit - decrease caused by natural mortality-catch by fishing

Recruit = abundance of adults x reproduction rate

These are basic formula of population variation. Not only reproduction but also mortality are influenced by environmental conditions as external pressures. Laws between them are obtained mainly from correlation among them.

3-4-2 Quantity

Factors of both inner self-reactions and external pressures are all quantified and relations among them are expressed by mathematical representations. We usually treat problems in mathematic representation. Problems themselves are from ecology and physiology of fishes but we can solve the problems easily from mathematical equations. First, we set up an equational model which represents a temporal change of population related with factors. Next, it will be solved mathematically under the conditions of species, time, place. Then patterns of population change become clear.

3-4-3 Fundamental formula - I. Survival and reproduction

Survival process:

$$\frac{dN}{dt} = -(M + F)\,N \text{ -------- } (1)$$

where,

N : Population size in number

M : Natural mortality coefficient

F : Fishing mortality coefficient

Reproduction mechanism:

$$R = K(A) \times A \text{ ------------ } (2)$$

$$A = f(N) \text{ ---------------- } (3)$$

A : Abundance of adults

K : Rate of reproduction

3-4-4 Fundamental formula - II. General propagation

$$\frac{dN}{dt} = (\lambda - M) N \text{ ------------ } (4)$$

λ : coefficient relevant to reproduction

$(\lambda - M)$ is a coefficient of net recruit. It is not constant but changeable with population size. Putting $\mu(\nu - N)$ instead of $(\lambda - M)$, then, Eqn. (4) is converted to:

$$\frac{dN}{dt} = \mu (\nu - N)N \text{ ----------- } (5)$$

If μ and ν are taken as constants, the solution of Eqn. (5) is a sigmoidal curve. Eqn. (4) and (5), are used when we can not take consideration with survival process and reproduction mechanism separately.

5-4-5 Transient or stationary state

Put

x : age
t : time

Population size N is a function of x and t, represented as N (x, t): namely transient phenomena. As a specific situation of $\frac{dN}{dt} = 0$, we consider it stationary state. In the actual analysis of population, we often assume it stationary. However, it is very rare that the actual status of population of fisheries may be recognized as stationary. Be careful in applying analytical results to an actual problem.

5-4-6 Catch

C, catch in number during period o - t, is described as

$$C = \int_o^t FN\, dt$$

or

$$\frac{dC}{dt} = FN \quad \text{--------- } (6)$$

If we know temporal changes of F and N, eqn. (6) will be solved.

Catch in weight, Y, is represented as follows:

$$Y = \int_{0}^{t} FN\,w\,dt$$

or ------- (7)

$$\frac{dY}{dt} = FNw$$

5-5 Solution of the simple model

As mentioned in the section of modeling, it becomes very difficult to solve a problem analytically, if we introduce many factors into modeling. Usual formulae used in stock assessment are derived from much simplified models.

This means that we should not adopt the formulae imitatively and blindly, which were presented in the past. Always examine from the standpoint of modeling originally.

5-5-1 When parameters are constant

In the simplest case, all parameters of Eqns. (1) - (5) are constant.

Solution of Eqn. (1)

Under assumption that M and F in Eqn. (1) are constant, the solution of Eqn. (1), is:

$$N = No\,e^{-(M + F)t} \text{ ------------ } (8)$$

This is very often used in dynamical analysis. Denoting N (t = 1) as an abundance after unit time,

$$N\ (t = 1) = No\ e^{-(M + F)}$$

or

$$\frac{N\ (t = 1)}{No} = S = e^{-(M + F)} \text{ --- } (9)$$

where S is a survival rate.

Solution of Eqn. (2)

Simpliest example of Eqn. (2) is

$$R = KA \quad \text{--------------} \quad (10)$$

K : constant

that is, the recruitment is proportional to the abundance of mature fishes. However, limited by spatial factors, the value of K decreases as A increases. Later in the Chapter on reproduction curve, this problem will be explained.

Solution of Eqn. (3)

If we know N as well as age at maturity, Eqn. (3), will easily be solved.

Solution of Eqn. (4)

If λ and M are constant,

$$N = Noe^{-(\lambda - M)\, t}$$

No.: initial population at time t = 0

If those are not constant solution would become complicated.

Solution of Eqn. (5)

If we take μ and ν as constant,

$$\frac{1}{N} = \left(\frac{1}{No} - \frac{1}{\nu}\right)e^{-\mu\nu t} + \frac{1}{\nu}$$

or $\qquad$ ----------- (12)

$$N = \frac{\nu}{1 + \left(\frac{\nu}{No} - 1\right)e^{-\mu\nu t}}$$

This graphical representation is sigmoidal as well-known for one example of spatial effect on population growth.

Solution of Eqn. (6)

$$C = \frac{NoF}{M+F}\left[1 - e^{-(M + F)t}\right] \qquad \text{-----------} \quad (13)$$

when t = 1 (unit time),

$$C(t = 1) = \frac{NoF}{M+F} \left(1 - e^{-(M + F)}\right)$$

or

$$\frac{C(t = 1)}{No} = E = \frac{F}{M + F} \left(1 - e^{-(M + F)}\right) \text{------} (14)$$

where E is a rate of exploitation.

Eqn. (14) is most frequently used in dynamical analysis. Nevertheless, do not forget the assumption that F and M are constant (simplified model) during period under consideration.

If a period of fishing season is less than 1 year, the rate of exploitation becomes, instead of Eqn. (14):

$$E = \frac{F}{M+F} \left(1 - e^{-(M + F)T}\right) \text{------------} (15)$$

where T: period of fishing season.

5-5-2 When parameters are variable

When M and F are functions of t, solution of Eqn. (1) is:

$$N = Noe^{-\int_o^t (M + F)\, dt} \text{------------} (16)$$

Unless M (t) and F (t) are represented concretely, we can not obtain a further formula than Eqn. (16).

Changes of M (t) and F (t) are complex, we must adopt a numerical method in solving.

(Practical excercise)

Find the theoretical formula when fishing efforts are not same during fishing season but change such as F_1 in the first half and F_2 in the latter half of the season.

4. Tagging and marking experiment

The capture-recapture experiment is a survey to observe special movements or temporal lapse of fishes which are marked or tagged before release into the sea. Methods of marking and tagging are as follows:

(1) Tag;
(2) Arrow-typed vinyl tube;
(3) Metal insert by gun;
(4) Fin cut;
(5) Dying;
(6) Special diet with particular material.

