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COASTAL NAVIGATION

by

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and
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Preface

To carry out any activities at sea one needs to know how to fix positions, set a course and measure the distance. In the area of fisheries, navigational knowledge is obviously essential. For instance, when a fishing boat is preparing to leave a port for a fishing ground, the planned sea route (Course) must be drawn on the charts, and the course in degrees and arrival time at the fishing ground must be decided.

After leaving the port, position fixes are taken during coasting. Eventually, terrestrial objects to fix positions by will not be visible, then positions must be fixed by means of observing celestial objects and the use of electronic navigational aids, in order to reach the fishing ground and to navigate safely and economically.

When the ship arrives at the fishing ground, she will move in pursuit of fish schools and position must be checked on the fishing ground chart in order to continue effective fishing activities. On the other hand, when one sets-up a stationary trapnet, a place to set it must be carefully selected and the position and depth of water measured beforehand. This is so that one can design the net, its size and construction, according to the depth of water and place it will be set. Also when one sets-up a conservation or prohibited fishing area, fish shelter or artificial reef, one has to determine the correct and exact position by latitudes and longitudes on a chart and set seamarkers in the water.

To carry out the aforementioned activities at sea, there are many devices and instruments which can be used to fix position e.g. sextant, divider, protractor, triangle ruler, magnetic compass, gyrocompass, chronometer, radar, echosounder, satellite navigator, loran receiver, decca receiver, omega receiver, doppler-speed log, sonar, direction finder, xy-plotter, course recorder, theodolite distance meter etc. These devices and instruments will help you fix position but, first you have to know: what latitudes and longitudes are; how to express ship's course and distance to be travelled; what difference of latitudes and longitudes are; what lines of position given by terrestrial objects are; the celestial bodies; how to observe and measure the distance and altitude of landmarks; how to measure the altitude of heavenly bodies; how to operate electronic instruments; what index error, chronometer error, variation, deviation and compass error are; how to check and select marine charts for navigation; how to make your own charts of fishing grounds and the topography of the sea-bed.

Without navigational knowledge, no one would understand the positioning of fishing gear in the water or information given by a meteorological agency concerning marine weather. Such knowledge is important to fishing boats in ensuring that their activities at sea are effective and safe. Navigational knowledge is also important in understanding the international laws of the sea such as: "CONVENTION ON THE TERRITORIAL SEA AND CONTIGUOUS ZONE" and "CONVENTION ON THE HIGH SEAS" and thus preventing troubles with other countries.

Trainees learning NAVIGATION at the SEAFDEC Training Department will receive instruction and gain practical experience, including ship-board training. During the course, it is recommended that you practice by yourself both at your desk and while on board the training ships, especially since we have only one year to complete the course, and only practice makes perfect!

Thank you,

M. [signature]

Masterfisherman and
Instructor of Navigation,
SEAFDEC

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COASTAL NAVIGATION

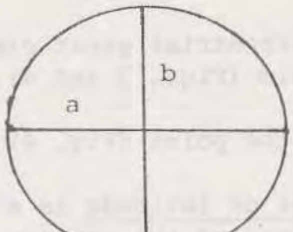
The earth

Nine planets orbit the sun, the inner four planets, Mercury, Venus, Earth and Mars, are mainly solid. The next four, Jupiter, Saturn, Uranus and Neptune, are large and mainly of gas. The outermost planet, Pluto, is the odd one out, it is quite small and believed to be solid.

a = Semimajor axis = 6378.3880 km
b = Semiminor axis = 6356.9119 km
$a - b \doteqdot 21.5$ km

$$f = \frac{a - b}{a} \doteqdot \frac{1}{297} \text{ (f = flattening)}$$

Fig. 1. The earth (Ellipsoid)



The earth is one of the planets in the Solar System, the third farthest from the sun and the earth orbits it in 365.24 days. The earth rotates on its axis in about 24 hours. The earth consists of three major parts: the core, the mantle and the crust. The core is mainly made up of iron and nickel and gives the earth the magnetic field and this is the cause of variation and deviation of a magnetic compass.

Oceans cover about 60 per cent of the earth's crust and the continents cover about 40 per cent of it. The earth has one satellite, the moon. The movements of tidal currents are mainly under the effect of the moon.

Basic Definitions

(1.) A GREAT CIRCLE AND A SMALL CIRCLE

A great circle is the line of intersection of a sphere and a plane through the center of the sphere (Fig. 2).

This is the largest circle that can be drawn on a sphere.

The shortest line on the surface of a sphere between two points on that surface is part of a great circle.

On the spheroidal earth the shortest line is called a *geodesic* (Fig. 2). $P_1 \sim P_2$

A small circle is the line of intersection of a sphere and a plane which does not pass through the center of the sphere (Fig. 2).

(2.) A MERIDIAN AND THE PRIME MERIDIAN

A meridian is a great circle through the geographical poles of the earth. Hence, all meridians meet at the poles, and their planes intersect each other in a line, the polar axis (Fig. 3).

The term meridian is usually applied to the upper branch only, that half from pole to pole which passes through a given point. The other half is called the lower branch (Fig. 4).

The prime meridian used almost universally is that through the original position of the British Royal Observatory at Greenwich, near London, U.K. (Figs. 4 and 5).

(3.) The equator is the terrestrial great circle whose plane is perpendicular to the polar axis (Figs. 2 and 4).

It is midway between the poles (Fig. 4).

(4.) A parallel or parallel of latitude is a circle on the surface of the earth, parallel to the plane of the equator (Fig. 2).

It connects all points of equal latitude.

(5.) The equator, a great circle, is a limiting case connecting points of 0^{o} latitude.

(6.) The poles, single points at latitude 90^{o}, are the other limiting case. All other parallels are small circles (Fig. 2).

(7.) Position on the earth

A position on the surface of the earth (except at either of the poles) may be defined by two magnitudes called coordinates (Figs. 5 and 6).

Those customarily used are latitude and longitude (Fig. 6).

A position may also be expressed in relation to known geographical positions.

(8.) Latitude (L, lat.) is angular distance from the equator, measured northward or southward along a meridian from 0^{o} at the equator to 90^{o} at the poles (Fig. 5).

It is designated north or south to indicate the direction of measurement.

(9.) The difference of latitude (D. lat.) between two places is the angular length of arc of any meridian between their parallels (Fig. 5).

It is the numerical difference of the latitudes if the places are on the same side of the equator, and the sum if they are on opposite sides.

It may be designated north or south when appropriate.

Exercise 1 (For solution see Fig. 17)

(1) Determine the difference of latitude between two places;

departure from 18° - 50!0 N.
arrival at 34° - 18!0 N.

(2) Determine the D. lat. between two places;

departure from 16° - 26!5 N.
arrival at 20° - 46!3 S.

(3) Determine the latitude of arrival, when a ship sails northward from the latitude 15° - 26!4 N. and difference of latitude is 259!4 nautical miles.

(4) Determine the latitude of arrival when a ship sails southward from the latitude 5° - 36!2 N., and the difference of latitude is 586!8 nautical miles.

(10) Longitude (λ. long) is the arc of a parallel or the angle at the pole between the prime meridian and the meridian of a point on the earth, measured eastward or westward from the prime meridian through 180° (Fig.5).

It is designated east or west to indicate the direction of measurement.

(11) The difference of longitude (D. Lo.) between two places is the shorter arc of the parallel or the smaller angle at the pole between the meridians of the two places (Fig. 5).

If both places are on the same side (east or west) of Greenwich, D. Lo. is the numerical difference of the longitudes of the two places; if on opposite sides, D. Lo. is the numerical sum unless this exceeds 180°, when it is 360°minus the sum.

The distance between two meridians at any parallel of latitude, expressed in distance units, usually nautical miles, is called departure (p. Dep.). (Fig. 15).

It represents distance made good to the east or west as a ship proceeds from one point to another (Fig.15).

Its numerical value between any two meridians decreases with increased latitude, while D. Lo. is numerically the same at any latitude (Fig. 15).

Either D. Lo. or Dep. may be designated east or west when appropriate.

Exercise 2 (For solution see Fig. 17)

(5) Determine the difference of longitude (D. Lo.), when a trawler departs from the longitude 135° - 30'E and ends its journey at 145° - 16'E.

(6) Determine the D. Lo., when a longliner starts from 164° - 28'E and ends up at 174° - 13'W.

(7) Determine the longitude of arrival, when a purse-seiner starts from the longitude 137° - 28'E, and proceeds 354' eastward (that is, D. Lo. is 354').

(8) Determine the longitude of arrival, when a fishery patrol boat starts from the longitude 2° - 27'E and proceeds 425' westward (that is, D. Lo. is 425').

(9) Determine the D. Lat. and the D. Lo.

a. Departure $\left\{ \begin{array}{l} \ell.\ 34^{\circ} - 26'N \\ \lambda.\ 138^{\circ} - 39'W \end{array} \right\}$ arrival $\left\{ \begin{array}{l} \ell.\ 16^{\circ} - 57'N \\ \lambda.\ 152^{\circ} - 16'W \end{array} \right\}$

ℓ : latitude, λ : longitude.

b. Departure $\left\{ \begin{array}{l} \ell.\ 18^{\circ} - 42'N \\ \lambda.\ 14^{\circ} - 26'E \end{array} \right\}$ arrival $\left\{ \begin{array}{l} \ell.\ 13^{\circ} - 35'S \\ \lambda.\ 17^{\circ} - 40'W \end{array} \right\}$

(10) Determine the position of arrival.

a. Departure $\left\{ \begin{array}{l} \ell.\ 32^{\circ} - 27'N \\ \lambda.\ 114^{\circ} - 30'E \end{array} \right\}$ where : D. Lat. = 328'S
D. Lo. = 256'W

b. Departure $\left\{ \begin{array}{l} \ell.\ 7^{\circ} - 38'N \\ \lambda.\ 5^{\circ} - 47'W \end{array} \right\}$ where : D. Lat. = 623'S
D. Lo. = 528'E

c. Departure $\left\{ \begin{array}{l} \ell.\ 12^{\circ} - 10'N \\ \lambda.\ 94^{\circ} - 16'E \end{array} \right\}$ where : D. Lat. = 250'S
D. Lo. = 224'E

d. Departure $\left\{ \begin{array}{l} \ell.\ 7^{\circ} - 40'N \\ \lambda.\ 98^{\circ} - 20'E \end{array} \right\}$ where : D. Lat. = 90'S
D. Lo. = 180'W

e. Departure $\left\{ \begin{array}{l} \ell.\ 8^{\circ} - 14'N \\ \lambda.\ 103^{\circ} - 56'E \end{array} \right\}$ where : D. Lat. = 134'S
D. Lo. = 56'W

(11) Distance, as customarily used by the navigator, refers to the length of the rhumb line connecting two places.

This is a line making the same oblique angle with all meridians.

Meridians and parallels (including the equator) which also maintain constant true directions, may be considered special cases of the rhumb line.

Any other rhumb line spirals toward the pole, forming a loxodromic curve or loxodrome (Fig. 9).

Distance along the great circle connecting two points is customarily designated the great circle distance.

(12) Speed is rate of motion, or distance per unit of time. A knot, the unit of speed commonly used in navigation, is a rate of one nautical mile per hour. The expression "knots per hour" refers to acceleration, not speed.

(13) Direction on the earth

Direction is the position of one point relative to another, without reference to the distance between them. In navigation, direction is customarily expressed as the angular difference in degrees from a reference direction, usually north of the ship's head.

Compass directions (north, northeast, south by west etc.) or points (of $11\frac{1}{4}^{o}$ or $\frac{1}{32}$ of a circle) are seldom used by modern navigators for precise directions.

(14) Course is the horizontal direction in which a vessel is steered or intended to be steered, expressed, as angular distance from north, usually from 000^{o} north, clockwise through 360^{o}.

The course is often designated as true, magnetic, or compass as the reference direction is true north, magnetic north, or compass north, respectively.

Course made good is the single resultant direction from the point of departure to the point of arrival at any given time (Fig. 10).

Course over ground is used to indicate the direction of the path actually followed, usually a somewhat irregular line (Fig. 10).

Course line is a line extending in the direction of a course.

In making a computation, it is sometimes convenient to express a course as an angle from either north or south, through 90° or 180°.

In this case, it is designated course angle and should be properly labelled to indicate the origin (prefix) and direction of measurement (suffix).

Thus, N 45°E (= Co. 045°), N 160°W (= Co. 200°)

S 26°E (= Co. 154°)

(15) Track is the intended or desired horizontal direction of travel with respect to the earth and also the path of intended travel (Fig. 10).

The track consists of one or a series of course lines from the point of departure to the destination, along which it is intended the vessel will proceed.

(16) Heading is the direction in which a vessel is pointed, expressed as angular distance from north, usually from 000° north, clockwise through 360°.

Heading should not be confused with course. Heading is a constantly changing value as a vessel oscillates or yaws, back and forth across the course due to the effects of sea, wind, and steering error.

(17) Bearing is the direction of one terrestrial point from another, expressed as angular distance from a reference direction, usually from 000° at the reference direction, clockwise through 360°.

When measured through 90° or 180° from either north or south, it is called bearing angle, which has the same relationship to bearing as course angle does to course.

Bearing and azimuth are sometimes used interchangeably, but the azimuth is better reserved exclusively for reference to the horizontal direction of a point on the celestial sphere from a point on the earth.

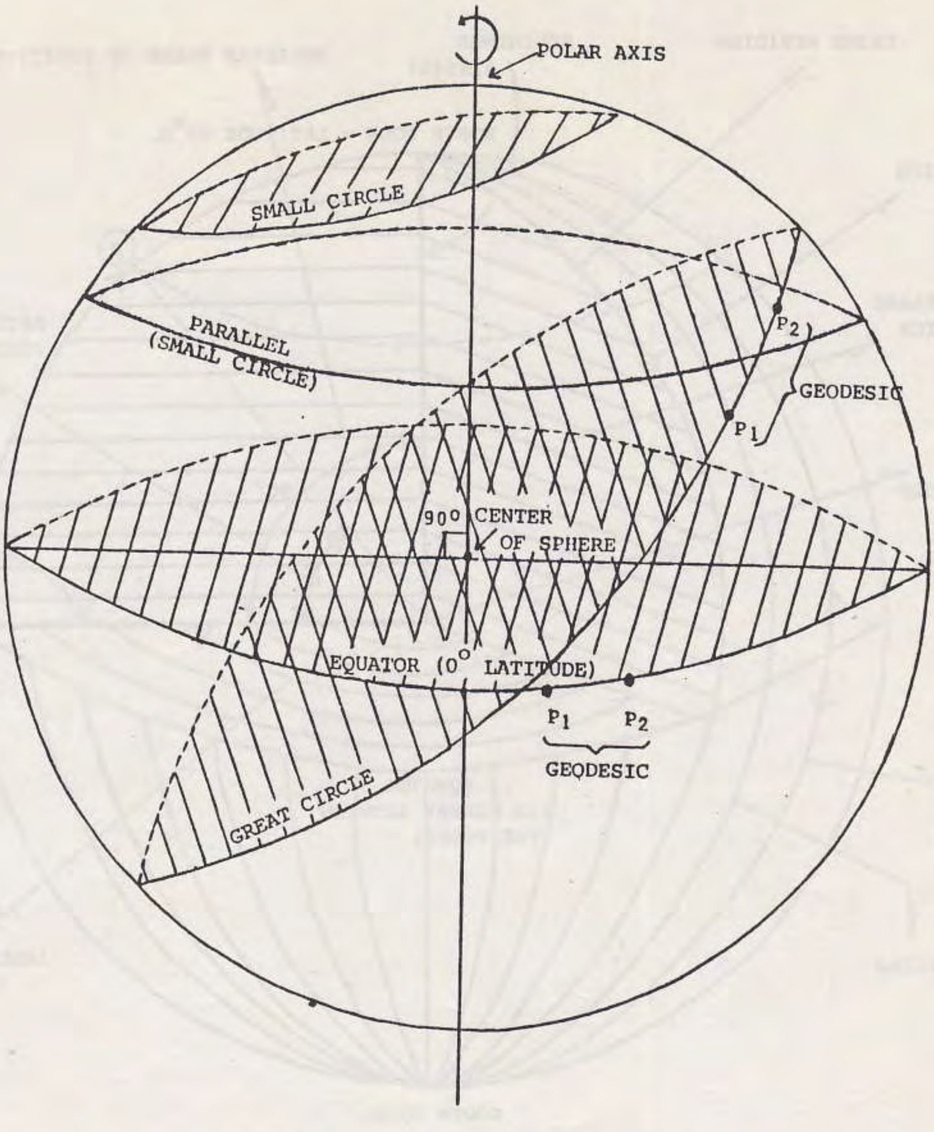


Fig. 2 Circles on the Earth.

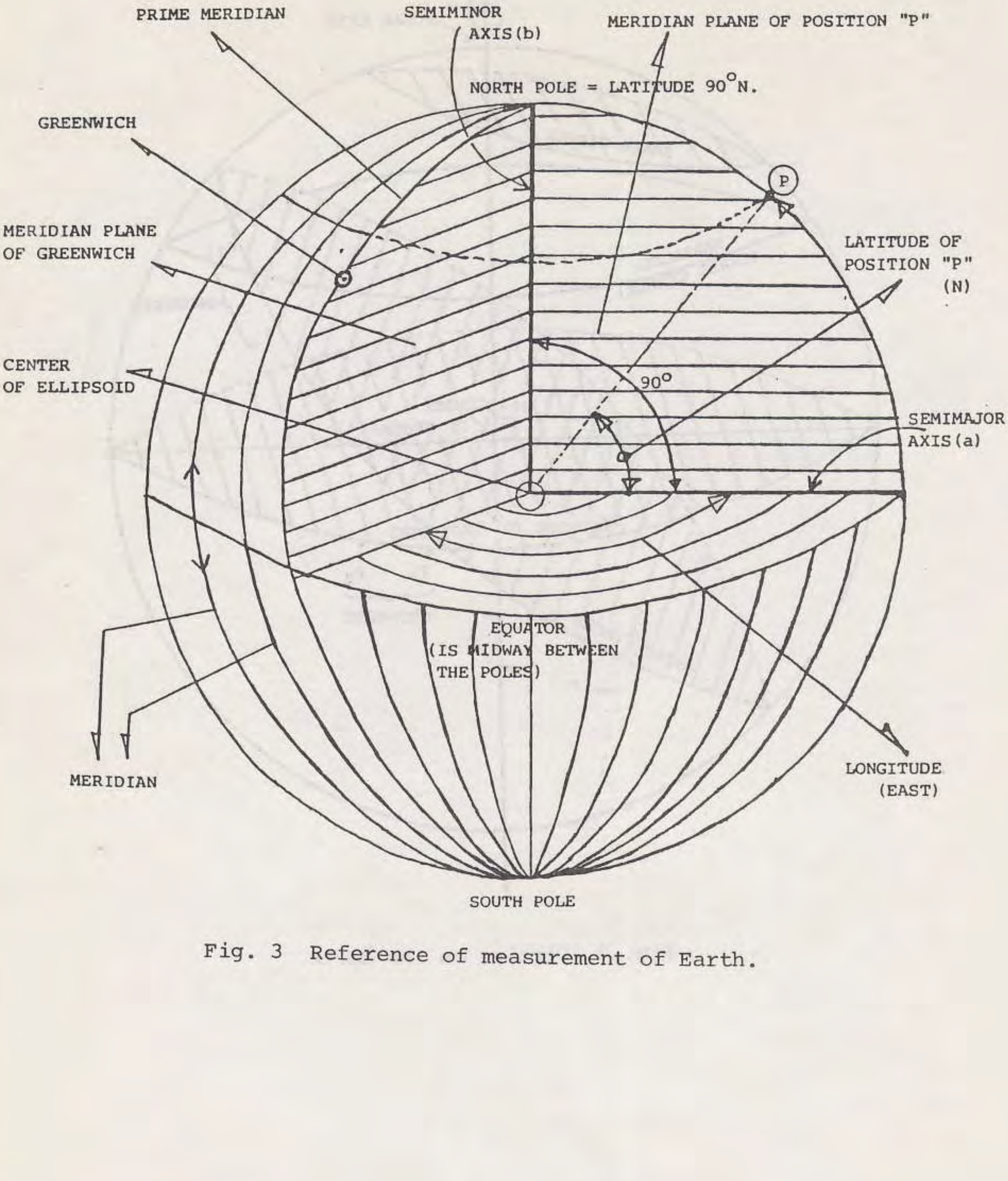


Fig. 3 Reference of measurement of Earth.

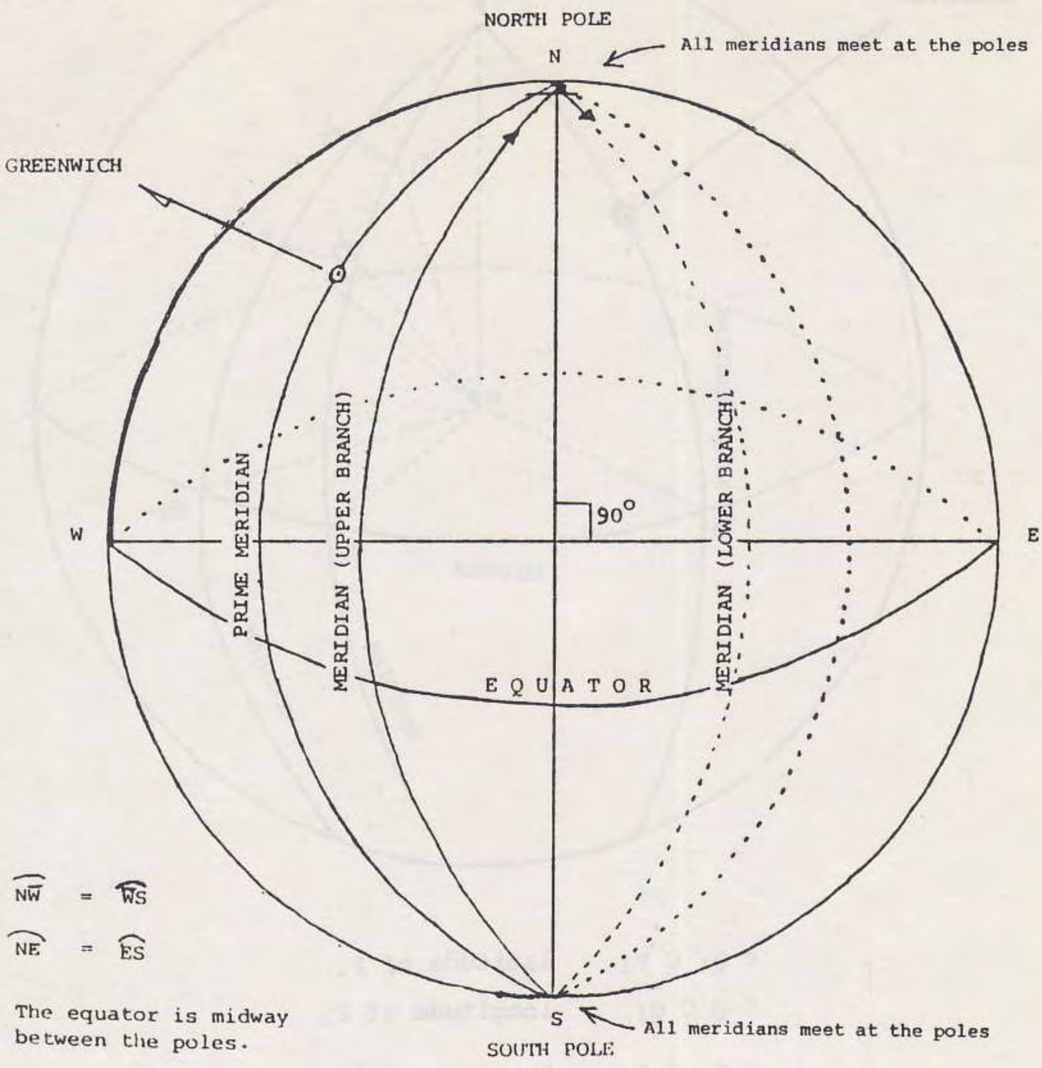
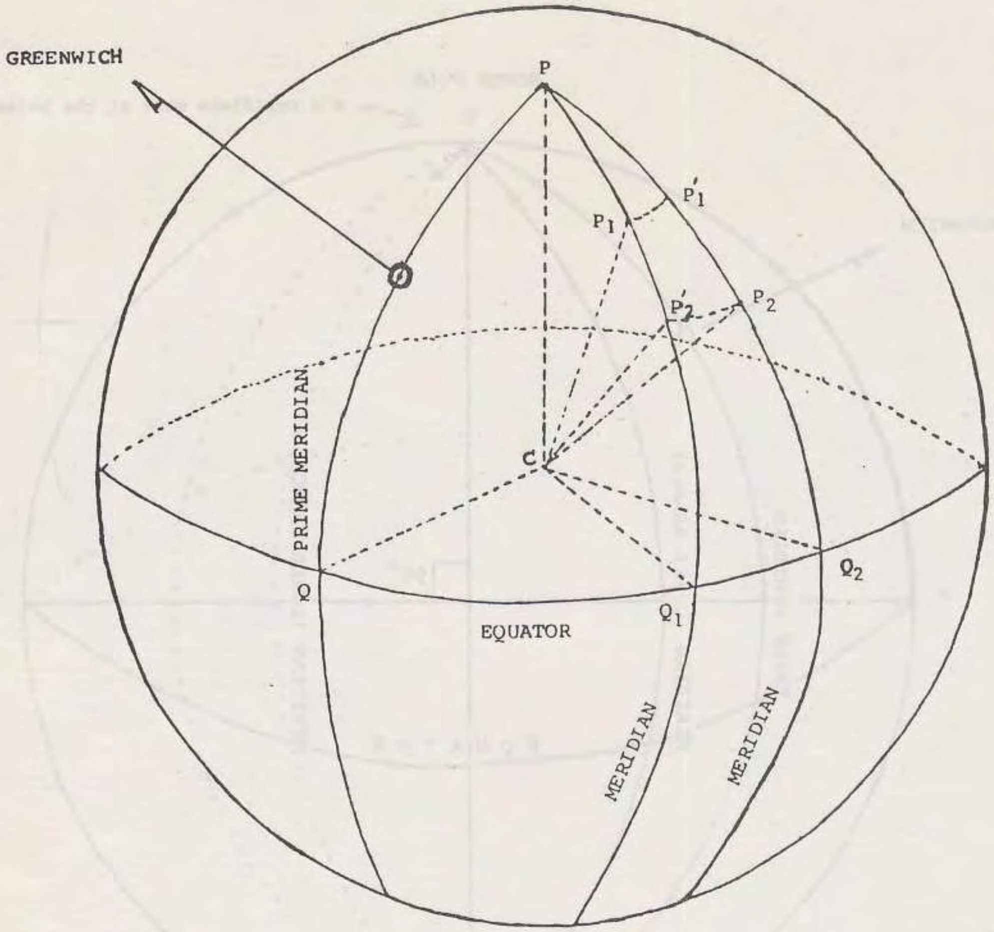


Fig. 4 Meridians and Poles.



$< Q_1 C P_1$	latitude of P_1
$< Q C Q_1$	longitude of P_1
$< Q_2 C P_2$	latitude of P_2
$< Q C Q_2$	longitude of P_2
$< P_1 C P'_2$	D. lat between P_1 and P_2
$< Q_1 C Q_2$	D. Lo between P_1 and P_2

Fig. 5 Latitude and Longitude.

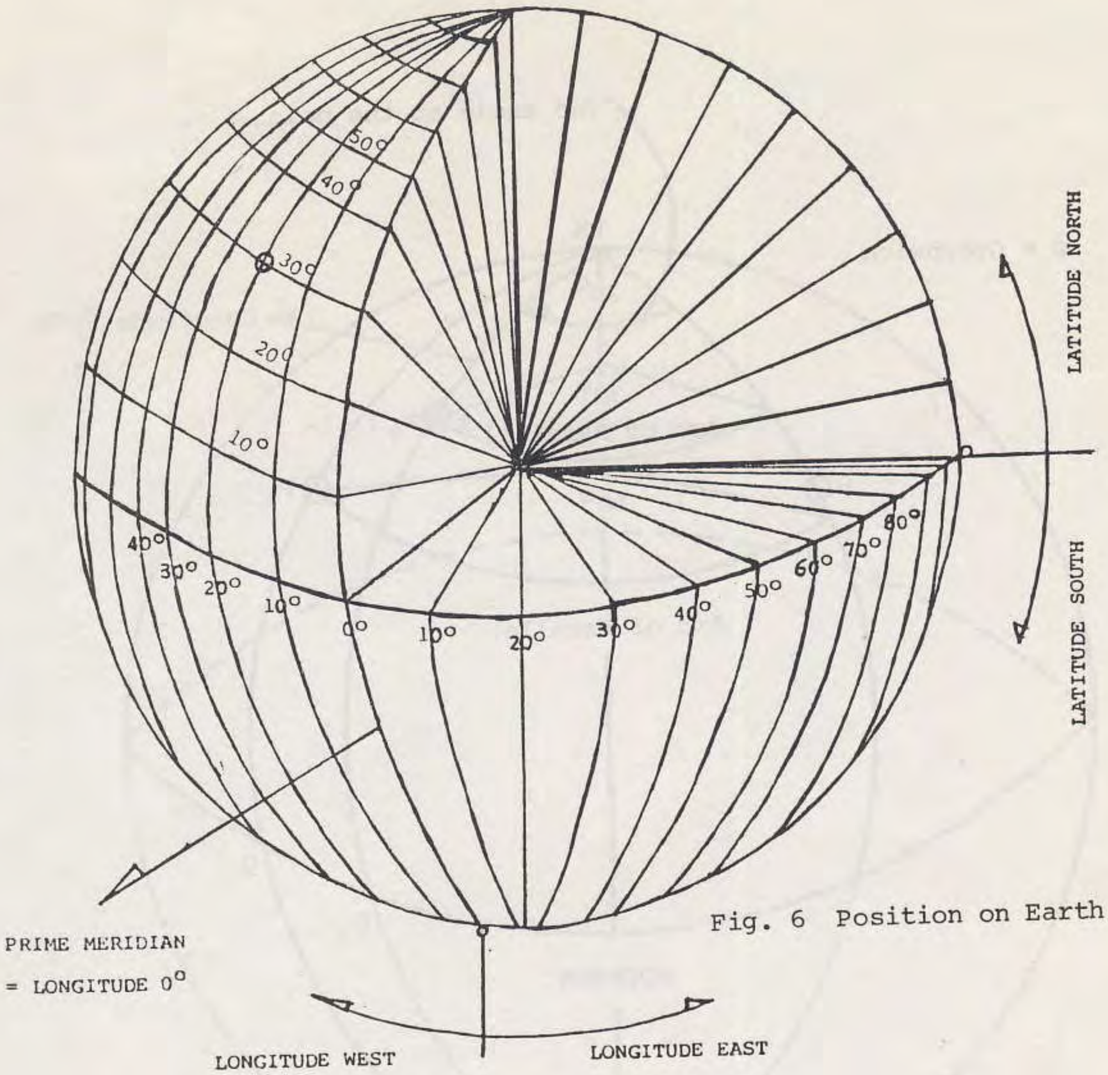


Fig. 6 Position on Earth

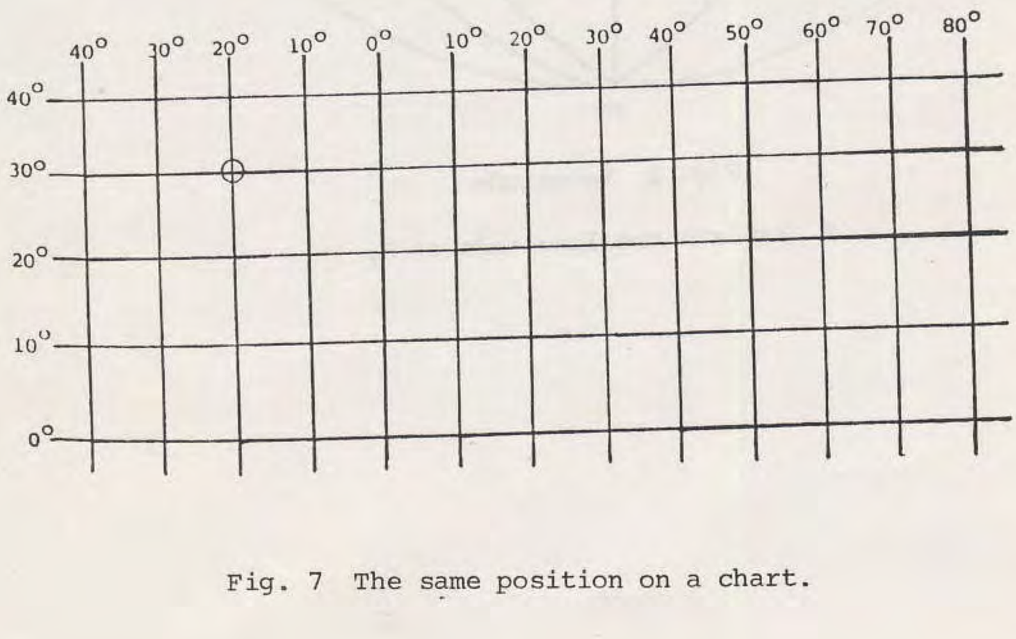


Fig. 7 The same position on a chart.