Data on such experiments are widely utilized in studying growth of body, migration, stock unit, rate of exploitation, estimate of abundance and so on. But we cannot neglect a lot doubts such as: Are there drop-out of tags? Behaviour of tagged fish is the same with natural fish? Reports of racapture are perfect and reliable?, etc. Capture-recapture process is only experimental method in population dynamics. Although theorical treatment has been developed, actual applications to the fisheries have not been put to practical use enough, because of many factors such as damage of function of fish body, change of survival, change of behaviour, fall out of tags, imperfect report of recapture, etc. And also a lot of meney is necessary to do a tagging experiment. Therefore, it becomes necessary to make a good plan in advance, considering how to analyse. Sometimes it is more useful of study on migration, rather than for estimate of population size.

In planning a tagging experiment as well as analysing recovery data, calculations may be done as explained in Chapter 1 of this Part IV. As a result of analysis we obtain population size N and accordingly;

$$E = \frac{C}{N}$$

(Practical excercise)

The following two tables indicate the records of caputre-recapture on red sea bream in the Inland Sea on ridly sea turtle in Mextle. Find the interval estimates of populations.

(red sea bream)

Number of release	Number of recaptured	Catch
3000	309	887

(ridly sea turtle during 1967-1971)

Number of release	Number of recaptured	Catch
990	63	9750

5. Reproductive mechanism

Reproductive mechanism is a most important element for any aim of population analysis - diagnosis, prediction and management. Unless aspects of reproduction are not clarified, it is impossible to do stock assessment theoretically and exactly.

5-1 Mature fishes and recruitment

If an age composition Nx is known in a considerably long period, the number of adult fishes can be calculated by the following equations:

$$A = \sum_{x_m} Nx$$

Putting R is an abundance at x_r (age at maturity), relations between R and A can be estimated. If we plot in a scatter diagram of R - A relation, points do not always lie on a curve but usually scatter very widely. So, reproductive curves are inductively drawn by eye or by curve fitting methods.

5-2 Survival process of juvenile fishes

Survival process is already expressed as

$$\frac{dN}{dt} = - MN \qquad \text{-------------} \ (1)$$

In the stage of juvenile, M is not constant but changes with spatial effect. Accordingly we can put:

$$M = M (x, N) = a (x) + b (x) N \qquad \text{----------} \ (2)$$

or

$$M = M (x, B) = a (x) + b (x) B \qquad \text{----------} \ (3)$$

where B is an abundance at specific time, for instance, the number of eggs spawned. Solving Eqn. (1) under the conditions of Eqn. (2) or Eqn. (3).

$$\frac{1}{N} = \frac{1}{P(x)} \left[Q\ (X) + \frac{1}{B} \right] \quad \text{------------- (4)}$$

or

$$N = P_{(x)}\ Be^{-Q(x)B} \quad \text{------------- (5)}$$

are derived. $P(x)$ and $Q(x)$ are functions of $a(x)$ and $b(x)$, not presented here.

Where,

$$N = \begin{cases} B & \text{at } x = o \\ R & \text{at } x = x_r \end{cases}$$

by substitution of x_r for x, Eqns. (4) and (5) are transformed into:

$$\frac{1}{R} = \frac{1}{P} \left(Q + \frac{1}{B}\right) \quad \text{------------- (6)}$$

$$R = PBe^{-QB} \quad \text{------------- (7)}$$

In this case P and Q become constants.

We can assume

$$B = hA$$

because the number of eggs is proportional to A as well as coefficient of fecundity h. Accordingly Eqns. (6) and (7) as functions of A become

$$\frac{1}{R} = \frac{1}{p} \left(q + \frac{1}{A}\right) \quad \text{------------- (8)}$$

$$R = pAe^{-qA} \quad \text{------------- (9)}$$

p and q are constants. The reproduction curves used usually are summarized into these two equations.

We must pay attention to the assumptions introduced in obtaining formula.

In Eqn. (8), the relation between $\frac{1}{R}$ and $\frac{1}{A}$ is linear regression and, in Eqn. (9), the relation between $\frac{R}{A}$ and A is linear regression.

Therefore, we can estimate p and q from analysis of linear regression.

5-3 Fecundity and reproduction

The rate of reproduction ($K = \frac{R}{A}$) is not indifferent into population size. This is the origin of species conservation. From this standpoint,

$$h = a - bA \qquad \text{--------------} \ (10)$$

is recognized. And natural mortality M' in juvenile stage also varies as population changes. So

$$M' = \alpha + \beta A \qquad \text{--------------} \ (11)$$

is assumed. Putting sex ratio is 50:50,

$$R = \frac{A}{2} he^{-M' x_m} \qquad \text{--------------} \ (12)$$

is derived, and substitute Eqns. (10) and(11) for h and M' of Eqn. (12). It is transformed into:

$$R = \frac{(a-bA)\ A}{2} e^{-x_m(\alpha + \beta A)} \qquad \text{-------} \ (13)$$

or

$$K = \frac{a-bA}{2} e^{-x_m} (\alpha + \beta A) \qquad \text{-------} \ (14)$$

This is a kind of reproduction curve obtained elegantly from deductive method.

If we can estimate h and M' anyhow, this method is convenient and useful.

(Practical excercise)

The relations between adults and returns are as follows about the pink salmon stock in the East Kamtchatka. Find the reproduction curve and draw the sustainable curve from which the maximum sustainable yield can be estimated.

Year class	Adults in thousand tons	Return in thousand tons
1947	3.0	16.6
48	5.0	14.6
49	8.3	18.8
50	7.3	14.3
51	9.4	30.3
52	7.1	10.9
53	15.1	25.8
54	5.3	1.5
55	12.5	56.2
56	0.5	3.9
57	22.8	55.2
58	1.7	11.4
59	18.0	29.3
60	4.7	10.3
61	13.7	32.9
62	4.9	14.4

(Practical excercise)

Construct the reproductive mechanism of green sea turtle in Mexico, using the following information on biology and ecology.

Age at maturity	6 age
Breeding cycle	b years
Number of eggs spawned by a female turtle	h
Natural mortality of mature turtle	M
Natural mortality of immature immature turtle	M'
Rate of hatch-out	a
Sex ratio	1:1

Note 1, b, h, M', M vary as population size changes. At the first approximation, we may suppose they will change linearly as population size.

Note 2. In the status of virgin stock the most probable estimates of b, h, M, and M' are 2 years, 200, 0.04583 and i.2. a is supposed to be about 80%.

6. Sustainable yield and maximum sustainable yield

The aim of diagnosis on fish populations is to judge their healthy or sick states; namely those populations are on the optimum level or not. If population levels are below the optimum level (over fishing) or above the optimum level (under exploited), it becomes practical and necessary to take adequate measures to recover their optimum levels or approach to the optimum level.

In order to diagnose a status of population, a sustainable yield and maximum sustainable yield are inevitable. There are many mathematical models and methods to solve. The most important factor is, of course, mechanism of reproduction. Unless we have no information about reproduction, we can not do an exact analysis. Mechanism of reproduction, as described above, depends upon changes of natural mortality, fecundity, pregnancy rate and growth of body with variations of size of population.

6-1 Sustainable yield

It is not a diagnosis to say that a population is increasing or decreasing, regardless of sustainability. Population size and catch should be maintained and sustained. A sustainable yield can be obtained considering balance of rate of reproduction to fishing.

Mathematically, it can be calculated, solving simultaneous equations relevant to both reproduction and process of survival.

6-2 Solution in sigmoidal curve

Sometimes growth law of biomass is represented by sigmoidal curve, specially when we can not pursuit each year-class, because of lack of knowledge about not only age composition but also reproductive mechanism. Representation by differential equation is

$$\frac{dN}{dt} = \mu\,(\nu - N) \qquad \text{----------- (1)}$$

and analytical solution is;

$$N = \frac{1}{a + b\,e^{-kt}} \qquad \text{------------ (2)}$$

where

$$\nu = \frac{1}{a}$$

$$\mu = ak$$

b: integral constant

a and k (accordingly μ and ν) can be obtained from the difference equation analysis. The b is to be calculated from actual values of N_t and estimates of both μ and ν, by the least square method. N does not exceed ν which is a saturated amount of population in virgin stock. Temporal increment in unit time is $\frac{dN}{dt}$. If we catch $\frac{dN}{dt}$ or $\mu\ (\nu - N)N$, the population level does not change. This is a sustainable yield C_s. Therefore,

$$C_s = \mu\ (\nu - N)N \qquad \text{-------------} \ (3)$$

Graphical representation of Eqn. (3) is parabola, where $C_s = o$ at $N = o$ and $N = \nu$. C_s takes the greatest value of $\frac{\mu\ \nu^2}{4}$ at $N = \frac{\nu}{2}$, which is the maximum sustainable yield $C_m = \frac{\mu\ \nu^2}{4}$ on the optimum level of population ($\frac{\nu}{2}$).

Such analysis is used to be applied for a population which is not clarified in detail on survival process and reproduction mechanism. When survival process and reproduction are evaluated from investigations, the method of analysis in the following section must be adopted.

6-3 Solution in which age composition is taken into consideration

The rate of reproduction K is necessary to obtain a sustainable yield. K is a function of M' (natural mortality in juvenile stage), x_m (age at maturity) and p (fecundity or pregnancy). Generally speaking, K is decided at the maturity age. But age at first capture or age at recruitment is not same as age at maturity. In the actual problem, therefore, K should be considered at a certain age conveniently, indifferent from biological meaning.

Fig. 1 shows a schematic age composition with population parameters in each stage. Mathematical expressions become easy from such a figure.

The rate of reproduction K is given by:

$$K = \frac{R_m}{A} \qquad \text{-------------} \ (4)$$

We can calculate easily from Fig. 7, the following amounts

$$R_c = R_m e^{-(t_c - t_m)M} = R_m So^{(t_c - t_m)} \quad \text{------- (5)}$$

where,

$$S_o = e^{-M} \quad \text{------- (6)}$$

The amount of adults between t_m and (t_{c-1})

$$= R_m \frac{1-S^{(t_c - t_m)}}{1 - S_o} \quad \text{------- (7)}$$

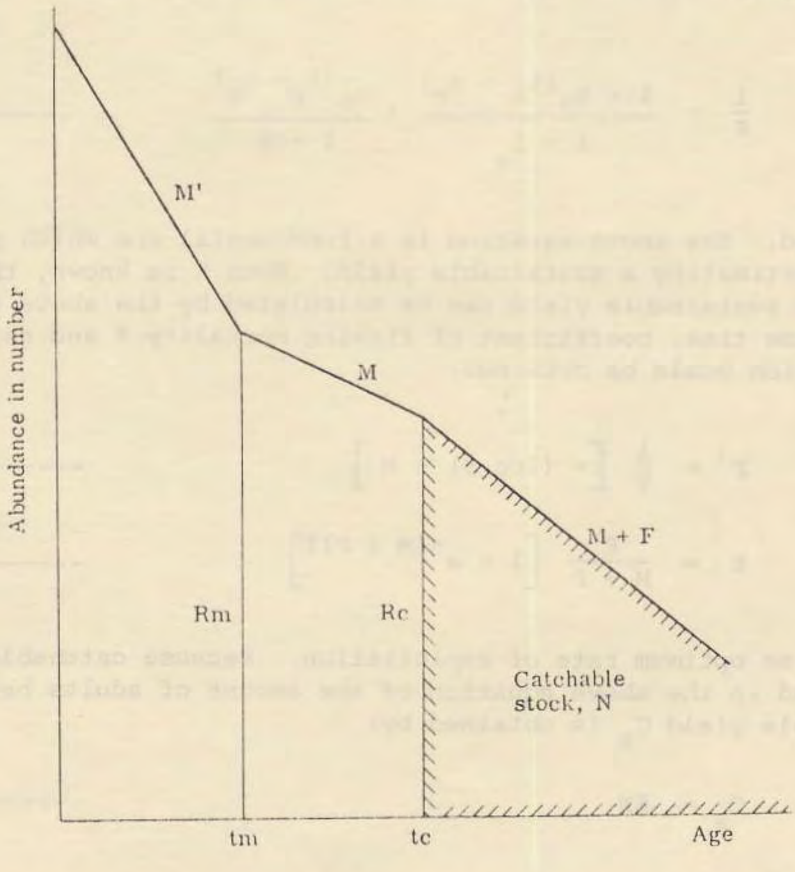


Fig. 1 Age composition and population parameters

M'	:	Natural mortality of juvenile fishes
M	:	Natural mortality of adults
tm	:	Age at maturity
tc	:	Age at first capture
A	:	Number of adult (over tm)
N	:	Catchable stock (over tc)
Rm	:	Recruitment at age tm
Rc	:	Recruitment at age tc

The amount of adults beyond x_c (catchable stock N)

$$= R_c \frac{1}{1-S} = R_m \frac{S^{(t_c - t_m)}}{1 - S} \qquad \text{---------- (8)}$$