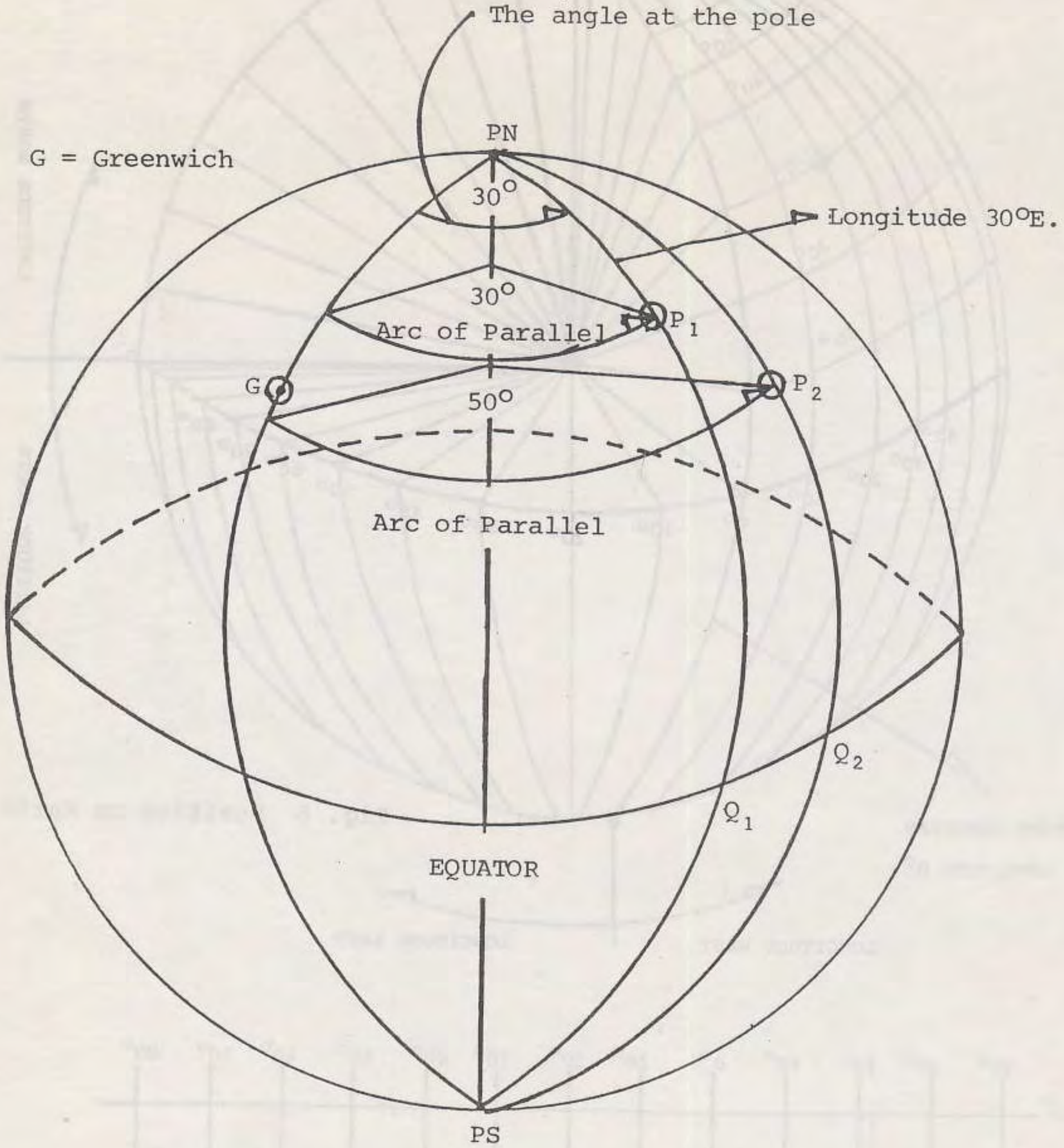
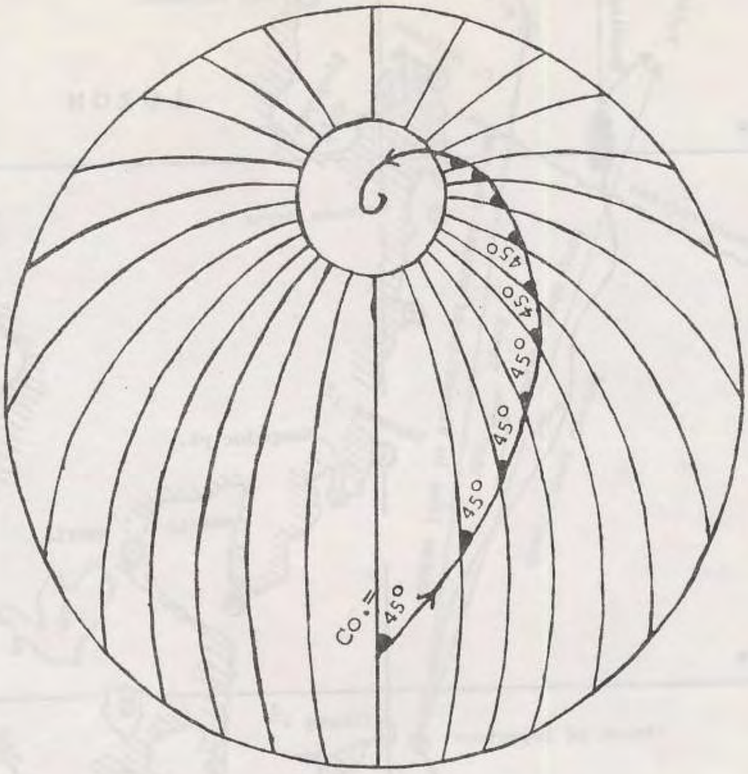


Fig. 8 Longitude

X Explain the longitude of P_2



RHUMB LINE

= LOXODROMIC CURVE

= LOXODROME

Fig. 9 A rhumb line spirals towards the pole, forming a loxodromic curve or loxodrome except meridians and parallels.

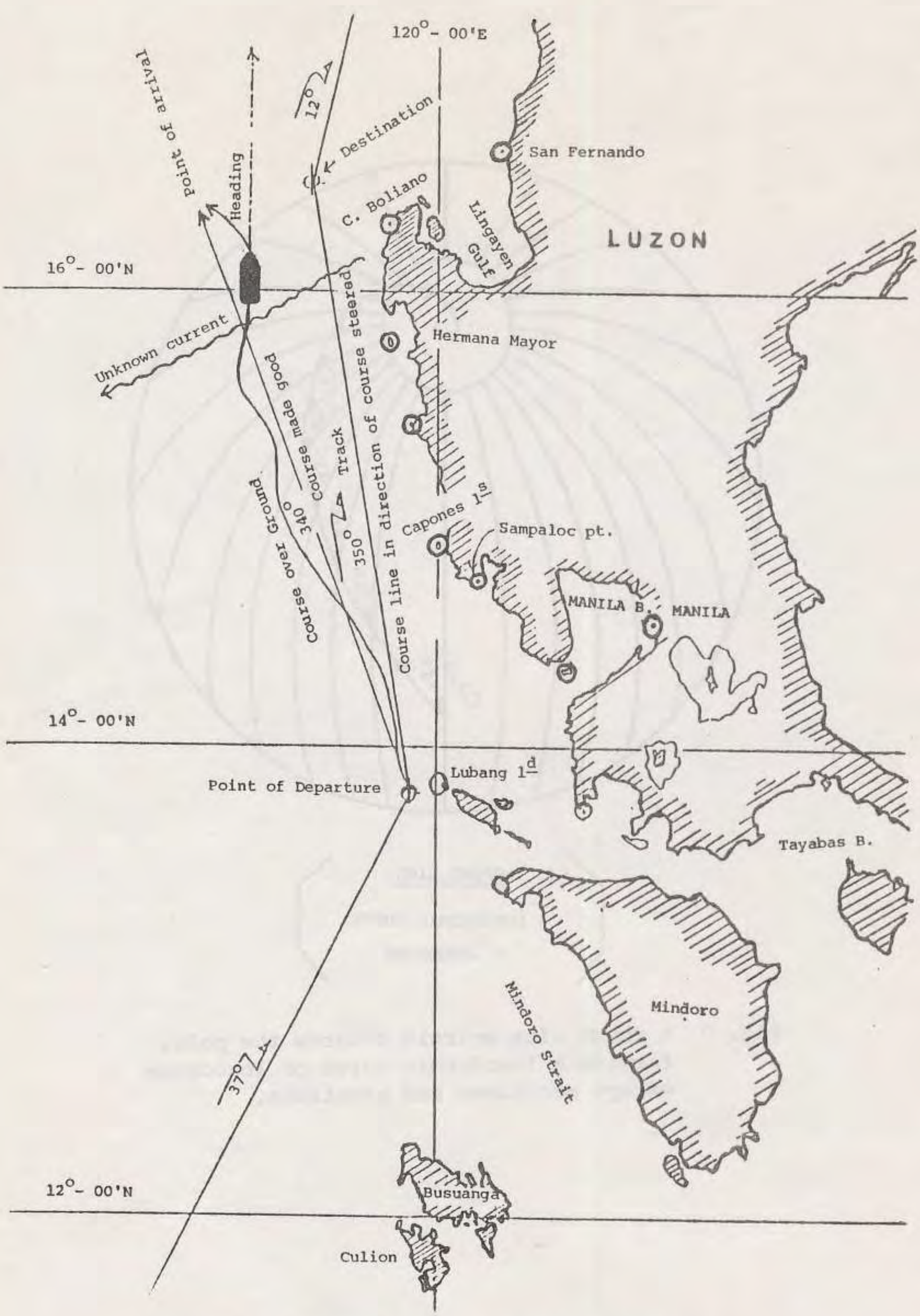
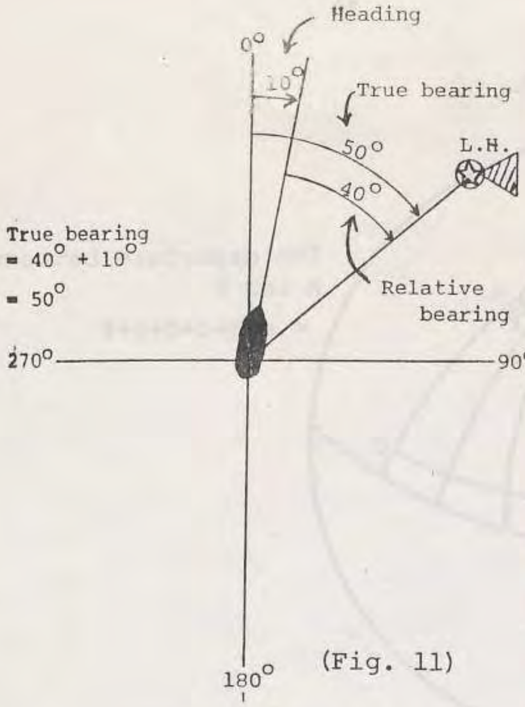
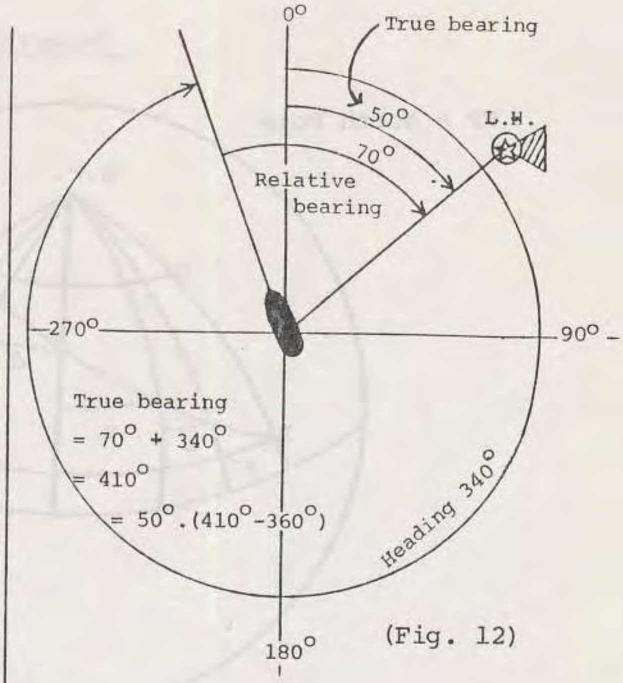


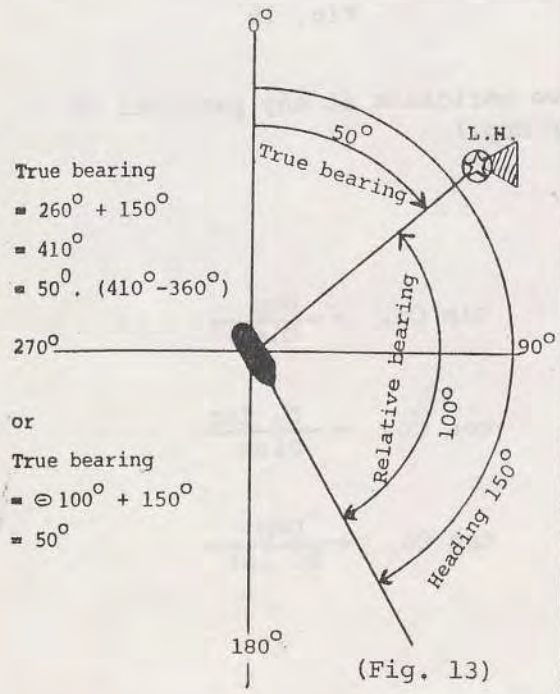
Fig. 10 Course made good, Track etc...



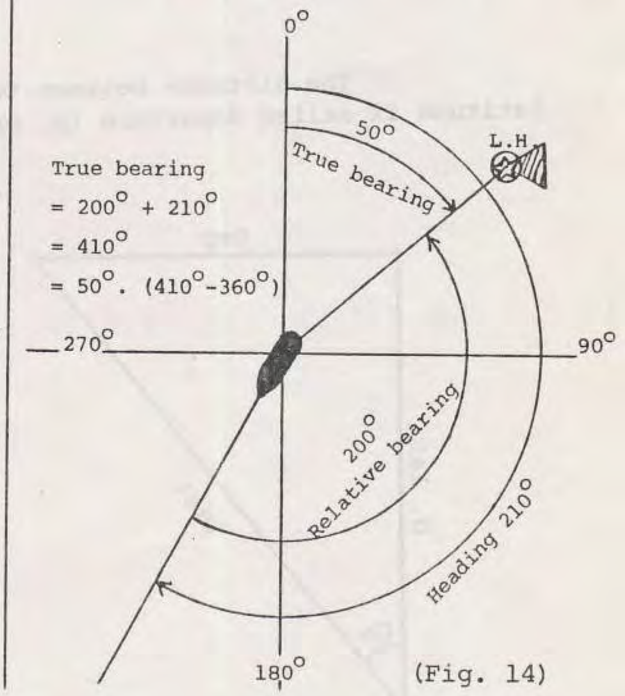
(Fig. 11)



(Fig. 12)



(Fig. 13)



(Fig. 14)

True Bearing = Relative Bearing + True Heading.

A relative bearing is conveniently measured right or left from 0° at the ship's head through 180°.

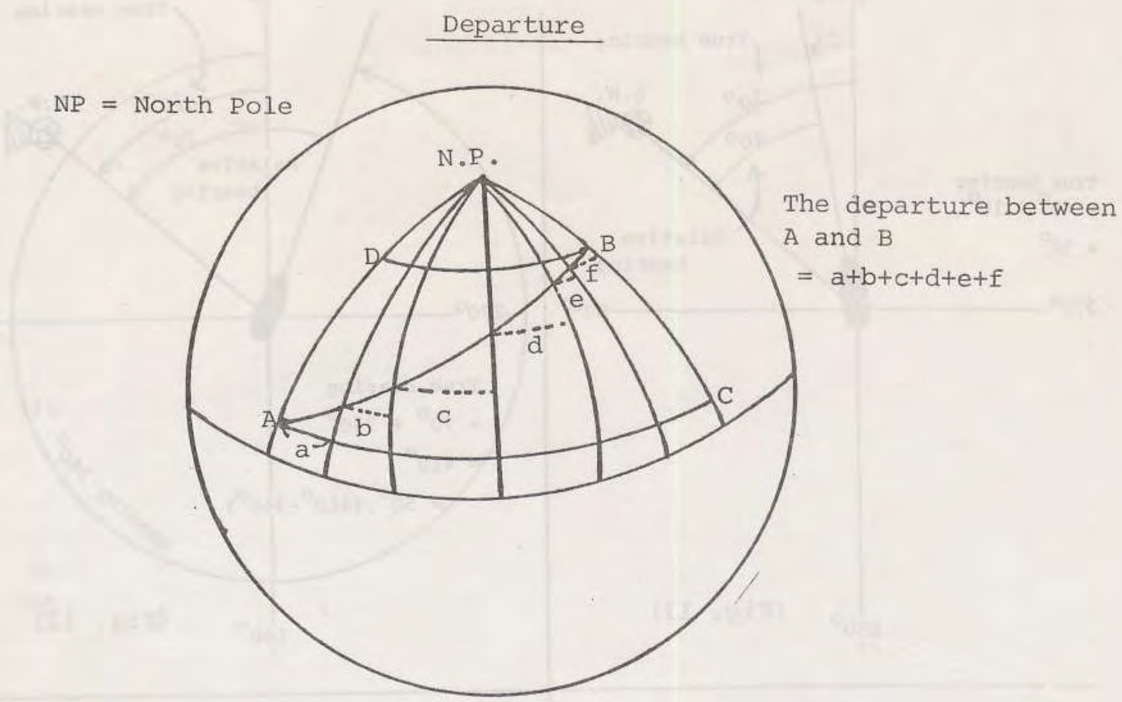


Fig. 15

The distance between two meridians at any parallel of latitude is called departure (p. or Dep.)

$$\sin \text{Co.} = \frac{\text{Dep}}{\text{Dist}}$$

$$\cos \text{Co.} = \frac{\text{D. lat}}{\text{Dist}}$$

$$\tan \text{Co.} = \frac{\text{Dep}}{\text{D. lat}}$$

Fig. 16 Departure, D. Lat. and Dist.

(18) A relative bearing is one relative to the heading, or to the vessel itself. It is usually measured from 000^{o} of the heading, clockwise through 360^{o}. However, it is sometimes conveniently measured right or left from 000^{o} at the ship's head through 180^{o} (Fig. 11-14).

True Bearing = Relative Bearing + True Heading

(19) Magnetic compass error

Directions relative to the northerly direction along a geographic meridian are true. In this case, true north is the reference direction (Fig. 25-28).

If a compass card is horizontal and oriented so that a straight line from its center to 000^{o} points to true north, any direction measured by the card is a true direction and has no error (assuming there is no calibration or observational error).

If the compass card remains horizontal but is rotated so that it points in any other direction, the amount of the rotation is the compass error (Fig. 25-28).

Stated differently, compass error is the angular difference between true north and compass north (the direction north as indicated by a magnetic compass).

It is called east or west to indicate the side of true north on which compass north lies (Fig. 25-28).

Magnetic variation is the angle between the geographic and magnetic meridians at any place. This is measured in angular units and called east or west to indicate the side of true north on which the northerly part of the magnetic meridian lies (Fig. 25-28).

For computational purposes, easterly variation is sometimes designated positive (+), and westerly variation negative (-) (Fig. 25-28).