The total amount of adults A

$$= R_m \left[\frac{1-S_o^{(t_c - t_m)}}{1 - S_o} + \frac{S_o^{(t_c - t_m)}}{1 - S} \right] \qquad \text{---------- (9)}$$

Solving Eqns. (4) and (5) as in simultaneous equations:

$$\frac{1}{K} = \frac{1 - S_o^{(t_c - t_m)}}{1 - S_o} + \frac{S_o^{(t_c - t_m)}}{1 - S} \qquad \text{---------- (10)}$$

is derived. The above equation is a fundamental one which plays a great role in estimating a sustainable yield. When K is known, the survival rate in a sustainable yield can be calculated by the above equation. At the same time, coefficient of fishing mortality F and rate of exploitation would be obtained:

$$F = \frac{1}{T} \left[- (\log S) - M \right] \qquad \text{----------- (11)}$$

$$E = \frac{F}{M + F} \left[1 - e^{-(M + F)T} \right] \qquad \text{----------- (12)}$$

Now E is an optimum rate of exploitation. Because catchable stock N is calculated in the above equation of the amount of adults beyond x_c, sustainable yield C_s is obtained by:

$$C_s = EN \qquad \text{----------- (13)}$$

The values of K, of course, changes as population size varies. Therefore, the above calculations must be carried out for various values of A. Thus the sustainable yield C_s can be calculated in various amounts of A. Plotting C_s in ordinate against A in abcisa, we have a dome-like curve, on which C_s = o at A = 1 (virgin stock). Between A = o and A = virgin stock, there is the highest point that is the maximum sustainable yield. The above equations are all derived from age composition shown in Fig. 1, where $x_m < x_c$. In actual fisheries $x_m > x_c$ sometimes occurs, because of catching juvenile fishes. In such cases the above equations should be modified.

In studying whale populations we may adopt pregnancy rate instead of fecundity. Fig. 2 shows a precedure to obtain the sustainable yield of sei whale in the Antarctic. Reproductive mechanism is estimated from a deductive method described in the previous Chapter 5, which is (3) in Fig. 2. The sustainable curve (5) in Fig. 2 can be calculated from the above equations, where $x_m = 10$ age $x_c = 17$ age.

6-4 Actual sustained catch

Sei whales become important species for whaling operations in the Antarctic because of a rapid decrease of fin whale population. We are afraid they follow the same fate with blue and fin whales. Therefore, we did stock assessment as shown in Fig. 2, taking into consideration with reproduction and recruit. The annual catch is, of course, known. So the abundance in the coming season can be calculated.

Results are shown in Table 1. This is an example in transient phenomena. The right column of Table 1, indicates the amount of catch which maintains the same level of abundance in the previous year. I call it "actual sustained catch". We can calculate it by solving in putting $N_{t+1} = N_t$.

The maximum sustainable yield of sei whale is 4300 individuals on the level decreased to 45% of the virgin stock.

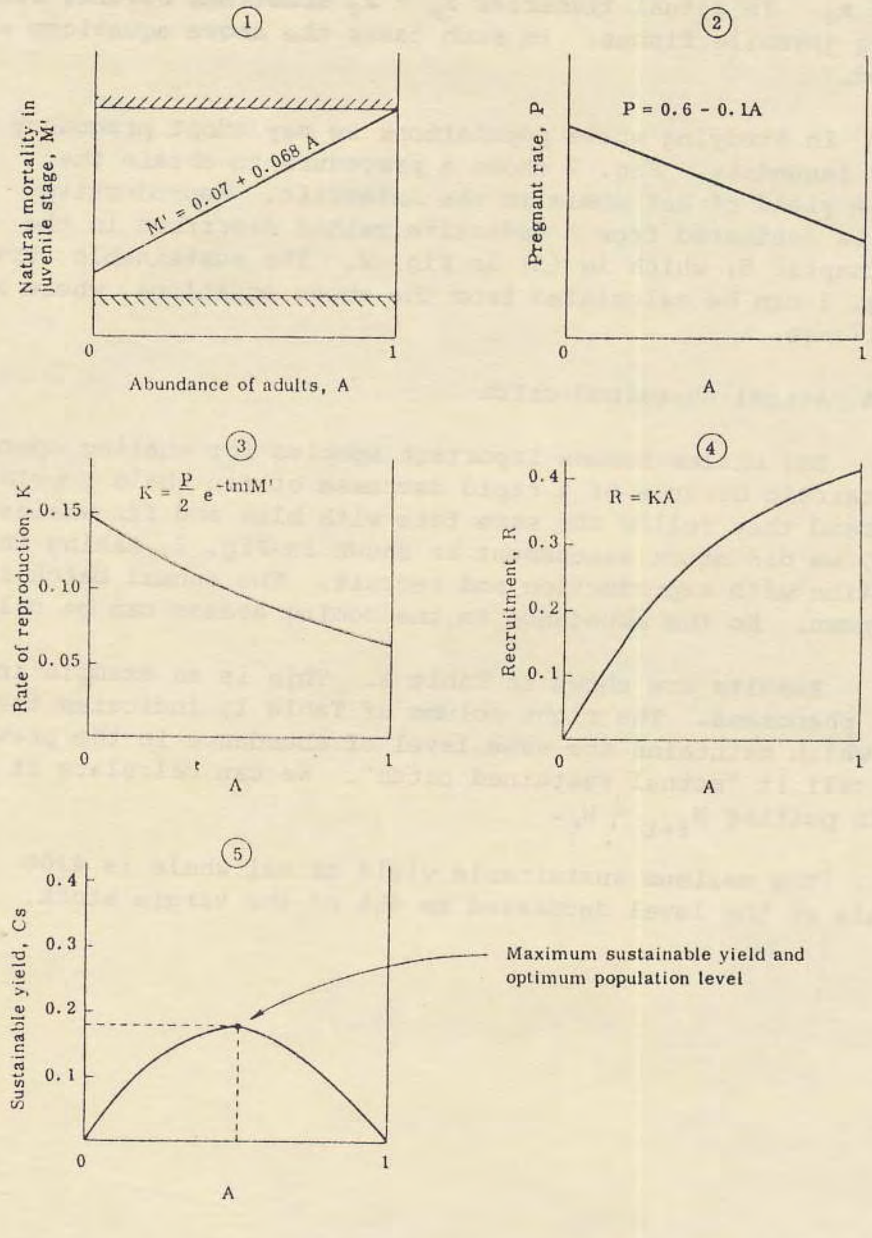


Fig. 2 How to obtain sustainable yield curve of sei whales in the Antarctic (Calculations proceed from ① to ⑤ in step)

Table 1 Calculation concerning population size, prediction and actual sustained catch of sei whales in the Antarctic

Season	Catch	Popula-tion	Uncatchable mature stock	Mature stock	Recruits	Population: After season	Population: In the coming season	Actual sustained catch
	C	N	A − N	A	R	N−C	$(N-C)e^{-M} + R$	
1955/56	276	150,000	86,426	236,426	9,440	149,724	149,741	0
1956/57	712	149,741	86,426	236,167	9,440	149,029	149,090	18
1957/58	2,385	149,090	86,426	235,516	9,440	146,705	146,912	61
1958/59	1,402	146,912	86,426	233,338	9,440	145,510	145,793	208
1959/60	3,234	145,793	86,426	232,219	9,440	142,559	143,027	233
1960/61	4,310	143,027	86,426	229,453	9,440	138,717	139,427	468
1961/62	4,749	139,427	86,426	225,853	9,440	134,678	135,642	710
1962/63	5,503	135,642	86,426	222,068	9,440	130,139	131,389	964
1963/64	8,286	131,389	86,426	217,815	9,440	123,103	124,796	1,250
1964/65	19,874	124,796	86,426	211,222	9,440	104,922	107,759	1,693
1965/66	17,583	107,759	82,615	190,374	9,440	90,176	93,941	2,837
1966/67	12,368	93,941	82,615	176,556	9,440	81,573	85,879	3,765
1967/68	10,357	85,879	82,615	168,494	9,440	75,522	80,209	4,306
1968/69	5,770	80,209	82,615	162,824	9,440	74,439	79,194	4,687
1969/70	5,857	79,194	82,615	161,809	9,440	73,337	78,162	4,756
1970/71		78,162						

6-5 Maximum sustainable yield

The sustainable yield can be calculated in any state of population size. The maximum sustainable yield is the greatest among them. It is nothing to say that the maximum sustainable yield is the most desirable status for human catching. The optimum level on which the maximum sustainable yield would be provided is always bellow the level of the virgin stock or unexploited stock. How the surplus for fishing increases by utilization of the properties of reproduction? This is the maximum sustainable yield. This does not mean there are so many fishes in the sea as the virgin stock.

The sustainable yield C_s is o at A = o as well as at A = virgin. In the middle of them it has a maximum point which is the maximum sustainable yield. In the analysis of sigmoidal curve, sustainable yield curve is a symetric and the optimum level is $\frac{1}{2}$ of the virgin stock. On the contrary, in the analysis based upon mechanism of reproduction, the curve is asymetric and the maximum point moves to the left hand side or right hand side of the middle, as shown in Figs. 2 and 3. Nevertheless, it usually lies near the middle, never bellow $\frac{1}{3}$. Therefore, we can make rough estimates of the optimum level in case of an unknown sustainable yield curve. In conclusion I emphasize that age composition of catchable stock and mechanism of reproduction should be paid attention to and accordingly mathematical model can be established.

6-6 Evaluation of present status of population

It is desirable to take adequate regulation measures, after we decide on which side of the optimum level the present status lies.

As for yellow croaker in the China Sea, sustainable yields and the maximum sustainable yield can be obtained, considering natural mortality, growth of weight and various values of x_c. Sustainable yield curves vary according to age at first capture, as shown in Fig. 3. In comparing with five curves in Fig. 3, the most effectual one among the maximum sustainable yield is obtained when x_c = 3 age and the optimum level is $\frac{1}{3}$ of the virgin stock. Fig 4 is a contour lines based upon Fig. 3.

Mark x represents the present status, □ indicates the maximum sustainable yield using the prevailing mesh size (54 mm) and △ is the most effectual maximum sustainable yield using nets with 80 mm mesh size.

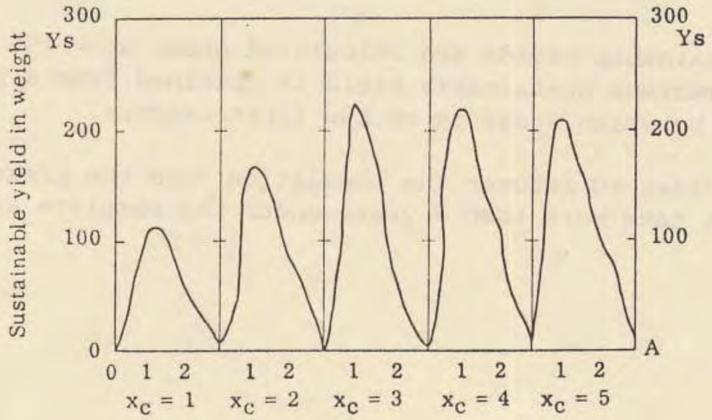


Fig. 3 Sustainable yield curves of yellow croaker population in the East China Sea, in various values of age at first capture

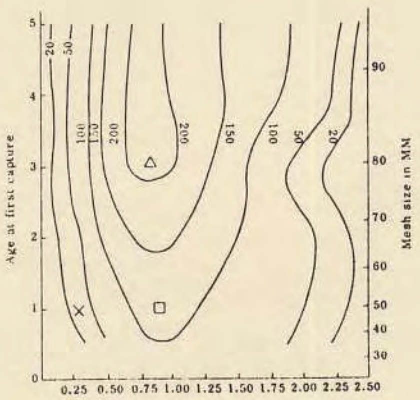


Fig. 4 Contour lines of sustainable yield of yellow croaker.

× : Present status (sustainable yield is 6,000 tons)

□ : Maximum sustainable yield in adopting $t_c = 1$ (30,000 tons)

△ : Most effective maximum sustainable yield (60,000 tons)

Summarized results from Figs. 3 and 4 are as follows:

(1) The optimum level of populations is $\frac{1}{3}$ of the virgin stock. The present status is less than $\frac{1}{10}$ of the virgin stock and extremely overfished.

(2) Sustainable yields are calculated under x_c = 1-5 age. The most effectual maximum sustainable yield is obtained from a net with 80 mm mesh size by which 3 age is of the first capture.

(3) In order to recover the population from the present to the optimum, it will take more than 4 years under the complete stop of catching.

(Practical excercise)

Find the sustainable yield and maximum sustainable yield of green sea turtle, using the reproductive mechanism obtained below.