When a compass is mounted in a vessel, it is generally subjected to various magnetic influences other than that of the earth. These arise largely from induced magnetism in metal decks, bulkheads, masts, stacks, boat davits, guns, etc., and from electro-magnetic fields associated with direction in electrical circuits.

Some metal in the vicinity of the compass may have acquired permanent magnetism. The actual magnetic field at the compass is the vector sum, or resultant, of all individual fields at that point.

Since the direction of this resultant field is generally not the same as that of the earth's field alone, the compass magnets do not lie in the magnetic meridian, but in a direction that makes an angle with it. This angle is called deviation (Dev. or D.).

Thus, deviation is the angular difference between magnetic north and compass north. It is expressed in angular units and named east or west to indicate the side of magnetic north on which compass north lies.

Thus, deviation is the error of the compass in pointing to magnetic north, and all directions measured with compass north as the reference direction are compass directions.

Since variation and deviation may each be either east or west, the effect of deviation may be to either increase or decrease the error due to variation alone (Fig. 25-28).

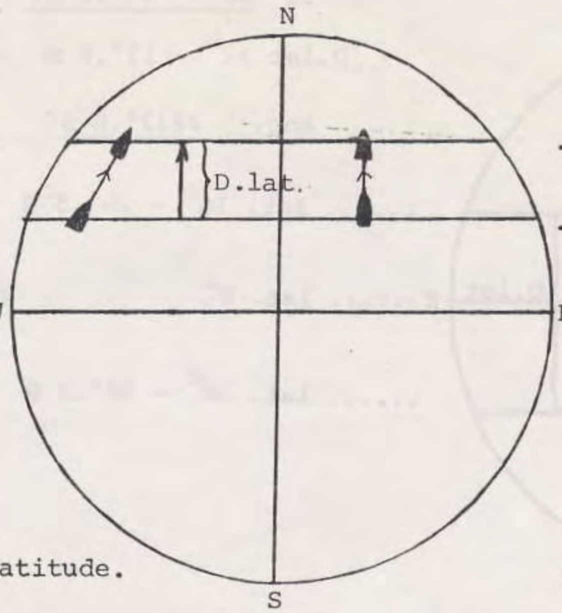
The algebraic sum of variation and deviation is the total compass error (Fig. 25-28).

For computational purposes, deviation and compass error, (like variation), may be designated positive (+) if east and nagative (-) if west.

Variation changes with location. Deviation depends upon the magnetic latitude and also upon the individual vessel, its trim and loading, whether it is pitching or rolling, the heading (orientation of the vessel with respect to the earth's magnetic field), and the location of the compass within the vessel. Therefore, deviation is not indicated on charts.

Solutions to Exercise 1

(1)



l_2 .... latitude = $34^{\circ} - 18'$ N

l_1 .... latitude = $18^{\circ} - 50'$ N

latitude = 0°

l_1 $18^{\circ} - 50'$ N

l_2 $34^{\circ} - 18'$ N (−)

D-lat $15^{\circ} - 28'$ N -----> 928' N

Fig. 17 D. Latitude.

l_1 = the latitude of departure

l_2 = the latitude of arrival

Ans. 928' N

Attention :

If the ships are moving northward, D.lat should be designated "N". If the ships start from $34^{\circ} - 18'$ N and proceed to $18^{\circ} - 50'$ N, then D.lat should be designated "S", because the movement is southward.

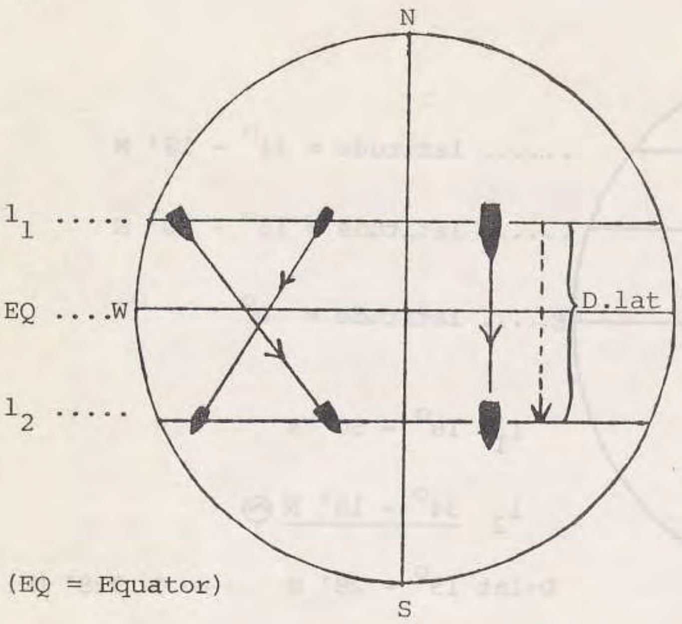
(2)

l_1 $16^{\circ} - 26'.5$ N

l_2 $\underline{20^{\circ} - 46'.3}$ S+

D.lat $37^{\circ} - 12'.8$ S ----→ 2232'.8 S

Ans. $\underline{2232'.8 \text{ S}}$



(lat.=latitude)

(EQ = Equator)

Fig. 18 D. Latitude

(3)

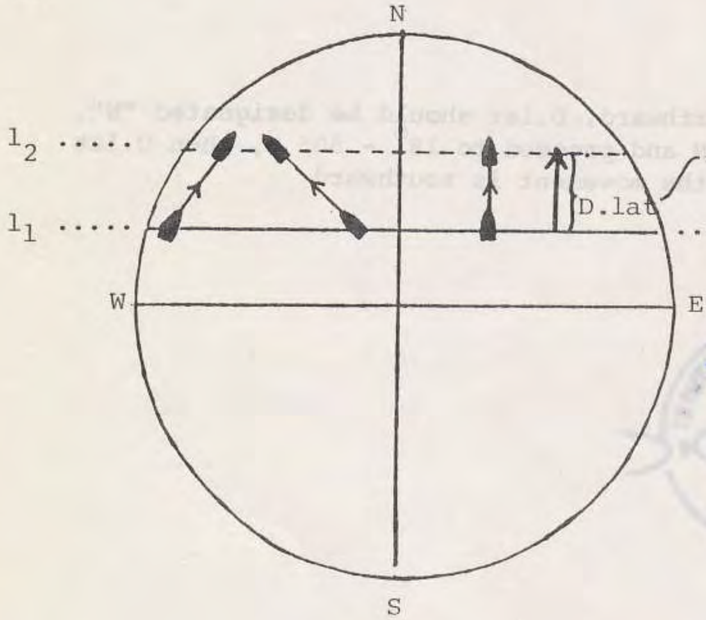


Fig. 19 D. Latitude.

l_1 $15^{\circ} - 26'.4$ N

D.lat $\underline{4^{\circ} - 55'.4}$ N + ←--- 295'.4 N

$19^{\circ} - 81'.8$ N

l_2 = $20^{\circ} - 21'.8$ N

Ans. $\underline{20^{\circ} - 21'.8 \text{ N}}$

(4)

l_1 $5^{\circ} - 36'.2$ N

D.lat $9^{\circ} - 46'.8$ S ⊝ ←---586'.8 S

l_2 $4^{\circ} - 10'.6$ S

Ans. $4^{\circ} - 10'.6$ S

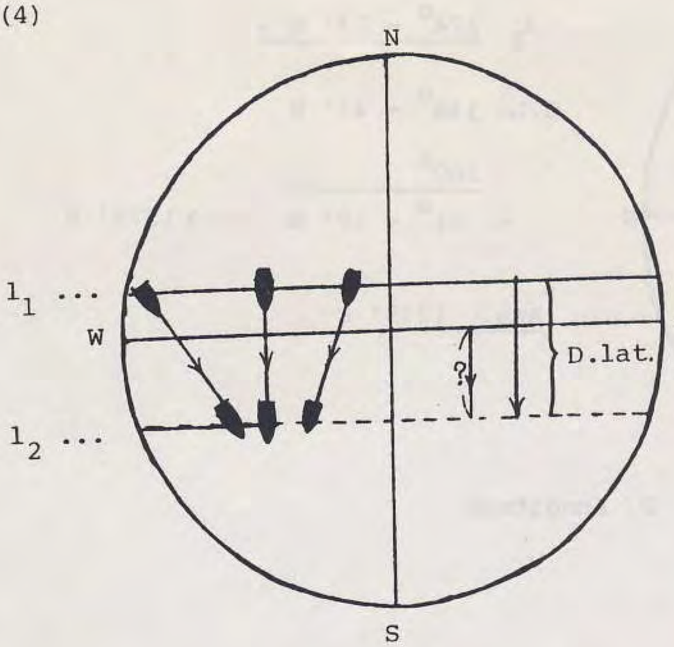
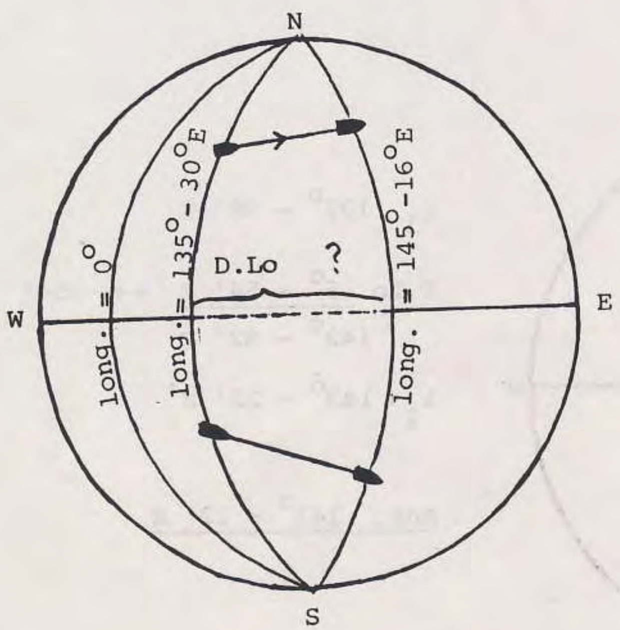


Fig. 20 D. Latitude.

Solutions to Exercise 2

(5)



λ_1 $135^{\circ} - 30'$ E

λ_2 $145^{\circ} - 16'$ E ⊝

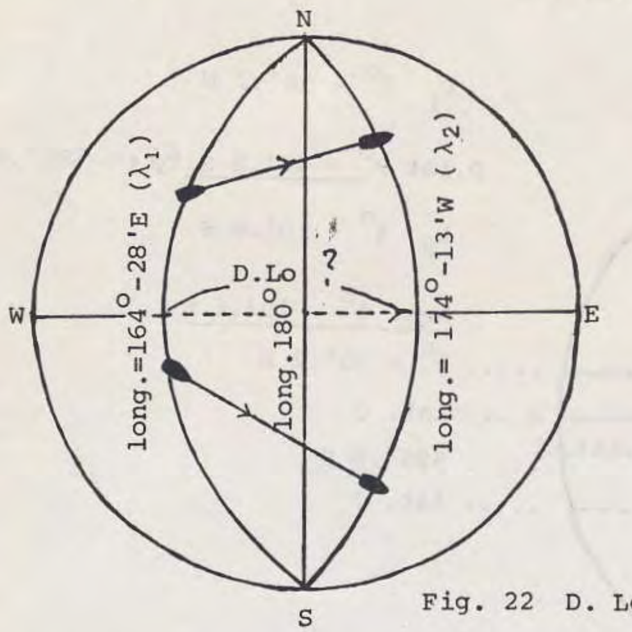
D.Lo $9^{\circ} - 46'$ E ---→ 586' E

Ans. 586' E

(long. = longitude)

Fig. 21 D. Longitude.

(6)



λ_1 $164^\circ - 28'$ E

λ_2 $174^\circ - 13'$ E +

D.Lo $338^\circ - 41'$ W

360°

$= 21^\circ - 19'$ W ---→ 1279' E

Ans. 1279' E

Fig. 22 D. Longitude

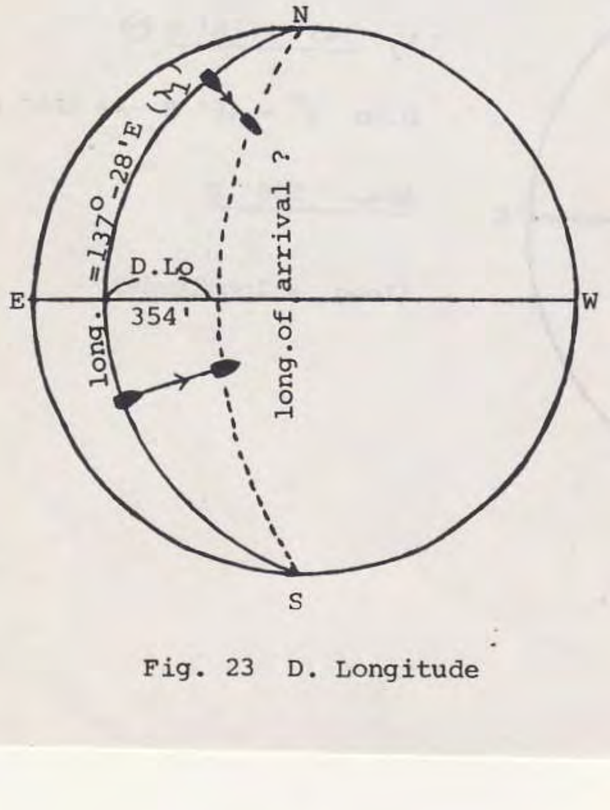
λ_1 = the longitude of departure

λ_2 = the longitude of arrival

Attention :

This ship proceeds eastward through longitude 180° and will enter into the area of longitude WEST, but its heading is still eastward, so that the D.Lo should be designated "E".

(7)



λ_1 $137^\circ - 28'$ E

D.Lo $5^\circ - 54'$ E + ←--354'

$142^\circ - 82'$ E

$\lambda_2 = 143^\circ - 22'$ E

Ans. $143^\circ - 22'$ E

Fig. 23 D. Longitude

(8)

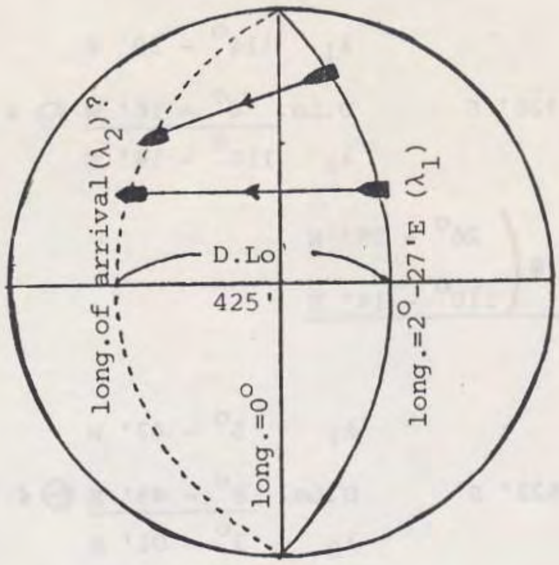


Fig. 24 D. Longitude.

λ_1 $2^\circ - 27'$ E

D.Lo. $7^\circ - 05'$ W ⊖ ←-- 425'

λ_2 $4^\circ - 38'$ W

Ans. $4^\circ - 38'$ W

(9)

(1) l_1 $34^\circ - 26'$ N

l_2 $16^\circ - 57'$ N ⊖

D-lat $17^\circ - 29'$ S ----→ 1049' S

λ_1 $138^\circ - 39'$ W

λ_2 $152^\circ - 16'$ W ⊖

D.Lo $13^\circ - 37'$ W ---→ 817' W

Ans. D.lat 1049' S

D.Lo 817' W

(2) l_1 $18^\circ - 42'$ N

l_2 $13^\circ - 35'$ S +

D-lat. $32^\circ - 17'$ S ------→ 1937' S

λ_1 $14^\circ - 26'$ E

λ_2 $17^\circ - 40'$ W +

D.Lo $32^\circ - 06'$ W ----→ 1926' W

Ans. D.lat. 1937' S

D.Lo. 1926' W

(10)

(1) l_1 $32^\circ - 27'$ N $\quad\quad$ λ_1 $114^\circ - 30'$ E

D.lat. $5^\circ - 28'$ S $\ominus$ ←---328' S $\quad\quad$ D.Lo. $4^\circ - 16'$ W $\ominus$ ←---256' S

l_2 $26^\circ - 59'$ N $\quad\quad$ λ_2 $110^\circ - 14'$ E

Ans. arrival ⊕ $\left(\begin{array}{l} 26^\circ - 59' \text{ N} \\ 110^\circ - 14' \text{ E} \end{array}\right.$

(2) l_1 $7^\circ - 38'$ N $\quad\quad$ λ_1 $5^\circ - 47'$ W

D.lat. $10^\circ - 23'$ S $\ominus$ ←---623' S $\quad\quad$ D.Lo. $8^\circ - 48'$ E $\ominus$ ←---528' E

l_2 $2^\circ - 45'$ S $\quad\quad$ λ_2 $3^\circ - 01'$ E

Ans. arrival ⊕ $\left(\begin{array}{l} 2^\circ - 45' \text{ S} \\ 3^\circ - 01' \text{ E} \end{array}\right.$

(3) l_1 $12^\circ - 10'$ N $\quad\quad$ λ_1 $94^\circ - 16'$ E

D.lat. $4^\circ - 10'$ S $\ominus$ ←---250' S $\quad\quad$ D.Lo. $3^\circ - 44'$ E + ←---224' E

l_2 $8^\circ - 00'$ N $\quad\quad$ λ_2 $98^\circ - 00'$ E

Ans. arrival ⊕ $\left(\begin{array}{l} 8^\circ - 00' \text{ N} \\ 98^\circ - 00' \text{ E} \end{array}\right.$