Year	Catch ton	Effort (Number of fishermen)	Reproductive mechanism (in relative index)		Population ton
			Adult A (6 years ago)	Recruit R	
1948	56.5	16	1.0	0.0448	8468
49	94.0	20	1.0	0.0448	
50	198.5	32	1.0	0.0448	
51	129.4	48	1.0	0.0448	
52	77.6	42	1.0	0.0448	
53	152.4	54	1.0	0.0448	
54	176.4	80	0.994	0.0450	
55	194.7	88	0.983	0.0451	
56	228.4	141	0.962	0.0459	
57	480.4	171	0.949	0.0464	
58	495.0	171	0.943	0.0466	
59	566.7	181	0.928	0.0469	
60	803.6	266	0.911	0.0473	
61	554.7	249	0.893	0.0474	
62	618.7	342	0.871	0.0478	
63	664.1	342	0.822	0.0480	
64	774.9	361	0.774	0.0485	
65	730.6	361	0.719	0.0477	
66	523.0	342	0.64	0.0472	
67	467.9	336	0.591	0.0455	
68	396.2	345	0.537	0.0442	
69	379.2	350	0.478	0.0421	
70	421.0	432	0.407	0.0385	

Initial population size was estimated previously.

7. Fisheries management

Studies on fisheries management become of use when results of studies are able to be applied for demands and needs of administrative department as well as fishermen. The conservation of resources can be carried out through regulation of fisheries, except cultivation fisheries such as net culture of young fishes and release of juvenile fishes artificially hatched out. How do the quality and quantity of fishing effort influence upon the changes of abundance and catch, relevant to conservation as well as utilization of fish resources?

Qualities of fishing effort are kinds of fishing gear, construction of net, size of net, method of fishing, fishing season and fishing ground, etc. Quantities of fishing effort are number of vessels, number of operations, number of fishermen, number of days operated and number of hours operated, etc. Regulation measures can be led from relations between fishing effort and population parameter. And also we can say they can be led from combination of diagnosis with prediction of future catch.

Regulations should be achieved by the following items:

A. Body lengths limit;
B. Mesh size regulation;
C. Sanctuary;
D. No fishing period;
E. Regulation of quantity of fishing effort;
F. Catch limit.

Sometimes scientific regulation measures are not adopted by political standpoint. Against such a situation, scientists should be strong with good and available results of studies. In my opinion, the quota limitation of catch is the most important technique of regulation measure. The others are supplementary or secondary.

If we know the relationship between regulation measure, catchability and availability, it is possible to expect the change of population due to change of management.

8. Prediction of future catch

8-1 Elements of catch and classification of prediction methods

In weather forecasting, we use meteorological elements of climate such as: fine, cloudy, rain, snow, temperature, humidity, wind, atmospheric pressure, front and path of typhoon, etc. In the same meaning we use the elements on fish population and fisheries described in Chapter 5, in predicting future catches.

There are two kinds of prediction, long-term prediction and short-period prediction. Usually the former is for wide seas and the later for local areas. However, the methods to be applied are same for both situations.

Numerical methods for prediction are classified into the followings:

- Method of Prediction
  - Projective
    - correlation
    - time series
  - logical (dynamical)
  - analogeous

8-2 Correlation

If we can find statistical correlation among phenomena on abundance of population, catch, environmental factors and so on, in the past records, prediction of future catch can be estimated under consideration that the same relationships would continue in the future, although the reason remains unknown. This is the principle of projection, which used to be often adopted popularly. The information necessary in methods of correlation is three points as follows:

(1) The past records of catch during a considerably long period.

(2) Environmental factors in the past which seem to influence catches.

(3) Coefficients of correlation among (1) and (2) are usually used as a measure of relationships.

The prediction by correlation method will be carried out according to the schematic flow shown in Fig. 1.

The method of correlation is easy to understand, and data are also considerably easy to collect. Therefore, many examples are reported. But sometimes relationships among phenomena are apt to be a desk theory and subjective.

In addition, ranges of time and space are limited to narrow and short, because of lack of biological mechanism.

In order to endeavour to approach to scientific evidence, the interval estimate is sometimes given by the method in Chapter 1.

If catch would be related only a certain one factor, for instance water temperature, problem is very easy: In the actual phenomena, however, a lot of environmental factors influence change of catch getting intertwined with each other. As for such a situation, the principal component analysis (described in Chapter 1) should be adopted. In the principal component analysis, not each factor but components estimated from combination with each factor give us useful information concerning its influence upon future catches.

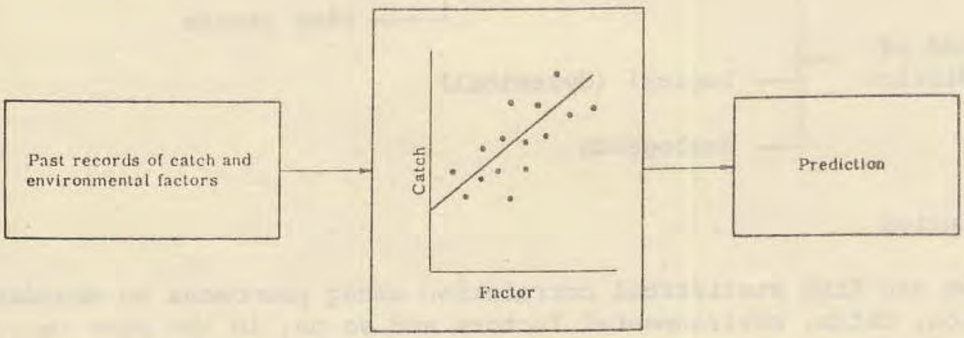


Fig. 1 Schematic flow chart of method of prediction by correlation

8-3 Time series

If there is no systematic survey in the past, a catch is only an available datum, without any other information. In such a case, catches in a considerably long period can be analysed by time series, which is one technique of information theory. This technique is useful to detect latent properties in random variations of natural phenomena.

There are several methods in time series analysis. One of them is a statistical treatment about components of:

1. long-term trend;
2. cyclic variation;
3. stationary stochastic variation

which are obtained by breakdown of temporal changes of phenomena.

A first long-term trend should be detected by temporal regression or moving average. Secondly cyclic changes are examined by periodical analysis. Thirdly, excluding a longterm trend and cyclic variations, stochastic variations are analysed by corelogram which is explained in Chapter 1.

Thus we can get laws which are burried in the random fluctuation of phenomena.

In predicting future catches, changes of long term trend as well as of cyclic variations are extrapolated. Stochastic component will be calculated statistically from the laws which are detected. Summing up the above three components, we can estimate future catches. Fig. 2 shows a schematic flow chart of analysis of time series. Computer programs are useful to calculate periodical analysis and stochatic analysis.

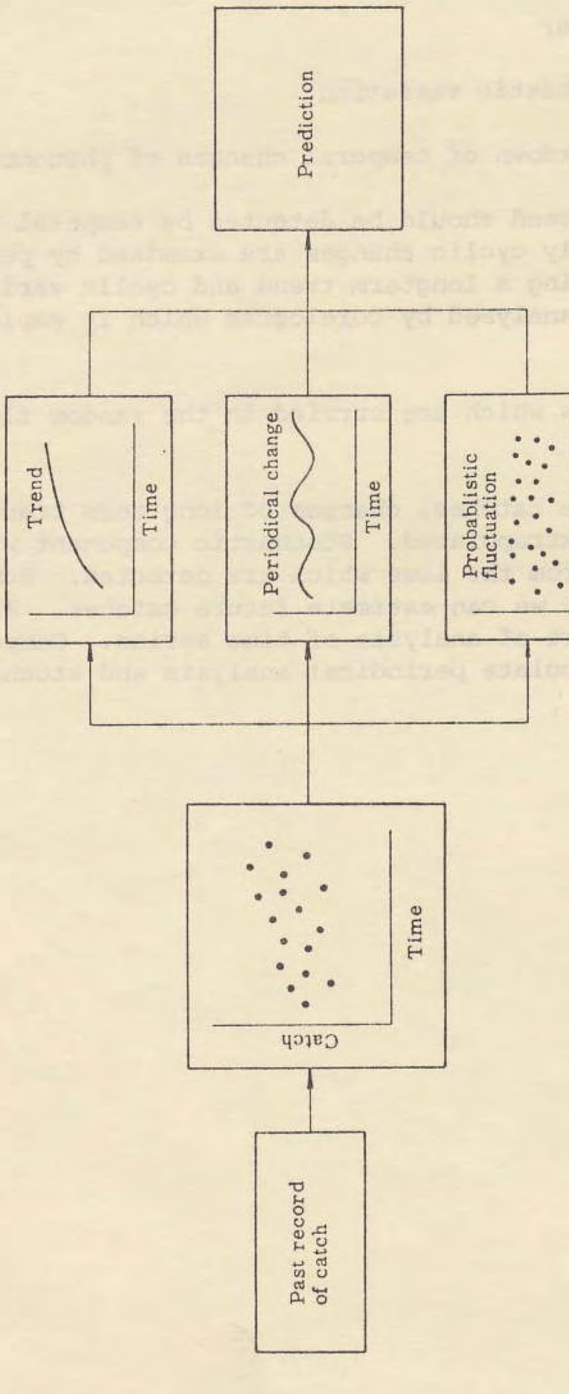


Fig. 2 Schematic flow chart of prediction by time series method

8-4 Logical method

In the projective methods we suppose that the aspects of variations of phenomena in the past will occur in the future too. But, if such premise is not good, the projective method is not available practically.

The movements of the earth, the moon and the sun can be obtained by solving equations of dynamics. For instance, we can know exactly the time of eclipse of the moon and the sun in advance. In the same meaning, if the system of population dynamics is correct, future catches will be able to be calculated numerically by, solving the equation of population change; this is a law of cause and effect. However, accuracy of the present population dynamics is not so strong as dynamics in physical study, because we can not grasp perfectly the ecology of marine resources. Fishes live in the wide ocean and man only sees a part of them. There are many species whose life cycles remain unknown yet. Although it is impossible to make mapping perfectly, we can establish a constructure for prediction, using survival process and reproduction mechanism described in Chapter 5.

Fig. 3 is a flow chart thus obtained. The abundances in Table 5 are calculated according to such a flow.