(4) l_1 $7^\circ - 40'$ N $\quad\quad$ λ_1 $98^\circ - 20'$ E

D.lat. $10^\circ - 30'$ S $\ominus$ ←---90' S $\quad\quad$ D.Lo. $3^\circ - 00'$ W $\ominus$ ←---180' W

l_2 $6^\circ - 10'$ N $\quad\quad$ λ_2 $95^\circ - 20'$ E

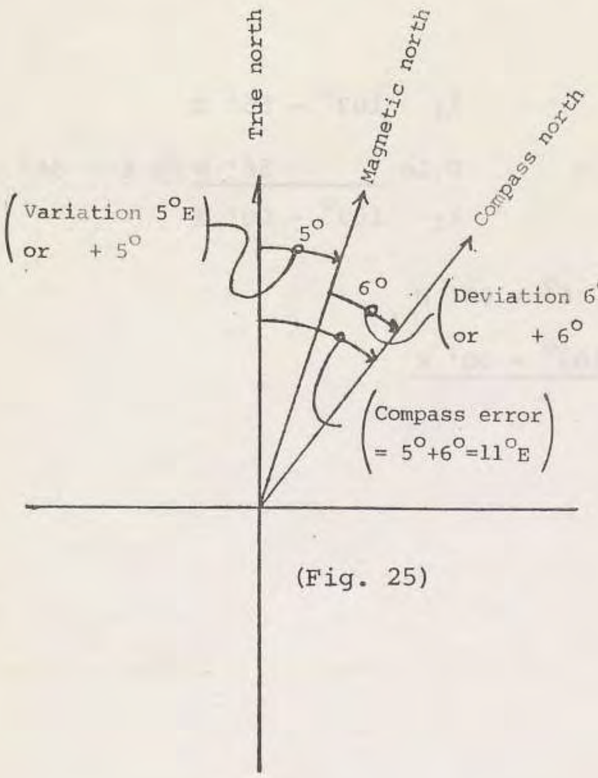
Ans. arrival ⊕ $\left(\begin{array}{l} 6^\circ - 10' \text{ N} \\ 95^\circ - 20' \text{ E} \end{array}\right.$

(5) l_1 $8^\circ - 14'$ N λ_1 $103^\circ - 56'$ E

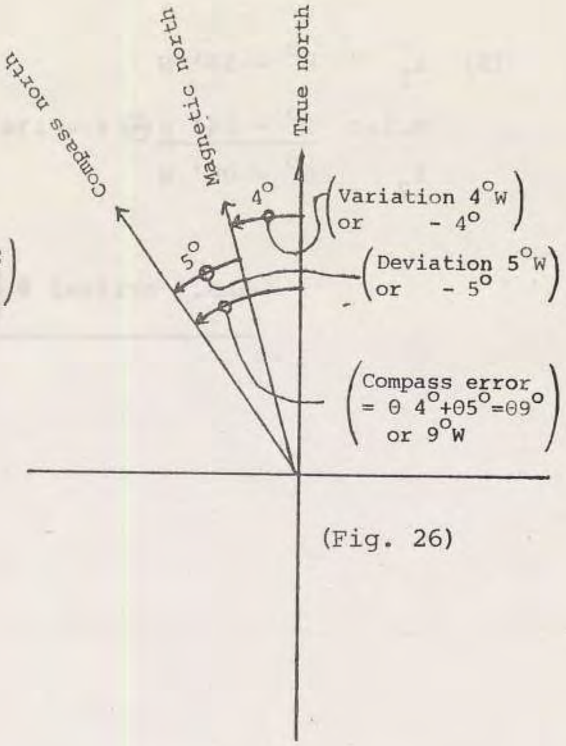
D.lat $2^\circ - 14'$ S (~) ←--134' S D.Lo 56' W (~) ←---56'

l_2 $6^\circ - 00'$ N λ_2 $103^\circ - 00'$ E

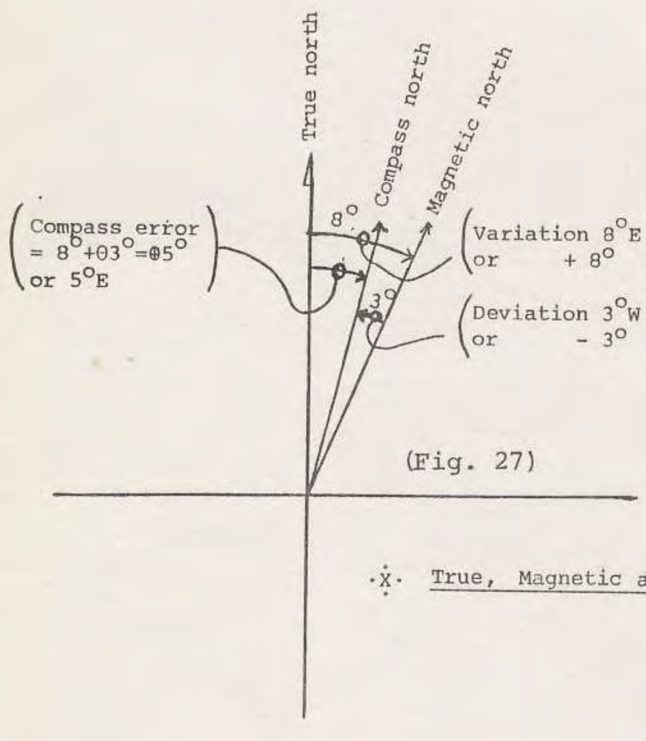
Ans. arrival ⊕ $\left(\begin{array}{l} 6^\circ - 00' \text{ N} \\ 103^\circ - 00' \text{ E} \end{array} \right.$



(Fig. 25)

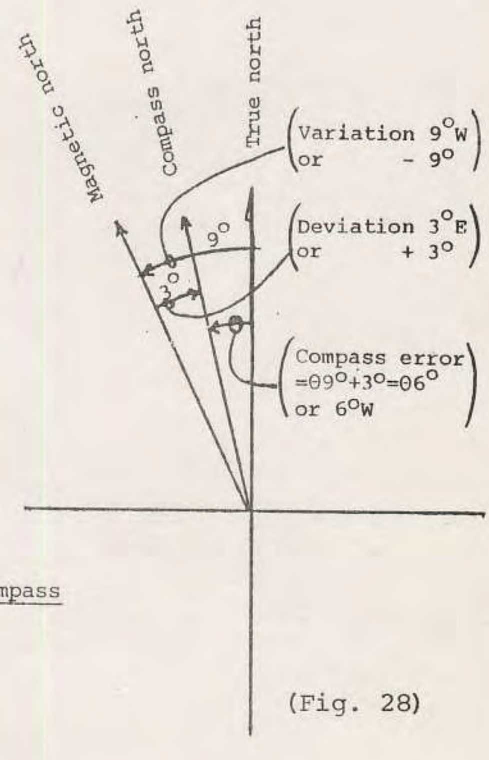


(Fig. 26)



(Fig. 27)

·X· <u>True, Magnetic and Compass</u>



(Fig. 28)

Exercise 3

(11) What is the true course ?

1. Compass course is N 25°E, when the deviation is 5°W, the variation is 2°E. (Ans. N 22°E)

2. Compass course is S 37°W, when the deviation is 2°E, the variation is 6°W. (Ans. S 33°W)

(12) What is the compass course ?

1. True course is S 10°E, when the deviation is 1°E the variation 3°W. (Ans. S 8°E)

2. True course is N 48°E, when the deviation is 3°E, the variation is 4°W, the leeway is 2° and the wind direction is N. (Ans. N 51°E)

(13) What is the true course ?

Where : the compass course is N 29°E, the variation is 6°E, and the deviation is 2°W. (Ans. N 33°E)

(14) What is the true course ?

Where : the compass course is S 46°W, the variation is 3°W, the deviation is 7°E, the leeway is 2°, and the wind direction is west. (Ans. S 48°W)

(20) The method of quadrant

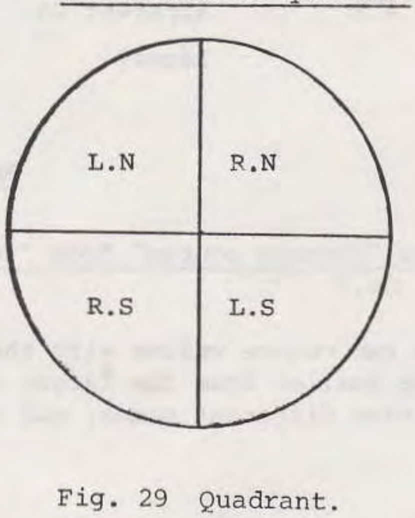


Fig. 29 Quadrant.

a. How to calculate "True Course" from "Compass Course" (Comp. Co. → T. Co.)

1. If variation or deviation is easterly, the name should be "R".

2. If variation or deviation is westerly, the name should be "L".

3. The reference of the course should be given by "A Quadrant".

4. The name of the leeway should be "R", when wind (or current) comes from the port side, and "L" when it comes from the starboard side.

5. However, if you have two values with different names, subtract the smaller from the larger one, the name in the result will be that of the larger one, and two values with the same names, should be added together, then the name in the result will be their name.

Example of calculation 1

Assume that the compass course is S 46°W, the variation is 3°W, the deviation is 7°E, leeway 2°, and the wind direction is westerly. What is the true course ?

Solution

Var.	3°L	Comp. Co.	46° R.S
Dev.	7°R (-)	Comp. error	4° R (+)
Comp. error	4°R	Apparent Co.	50° R.S
		Leeway	2° L.N
			48° R.S
			↓
			Ans. S 48°W

b. How to calculate "Compass course" from "True Course" (T. Co. → Comp. Co.)

1. If you have two course values with the same name, subtract the smaller from the larger one, and if the two values have different names, add them together.

Example of calculation 2

What is the compass course if the true course is N 68°E,: the variation is 4°W, the deviation is 3°E, the leeway is 2° and the wind direction is north ?

Solution

Var.	4°L	T. Co.	68°R.N
Dev.	3°R (-)	Comp. error	1°L (+)
Comp. error	1°L		69°R.N
		Leeway	2°R (-)
		Comp. Co.	67°R.N
			Ans. N 67°E.

Remark: Make sure that you understand the theory itself first, then you will be able to use "the quadrant method".

The Sailings

Dead reckoning involves the determination of position by means of course and distance from a known position. A closely related problem is that of finding the course and distance from one point to another. Although both of these problems are customarily solved by plotting directly on the chart, it occasionally becomes desirable to solve them by computation, frequently by logarithms or traverse tables.

The various methods of solution are collectively called the sailings.

There are various kinds of sailings as follows:

1. Plane sailing

2. Traverse sailing

3. Mercator sailing and chart

1. Plane sailing

Is a method of solving the various problems involving a single course and distance, difference of latitude, and departure, in which the earth, or that part traversed, is regarded as a plane surface. Hence, the method provides a solution for the latitude of the point of arrival, but not for the longitude of this point, for which one of the spherical sailings is needed.

Because of its basic assumption that the earth is flat, this method should not be used for distances of more than a few hundred miles.

Computation

(Fig. 30)

D-lat. = dist. x cos. co.

Dep. = dist. x sin. co.

$\frac{\text{Dep.}}{\text{D-lat.}}$ = tan. co.

dist. = D-lat. x sec. co.

= Dep. x cosec co.

Attention:

"Dep." = dist. x sin. co. is "a+b+c+ ..... g", neither $\overline{BD}$ nor $\overline{AC}$ on Page 16. Fig. 15.

Example of computation 3 (using trigonometry & logarithms).

A ship sails from 37°- 14!0 N., 128°- 38!0 E. Its course is S 28°E, and sailing distance is 295 nautical miles.
What is the arrival latitude and the departure ?

Solution

1. D-lat. = 295' x cos. 28° = 260!5 S. 4° - 20!5 S.

Dep. = 295' x sin. 28° = 138!5 E.

ℓ1	37°- 14!0 N.	
D-lat.	4°- 20!5 S.	~
ℓ2	32°- 53!5 N.	

Ans. The arrival latitude: 32°- 53!5

The departure: 138!5 E.

Attention:

Don't forget to include N. and E. in your answer.

~ means "absolute difference".

2\.

log.	295 ---------→	2.46982	+
log.	cos. 28°----→	9.94593	
D-lat.	260'.5 S. ←----	2.41575	

= 4°- 20'.5 S.

log.	295	2.46982	+
log.	sin. 28°	9.67161	
Dep.	138'.5 E.	2.14143	

ℓ1	37°- 14'.0 N.	−
D-lat.	4°- 20'.5 S.	
ℓ2	32°- 53'.5 N.	

Ans. The arrival latitude: 32°- 53'.5 N.

The departure: 138'.5 E.

Example of computation 4

A ship departs from 38°- 26'.0 N. and proceeds southwesterly, then its difference of latitude is 3°- 18' and the departure is 234 nautical miles. What is its sailing course and distance ?

Solution

$$\tan.\ co. = \frac{Dep.}{D\text{-}lat.} = \frac{234'}{198'}$$

log.	234	2.36922
log.	198	2.29667
log.	tan. co.	10.07255

co. = 49° - 45'.8

dist. = 198' x sec. 49°- 45'.8

log.	198	2.29667
log.	49°- 45'.8	10.18980
log.	dist.	2.48647

dist. = 306'.5

Ans. Co. = S. 49°- 45'.8 W.
dist. = 306'.5

Example computation 5 (Using traverse table)

What is the D-lat. and the Dep.?

Where: dist. = 374', co. - N. 39 W. (Ans. D-lat. = 290'.7 N.
Dep. = 235'.4 W)

Exercise 4

15. What is the distance and the course?

Where: D-lat. = 319!5 N. Dep. = 116!3 E.

(Ans. Dist. = 340'
Co. = N.20°E.)

16. What is the distance and course?

Where: D-lat. = 137!4 S. Dep. = 399!0 W.

(Ans. Dist. = 422'
Co. = S.71°W.)

17. What is the distance and the course?

Where: D-lat. = 43!1 N. Dep. = 18!3 W

(Ans. Dist. = 46.8'
Co. = N.23°W.)

18. A ship is sailing, its course is N.40°E., and the distance is 305 nautical miles. What is its D-lat. and Dep.?

(Calculate using logarithms).

(Ans. Dep. = 196'E
D-lat. = 233!6 N.)

19. A port is located on the latitude 43°- 07'N; M.V. PAKNAM departs from this port and sails at a speed of 8.5 knots and its course is S.25°E. How long does M.V. PAKNAM take to get to the latitude 37°- 59'N ?

(Ans. $39^h\ 57^m$)

20. In Figure 31, a navigator throws a log into the sea at Point A; this log moves from point B to point C, in 8 seconds. What is the ship's speed in knots: (1 nautical mile = 1852 m. Distance from B to C is 30 metres)

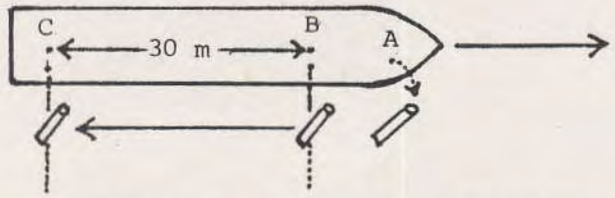


Fig. 31 Measurement of Speed with a log of wood.

21. What is the difference of longitude?

Longitude of departure	Longitude of arrival	Difference of longitude
165°- 32'E	138°- 56'E	26°- 36'W
178°- 26'W	178°- 26'E	3°- 04'W
173°- 13'E	170°- 47'E	2°- 26'W
7°- 03'E	3°- 59'W	11°- 02'W
135°- 43'E	141°- 21'E	5°- 38'E

22. What is the longitude of arrival?

Longitude of departure	Difference of longitude	Longitude of arrival
137°- 47'E	219'E	141°- 26'E
143°- 11'E	357'W	137°- 14'E
179°- 37'E	91'E	178°- 52'W
169°- 49'W	138'W	172°- 07'W
3°- 26'W	312'W	8°- 38'W

23. What is the difference of latitude?

Latitude of departure	Latitude of arrival	Difference of latitude
38°- 15'N	41°- 08'N	173'N
39°- 37'N	34°- 49'N	288'S
7°- 24'N	2°- 57'S	621'S
0°- 36'S	0°- 36'N	72'N
10°- 39'S	13°- 27'S	168'S

24. What is the latitude of arrival?

Latitude of departure	Difference of latitude	Latitude of arrival
33°- 53'N	113'N	35°- 46'N
43°- 11'N	237'S	39°- 14'N
1°- 43'S	192'N	1°- 29'N
2°- 26'N	233'S	1°- 27'S
5°- 08'S	139'N	2°- 49'S

25. What is the difference of latitude and the departure?

Course	Distance	Difference of latitude	Departure
N 16°E	44'	42'.3 N	12'.1 E
S 37°E	73'	58'.3 S	43'.9 E
S 41°W	41'	30'.9 S	26'.9 W
N 29°W	92'	80'.5 N	44'.6 W
N 56°E	65'	36'.3 N	53'.9 E
S 70°W	47'	16'.1 S	44'.2 W
S 34°E	163'	135'.1 S	91'.1 E

26. Calculate the deviation, when the compass bearing is N 63°W, and the magnetic bearing is N 55°W. (Ans. 8°E)

27. What is the deviation?

Compass bearing	Magnetic bearing	Deviation
N 37°E	N 45°E	8°E
S 43°E	S 39°E	4°E
S 28°W	S 36°W	8°E
N 73°W	N 81°W	8°W
N 46°E	N 39°E	7°W

28. M.V. PAKNAM is going to sail on true course 127°; what should be its compass course if direction of wind is NE, leeway 7°, variation 5°- 42'W, and deviation 7°- 18'W. (Ans. S 47°E)

29.

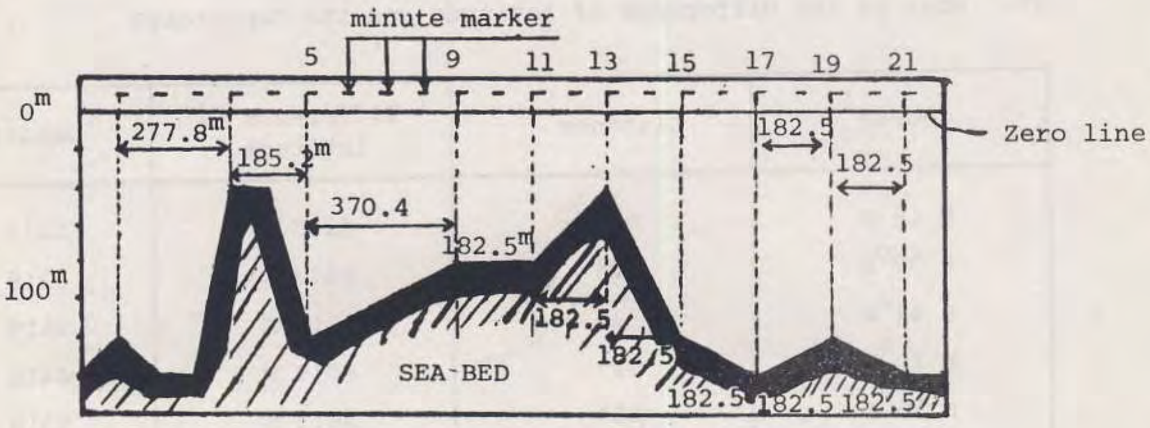


Fig. 32 Recording of Sea-bed.

The shape of the sea-bed surveyed is shown on the recording paper (Fig. 32), but this shape is deformed by the mechanical system of the echo sounder's receiver. Draw the actual shape of the sea-bed if the ship's speed is 3 knots.

30. What is the true course if:

The compass course is 036°, the direction of wind is NW, leeway is 11°.3, the deviation is 3°E, and the variation is 8°W?

(Ans. 42°.3)

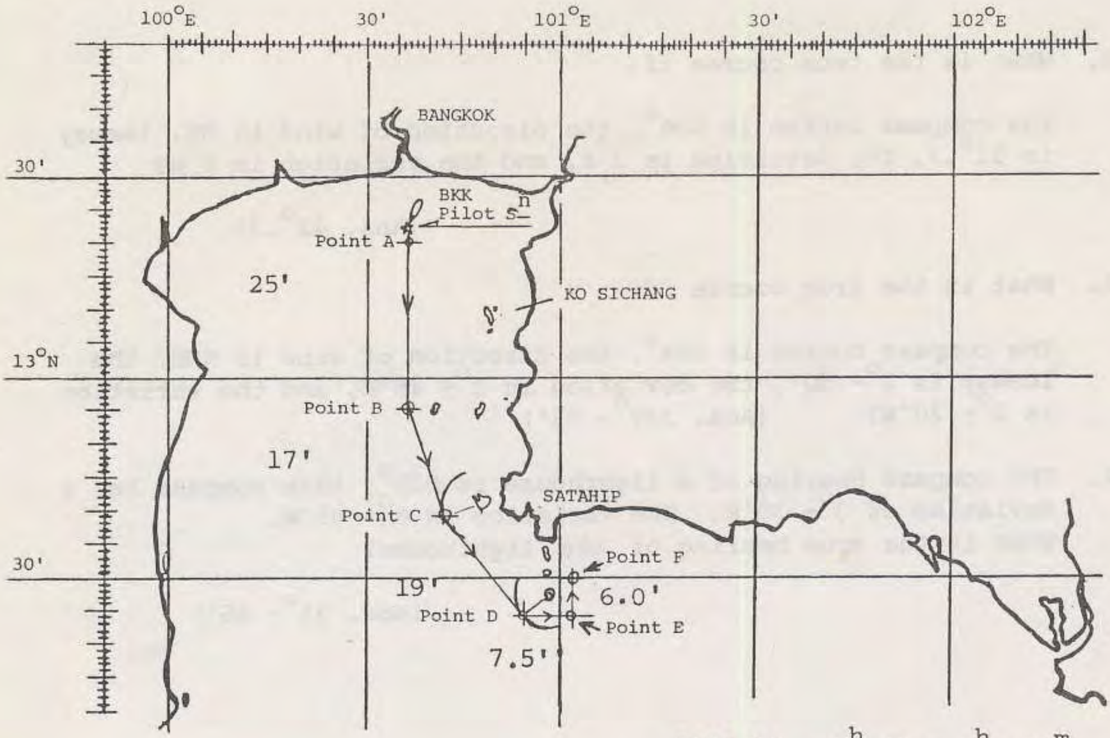
31. What is the true course if:

The compass course is 344°, the direction of wind is NNE, the leeway is 2° - 50', the deviation is 3° - 15'E, and the variation is 5° - 20'W? (Ans. 339° - 05')

32. The compass bearing of a lighthouse is 035°; this compass has a deviation of 3° - 30'E. The variation is 6° - 45'W. What is the true bearing of the lighthouse?

(Ans. 31° - 45')

33.



$74'.5/10 = 7^h.45 = 7^h - 27^m$

Fig. 33 Sailing.

A ship sails from Point A for Point F though Points B, C, D and Point E, with the speed of 10 knots on April 14, at 18:15 hours (Fig. 33). How long does it take to get to Point F ? and what is the time of arrival? What are the lattitudes and longitudes of each Point from A to F?

14/4, $14^d - 18^h - 15^m$
$\quad\quad 7 - 27 \ +$
$14^d - 25 - 42$
$\quad\quad 24 \ -$
time of arrival $15^d - 01 - 42$: April

	Lat.	Long.
A	$13^\circ - 20'$N	$100^\circ - 36'.8$ E
B	$12^\circ - 55'$N	$100^\circ - 36'.1$ E
C	$12^\circ - 39'.5$ N	$100^\circ - 42'.0$ E
D	$12^\circ - 24'.0$ N	$100^\circ - 54'.5$ E
E	$12^\circ - 24'.0$ N	$101^\circ - 02'.0$ E
F	$12^\circ - 30'.0$ N	$101^\circ - 02'.0$ E

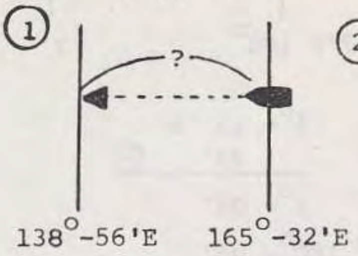
Solutions to Exercise 4 (P. 34)

(20)

$30 \div 8 \times 3600 \div 1852 = 7.28$ Ans. About 7.3 knots

(21)

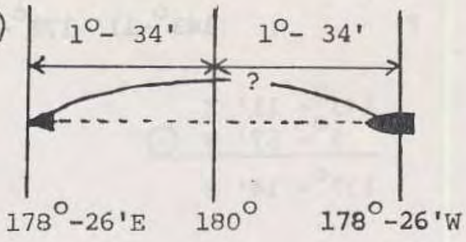
① (Fig. 34)



165° - 32'
138° - 56' ⊖
26° - 36' W.

Ans. 26° - 36' W
or 1596' W

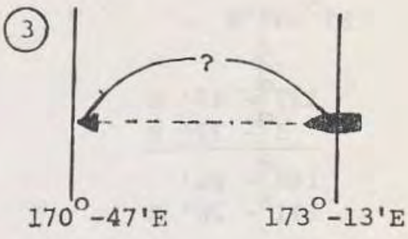
② (Fig. 35)



1° - 34'
1° - 34' ⊕
2° - 64'
= 3° - 04' W.

Ans. 3° - 04' W
or 184' W

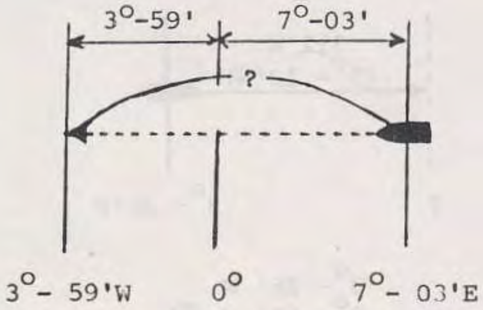
③ (Fig. 36)



173° - 13'
170° - 47' ⊖
2° - 26' W

Ans. 2° - 26' W
or 146' W

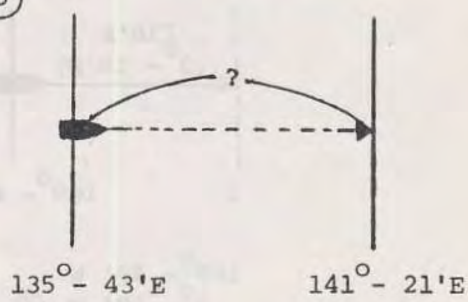
④ (Fig. 37)



3° - 59'
7° - 03' ⊕
10° - 62'
= 11° - 02' W

Ans. 11° - 02' W
or 662' W

⑤ (Fig. 38)

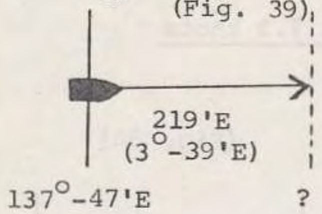


141° - 21'
135° - 43' ⊖
5° - 38' E

Ans. 5° - 38' E
or 338' E

(22)

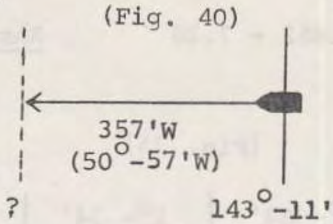
① (Fig. 39)



137°- 47' E
3°- 39' E
140°- 86'
141°- 26' E

Ans. 141°- 26' E

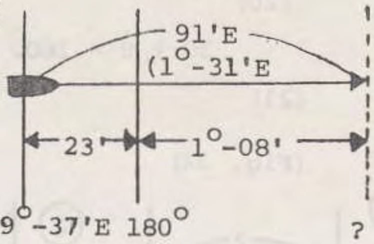
② (Fig. 40)



143°- 11' E
5°- 57' W ⊖
137°- 14' E

Ans. 137°- 14' E

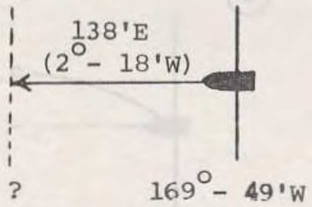
③ (Fig. 41)



1°- 31' E
23' ⊖
1°- 08'
180°- 08'
1°- 08' ⊖
178°- 52'

Ans. 178°- 52' W

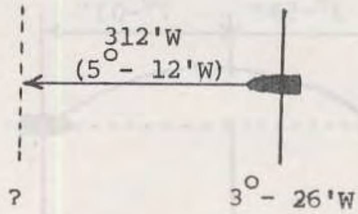
④ (Fig. 42)



169°- 49' W
2°- 18' W ⊕
171°- 67'
172°- 07'

Ans. 172°- 07' W

⑤ (Fig. 43)

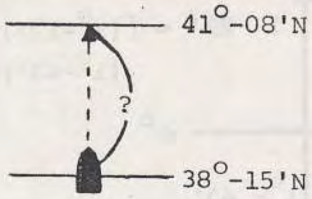


3°- 26' W
5°- 12' W ⊕
8°- 38'

Ans. 8°- 38' W

(23)

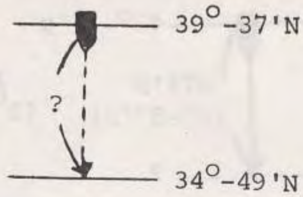
(1) (Fig. 44)



41° - 08' N
38° - 15' N (−)
2° - 53'

Ans. 2° - 53' N
or 173' N

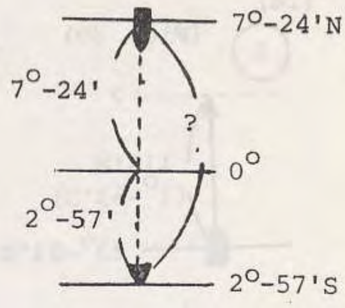
(2) (Fig. 45)



39° - 37' N
34° - 49' N
4° - 48' S

Ans. 4° - 48' S
or 288' S

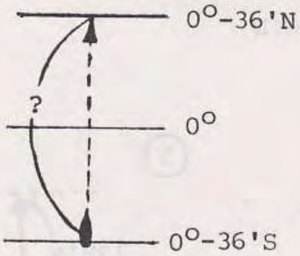
(3) (Fig. 46)



7° - 24' N
2° - 57' S (+)
9° - 81'
= 10° - 21'

Ans. 10° - 21' S
or 621' S

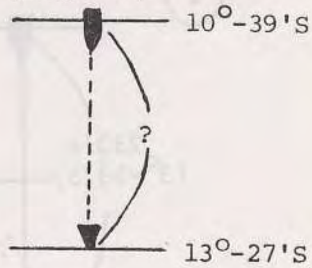
(4) (Fig. 47)



0° - 36' N
0° - 36' S (+)
72'

Ans. 1° - 12' N
or 72' N

(5) (Fig. 48)

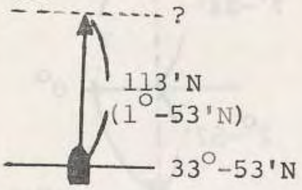


13° - 27' S
10° - 39' S (−)
2° - 48'

Ans. 2° - 48' S
or 168' S

(24)

(1) (Fig. 49)

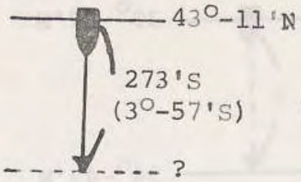


$33^{\circ} - 53'$ N
$1 - 53'$ N (+)

$34 - 106'$
$= 35^{\circ} - 46'$

Ans. $35^{\circ} - 46'$ N

(2) (Fig. 50)

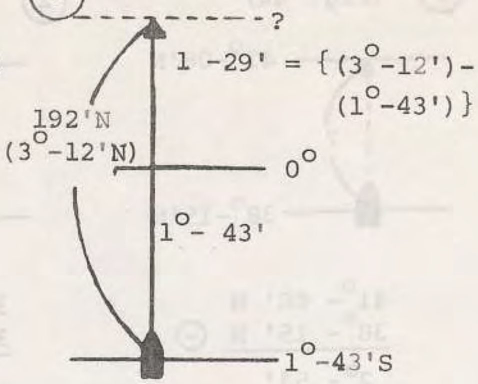


$43^{\circ} - 11'$ N
$3^{\circ} - 57'$ S (−)

$39^{\circ} - 14'$

Ans. $39^{\circ} - 14'$ N

(3) (Fig. 51)

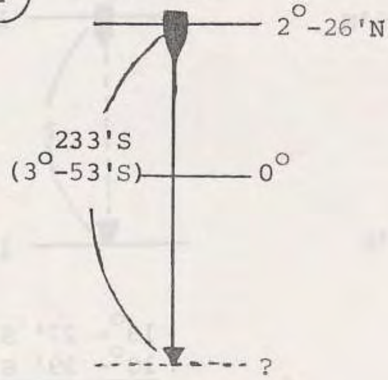


$3^{\circ} - 12'$ N
$1^{\circ} - 43'$ S (−)

$1^{\circ} - 29'$ N

Ans. $1^{\circ} - 29'$ N

(4) (Fig. 52)

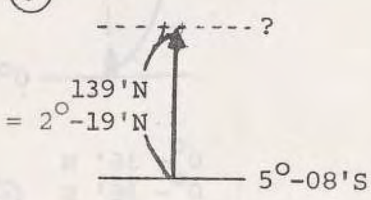


$3^{\circ} - 53'$ S
$2^{\circ} - 26'$ N (−)

$1^{\circ} - 27'$ S

Ans. $1^{\circ} - 27'$ S

(5)



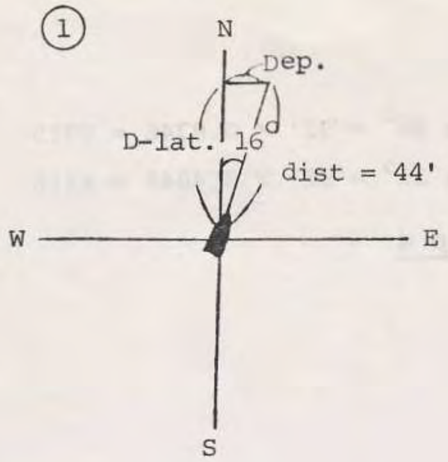
$5^{\circ} - 08'$ S
$2^{\circ} - 19'$ N (−)

$2^{\circ} - 49'$ S

Ans. $2^{\circ} - 49'$ S

(25)

(Fig. 53)

① 

D-lat. = dist. × Cos Co.

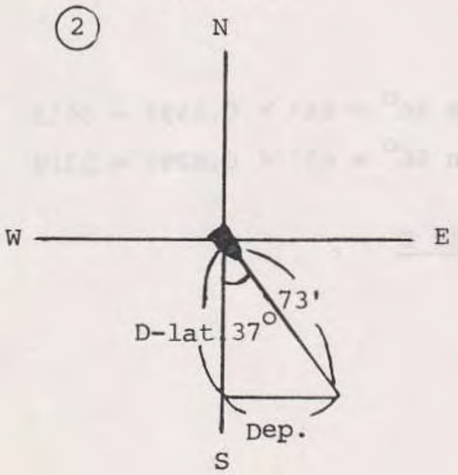
Dep. = dist × Sin co.