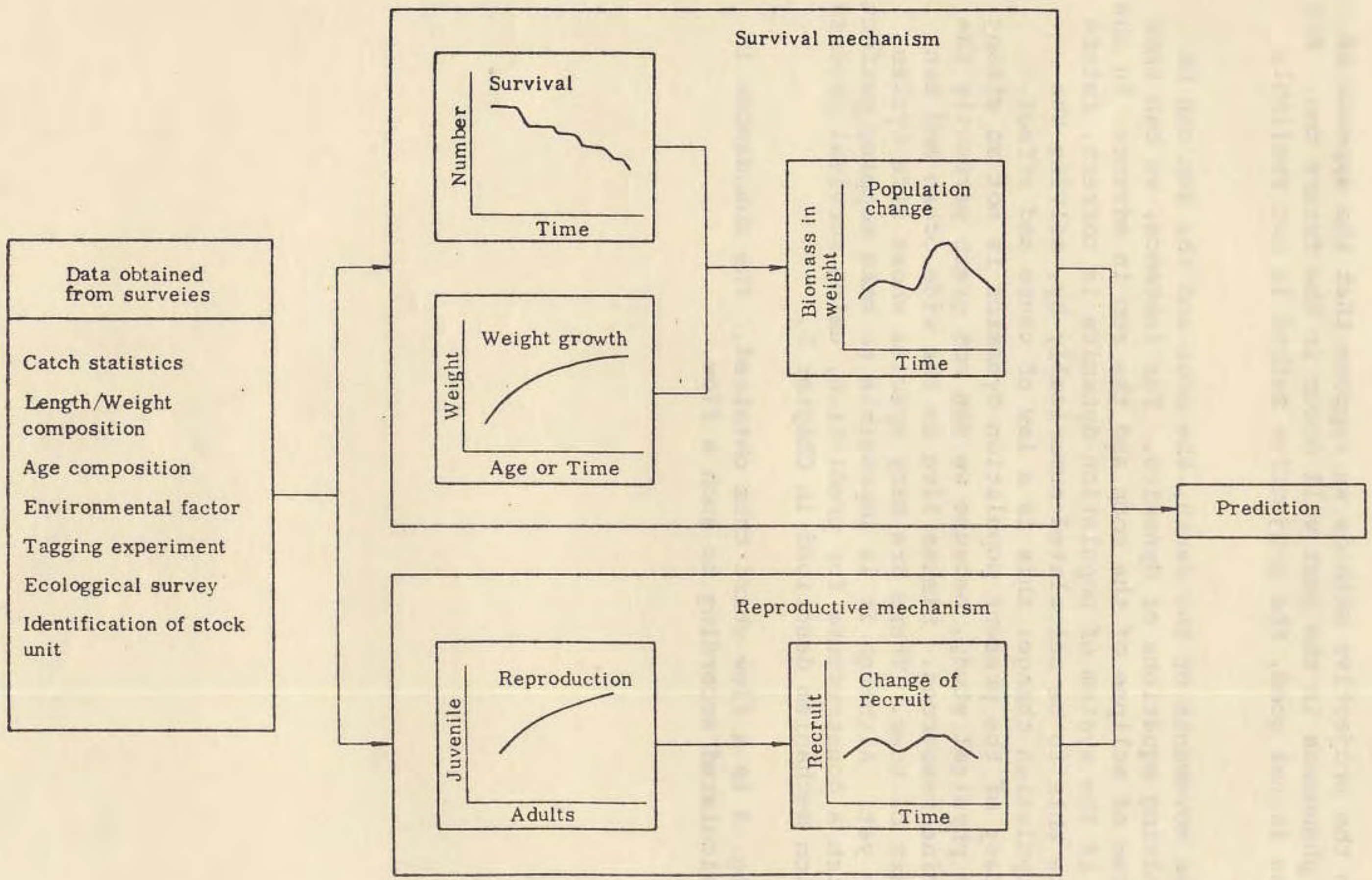


Fig. 3 Schematic flow chart of prediction by dynamical process

8-5 Analogy method

Consider a problem to forecast the path of a typhoon. The path of a typhoon may be solved mathematically as a problem of fluid dynamics, which is a dynamical method. However, partial differential equations of atmospheric pressure distribution to be solved are so complicated ones that it can not be solved analytically, or, it takes a lot of time even by use of computer. When we get a solution, a typhoon would be passed through. This is not prediction nor forecast: In such an occasion the analogy method becomes useful.

Namely, we search a similar atmospheric pressure distribution in the past records of typhoons. When we can find a similar one, we think that the present typhoon will go on the almost same path of the past. It is not absurd to expect the same effect from the same cause. This is a solution by analogy and then we get a prediction. Human beings cannot solve the problem but nature solves according to aerodynamics, as the fact. Although the method of analogy is a kind of logical method principally, solutions are obtained not through mathematical procedure but through a projective procedure.

Mathematical representation on survival process and reproduction mechanism are given in Chapters 3 and 5, but many times we cannot solve them analytically because of not only lack of conditions but also imperfectness of concrete figures. If we can find the same situation of fisheries and environmental factors in past records, prediction of future catches will become possible in following the past record.

In spring larva of anchovy are caught in Japan. And in Autum juveniles of anchovy (8-10 cm) appear in the fishing grounds. This is a survival process. When we know abundant larvae in spring, we can predict that in autumn an abundance of juveniles will be large, and vice-versa. However, we don't know population parameters such as coefficient of natural mortality, rate of availability and rate of exploitation. This is a feature of an analogy method.

8-6 Arrow diagram for prediction

Fig. 4 is a arrow diagram for prediction on yellow tail. I have example in correspondence to each job, but I have to omit here. Although it is very easy to say prediction, actual analysis of prediction is very difficult. A lot of study and earnest pursuit are necessary for prediction.

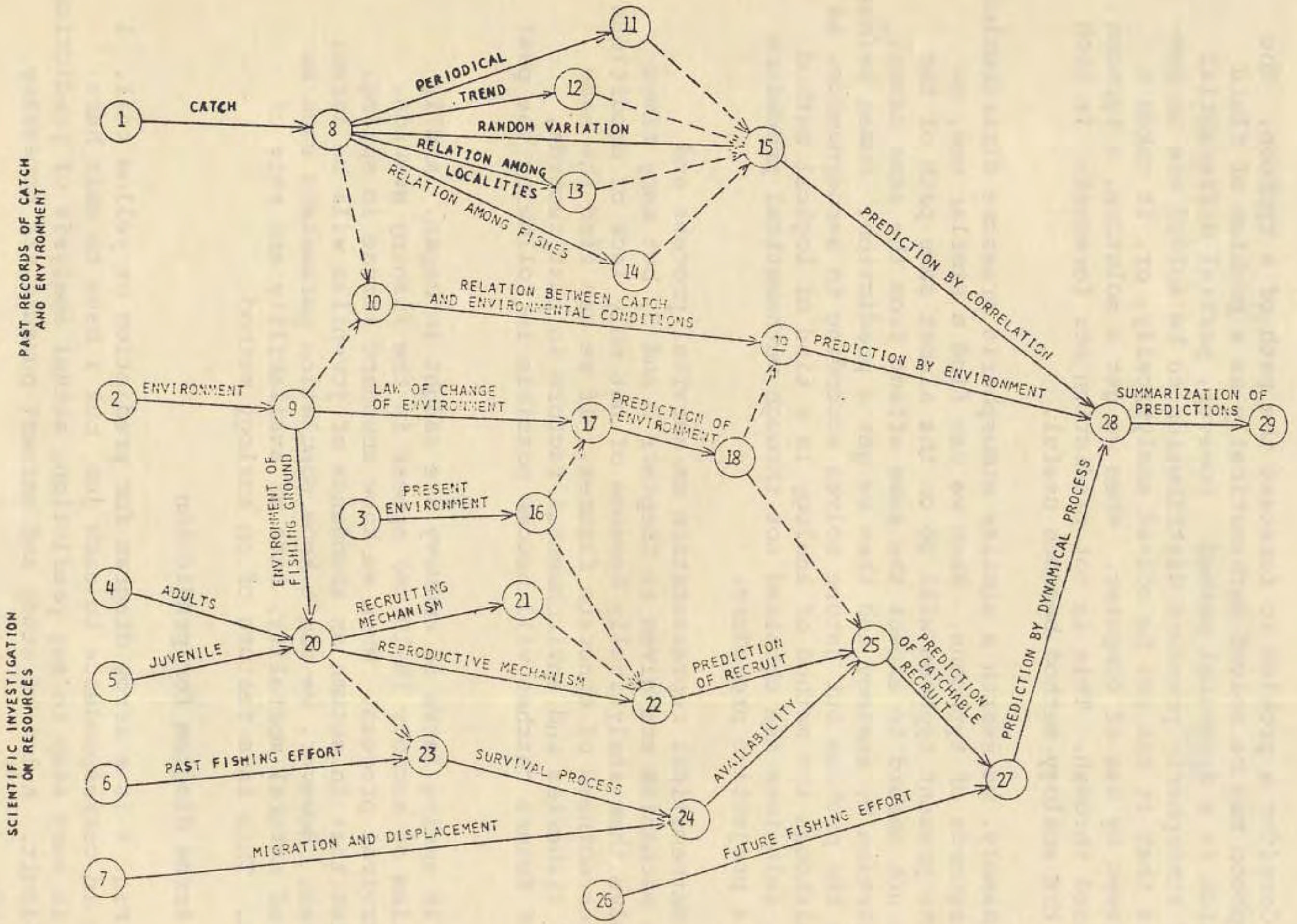


Fig. 4 Arrow diagram for prediction