D-lat = 44' × Cos 16° = 44' × 0.9613 = 42'.3

Dep. = 44' × Sin 16° = 44' × 0.2756 = 12'.1

<u>Ans. 42.3 N, 12.1'E</u>

(Fig. 54)

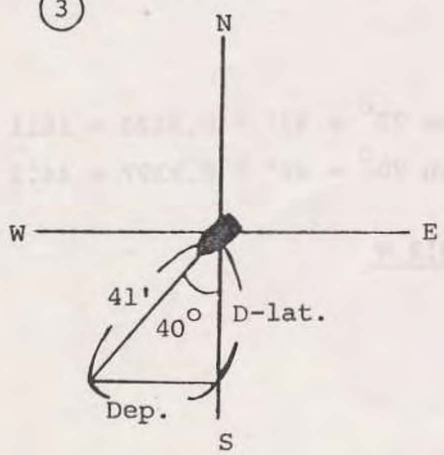
② 

D-lat. = 73' × Cos 37° = 73' × 0.7986 = 58'.3

Dep. = 73' × Sin 37° = 73' × 0.6018 = 43'.9

<u>Ans. 58'.3 S, 43'.9 E</u>

(Fig. 55)

③ 

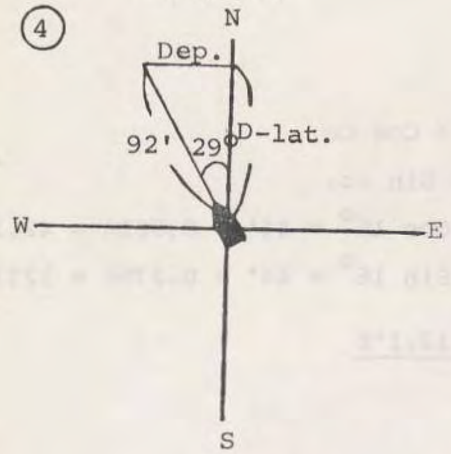
D-lat. = 41' × Cos 41° = 41' × 0.7547 = 30'.9

Dep. = 41' × Sin 41° = 41' × 0.6561 = 26'.9

<u>Ans. 30'.9 S, 26'.9 W</u>

(Fig. 56)

④



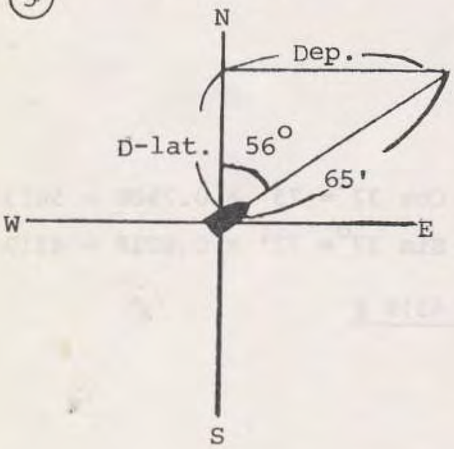
D-lat. = 92' × Cos 29° = 92' × 0.8746 = 80'.5

Dep. = 92' × Sin 29° = 92' × 0.4848 = 44'.6

Ans. 80'.5 N, 44'.6 W

(Fig. 57)

⑤



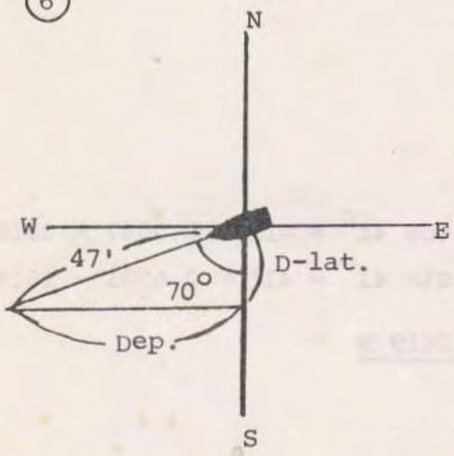
D-lat. = 65' × Cos 56° = 65' × 0.5592 = 36'.3

Dep. = 65' × Sin 56° = 65' × 0.8290 = 53'.9

Ans. 36'.3 N, 53'.9 E

(Fig. 58)

⑥

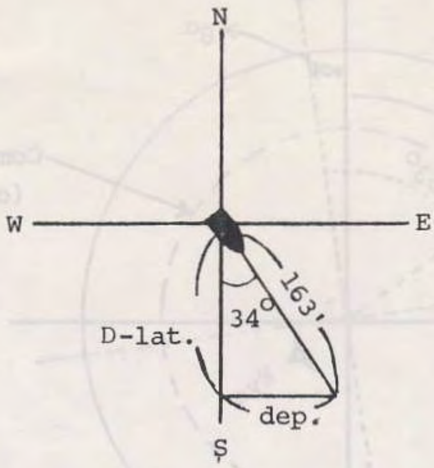


D-lat. = 47' × Cos 70° = 47' × 0.3420 = 16'.1

Dep. = 47' × Sin 70° = 47' × 0.9397 = 44'.2

Ans. 16'.1 S, 44'.2 W

(7)



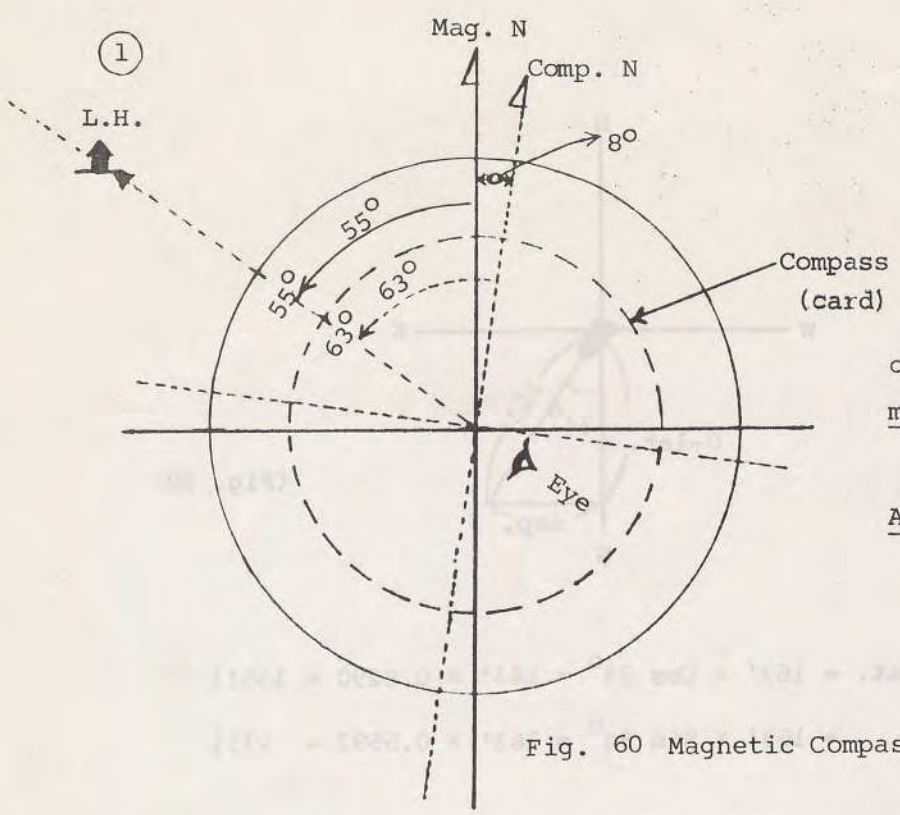
(Fig. 59)

$$\text{D-lat.} = 163' \times \cos 34^\circ = 163' \times 0.8290 = 135'.1$$

$$\text{Dep.} = 163' \times \sin 34^\circ = 163' \times 0.5592 = 91'.1$$

<u>Ans. 135'.1 S, 91'.1 E</u>

(26)

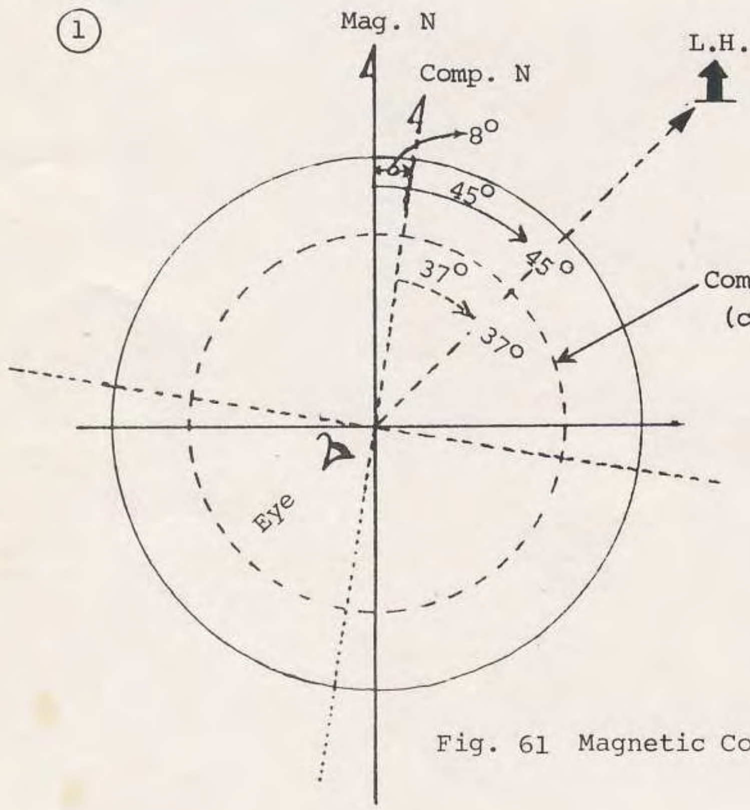


comp. b'g 63°
mag. b'g 55° ⊖
8°

Ans. 8°E

Fig. 60 Magnetic Compass.

(27)

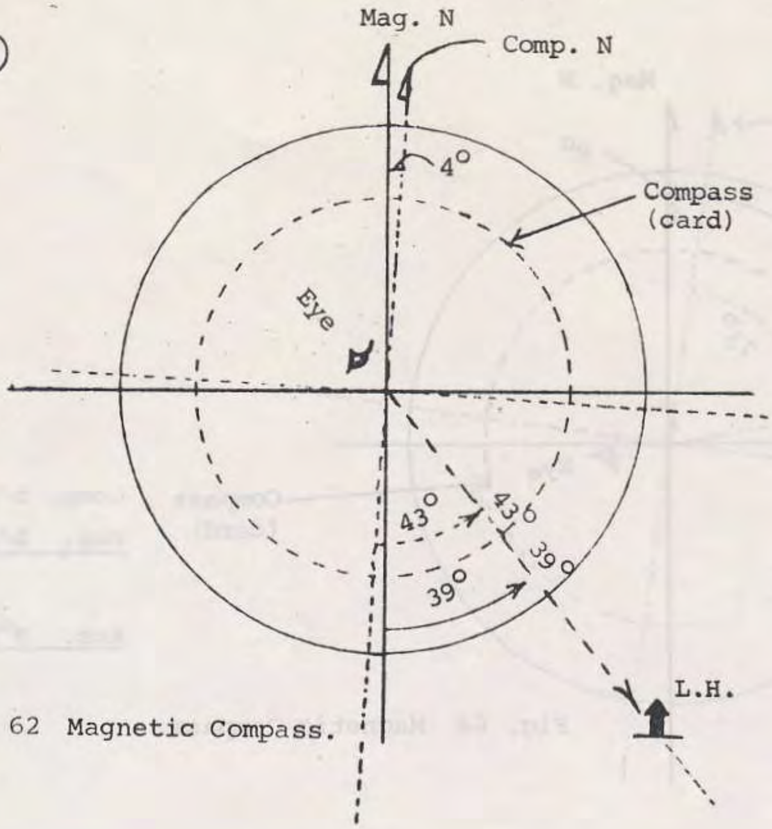


Comp. b'g 37°
mag. b'g 45° ⊖
8°

Ans. 8°E

Fig. 61 Magnetic Compass.

(3)

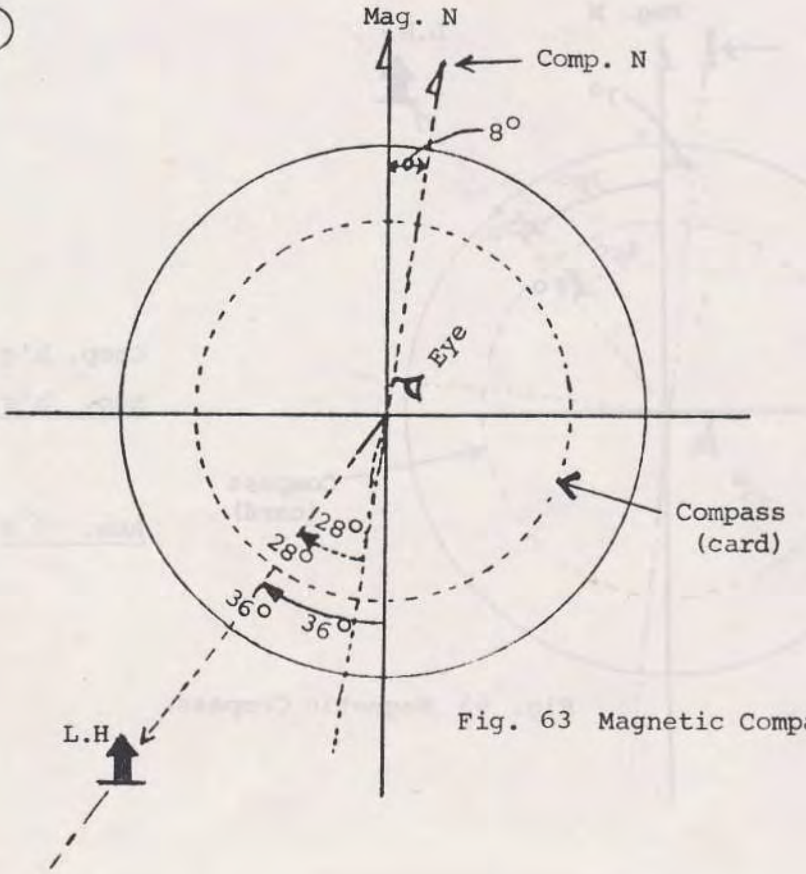


Comp. b'g 43°
mag. b'g 39° (−)
4°

Ans. 4°E

Fig. 62 Magnetic Compass.

(4)

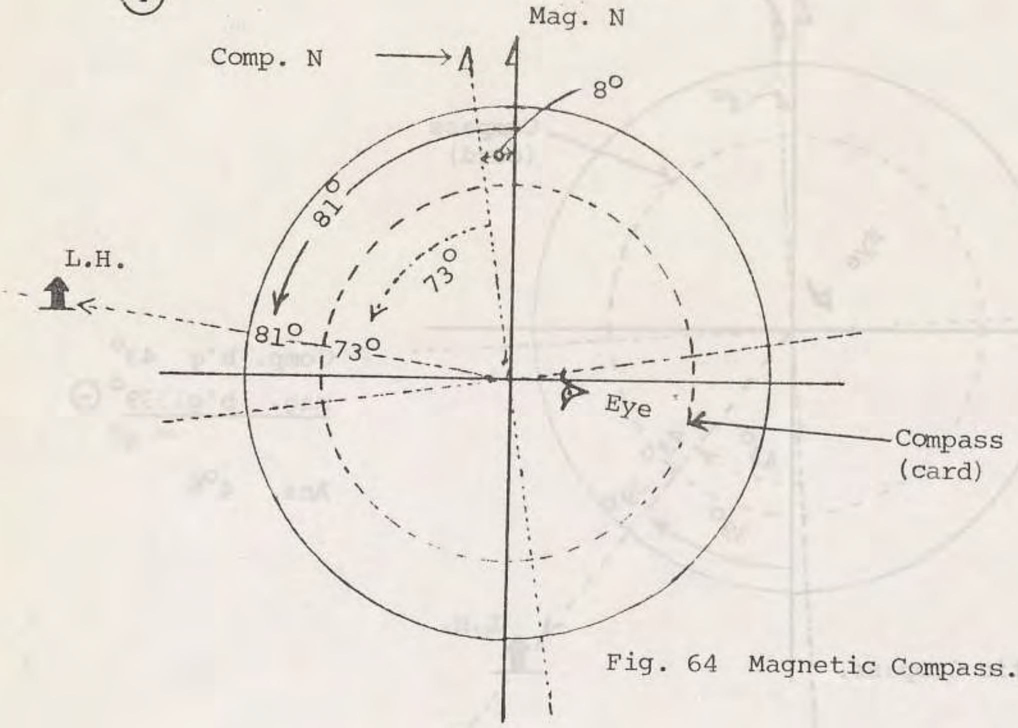


Comp. b'g 28°
mag. b'g 36° (−)
8°

Ans. 8°E

Fig. 63 Magnetic Compass.

④

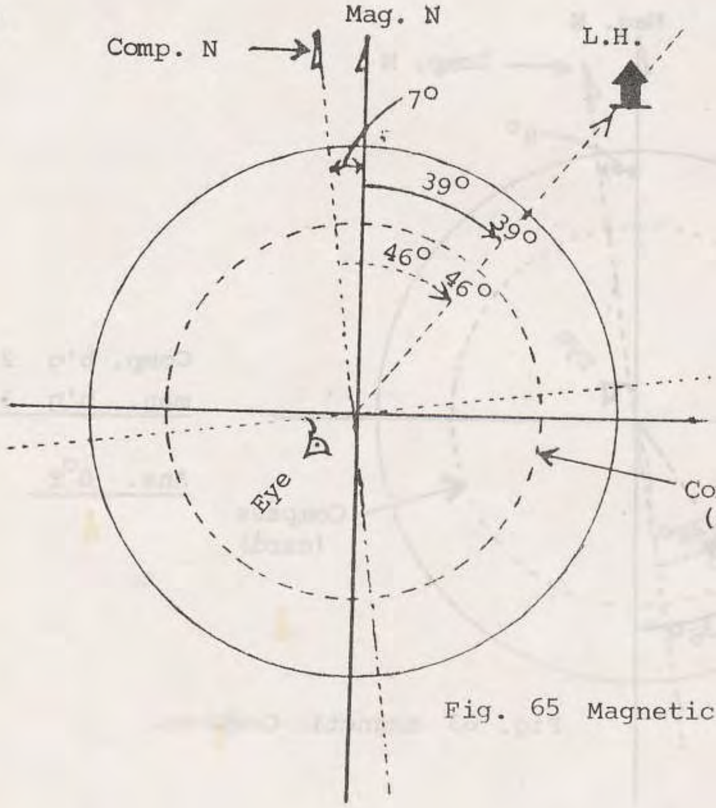


Comp. b'g 81°
mag. b'g 73°
8°

Ans. 8°W

Fig. 64 Magnetic Compass.

⑤



Comp. b'g 46°
mag. b'g 39° ⊖
7°

Ans. 7°W

Fig. 65 Magnetic Compass.

(28)

1. To compute the compass error first:

	var.	$5^\circ - 42'W$
	dev.	$7^\circ - 18'W$ ⊕
Comp.	error	$13^\circ - 00'W$

If there is no leeway, the compass course should be 140° ($127^\circ + 13^\circ$), but we have 7° leeway and the wind direction is northeast, then we have to steer the ship 7° more to the windward. Therefore, the compass course should be $140^\circ - 7^\circ = 133^\circ$ (or S 47°E).

Ans. 133° or S 47°E.

2.

	var.	$5^\circ - 42'W$ (L)
	dev.	$7^\circ - 18'W$ (L) ⊕
Comp.	error	$13^\circ - 00'W$ (L)

True Co.	$127^\circ - 00'$	L.S
Comp. error	$13^\circ - 00'$	L ⊕
Apparent Co.	$140^\circ - 00'$	L.S
Leeway	$7^\circ - 00'$	R.N ⊖
Comp. co.	$133^\circ - 00'$	L.S

Ans. 133° or S 47°E

(29)

$3' \quad 1852^m = 92^m.6$

1 min. $\Longrightarrow 92^m.6$

(see Fig. 67)

(30)

var. 8°L
dev. 3°R ⊖
Comp. error 5°L

Comp. co. 36° R.N
Comp. error 5° L ⊖
Apparent co. 31° R.N
Leeway 11.3° R ⊕
True co. 42.3° R.N

N 42.3° E

Ans. 42.3° or N 42.3° E

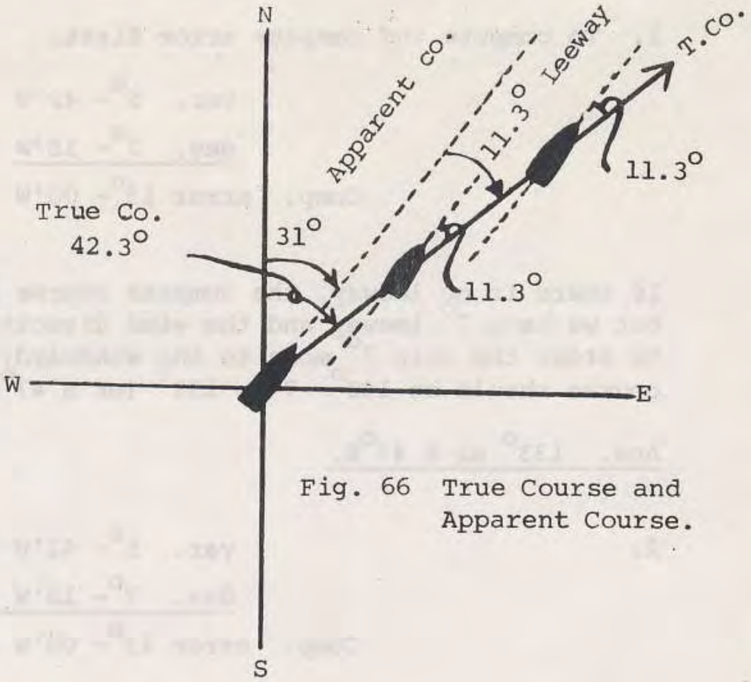


Fig. 66 True Course and Apparent Course.

(31)

var. 5° - 20'L
dev. 3° - 15'R ⊖
Comp. error 2° - 05'L

Comp. co 16° - 00' LN
Comp. error 2° - 05' L ⊕
Apparent co. 18° - 05' LN
Leeway 2° - 50' L ⊕
True co. 20° - 55' LN
= N 20° - 55' W
or 339° - 05'

Ans. N 20° - 55'W or 339° - 05'

(32)

var. 6° - 45'L
dev. 3° - 30'R ⊖
Comp. error 3° - 15'L

Comp. b'g 35° R.N
Comp. error 3° - 15'L ⊖
True b'g 31° - 45'

Ans. 31° - 45' or
N 31° - 45' E.

(33)

dist. (A → F) $= 74'.5$

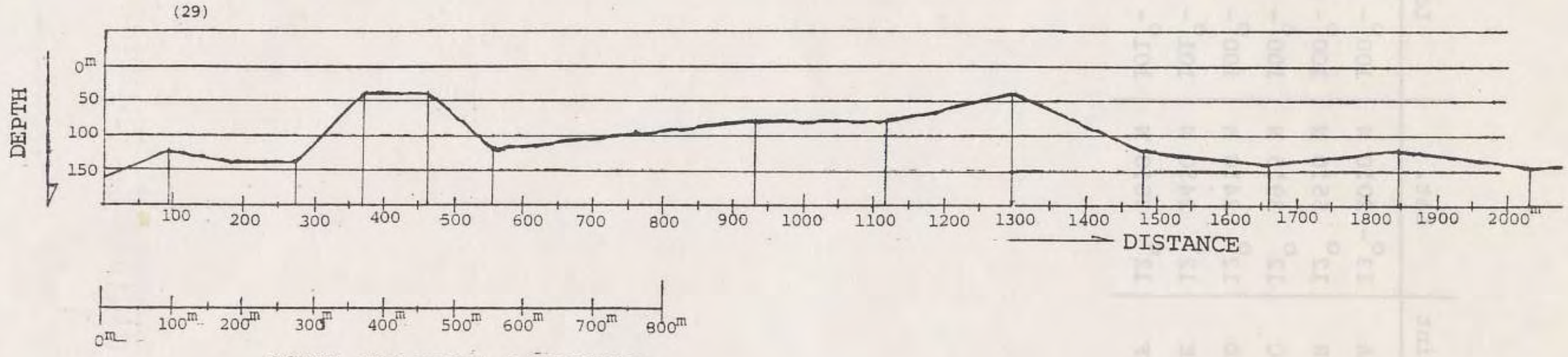
$74'.5 / 10' = 7^{h}.45 = 7^{h} - 27^{m}$

April $14^{d} - 18^{h} - 15^{m}$

$$\begin{array}{r} 14^{d} - 18^{h} - 15^{m} \\ 7^{h} - 27^{m} \; \oplus \\ \hline 14^{d} - 25^{h} - 42^{m} \\ \ominus \, 24 \\ \hline 15^{d} - 01^{h} - 42^{m} \end{array}$$

Ans. April 15, 01:42

Point	Lat.	Long.
A	$13^{\circ} - 20'.0$ N	$100^{\circ} - 36'.8$ E
B	$12^{\circ}\ 55'.0$ N	$100^{\circ} - 36'.1$ E
C	$12^{\circ}\ 39'.5$ N	$100^{\circ} - 42'.0$ E
D	$12^{\circ}\ 24'.0$ N	$100^{\circ} - 54'.5$ E
E	$12^{\circ}\ 24'.0$ N	$101^{\circ} - 02'.0$ E
F	$12^{\circ}\ 30'.0$ N	$101^{\circ} - 02'.0$ E



SCALE FOR DEPTH & DISTANCE

1 minute → 92.6 m

Fig. 67 Recording of sea-bed.

Exercise 5

(35) A ship has sailed 319.5 miles north and 116.3 miles east.

Find, - (1) course, and (2) distance. (Make use of Traverse Tables).

(Ans: Dist. = 340'
Co. N 20°E)

(36) A ship has sailed 137.4 miles south and 399.0 miles west.

Find, - (1) course, and (2) distance. (Make use of Traverse Tables).

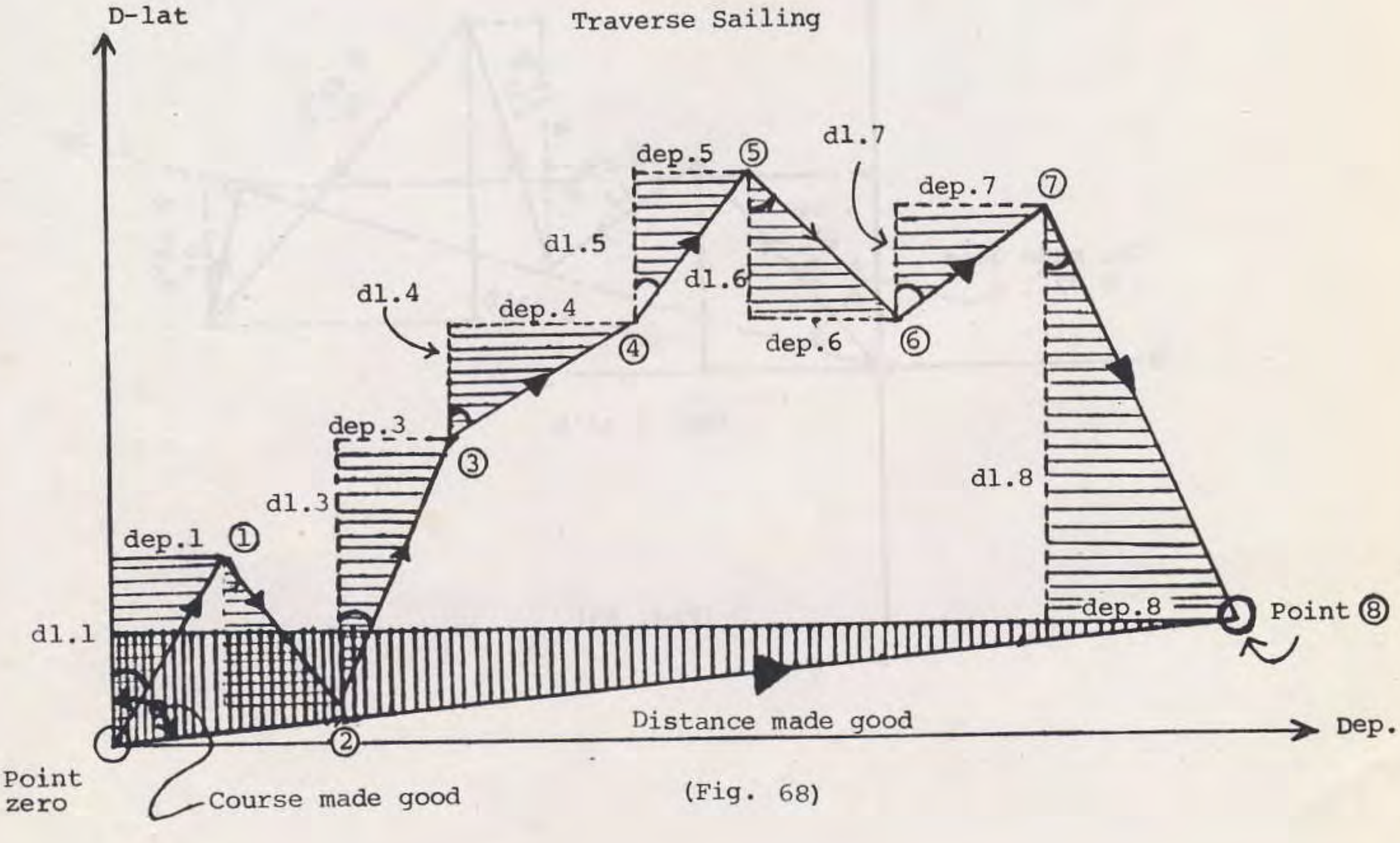
(Ans: Dist. = 422'
Co. = S 71°W)

(37) A ship has sailed 43.1 miles north and 18.3 miles west.

(Ans: Dist. = 47'
Co. = N 23°W)

2. Traverse Sailing

Is a combination of plane sailing solutions when there are two or more courses. This sailing is a solution to determine to equivalent course and distance made good, and to find an equivalent single course and distance (Fig. 68 and 69).



(Fig. 68)

Example 6

A ship has proceeded with the courses and distances tabulated as follows (see Fig. 69)

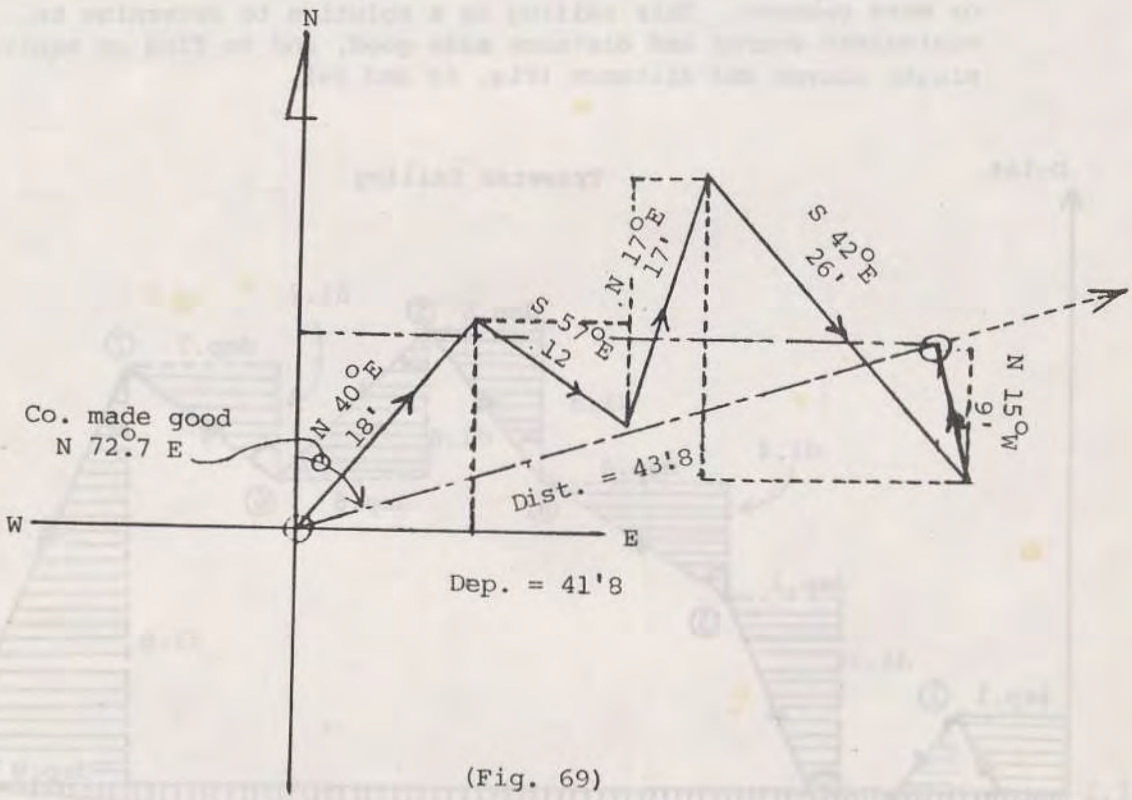
True Course	Distance
N 40°E	18 miles
S 57°E	12 miles
N 17°E	17 miles
S 42°E	26 miles
N 15°W	9 miles

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

(Solve by means of Traverse Tables.)

Solution to Example 6



(Fig. 69)

T. Co.	Dist.	D-lat. N	D-lat. S	Dep. E	Dep. W
N 40^{O}E	18'	13.8	-	11.6	-
S 57^{O}E	12'	-	6.5	10.1	-
N 17^{O}E	17'	16.3	-	5.0	-
S 42^{O}E	26'	-	19.3	17.4	-
N 15^{O}W	9	8.7	-	-	2.3
		38.8	25.8	44.1	2.3
		25.8 ⊖		2.3 ⊖	
		13.0 N		41.8 E	

Traverse Table

40	38.0	12.4
41	39.0	12.7
42	39.9	13.0
43	40.9	13.3
44	41.8	13.6
45	42.8	13.9
---	---	---
---	---	---
Dist.	Dep.	D-lat
72°		

Traverse Table

40	38.3	11.7
41	39.2	12.0
42	40.2	12.3
43	41.1	12.6
44	42.1	12.9
45	43.0	13.2
---	---	---
---	---	---
Dist.	Dep.	D-lat.
73°		

	Dist.	Dep.	D-lat.
73°	44'	42′.1	12′.9
?	?	(41.8)	(13.0)
72°	43'	40′.9	13′.3

44' − 43' = 1' ← 42.1 − 40.9 = 1′.2

42.1 − 41.8 = 0′.3 → X'

$1'.2 \times X = 0.3 \times 1'$

$X' = \frac{0.3}{1.2} = \frac{3}{12} = \frac{1}{4} = 0.25 \approx 0'.3$

Dist. = 44' − 0′.3 = 43′.7

73° − 72° = 1° ← 42′.1 − 40′.9 = 1′.2

42′.1 − 41.8 = 0′.3 → Y°

$1'.2 \times Y^\circ = 0'.3 \times 1^\circ$

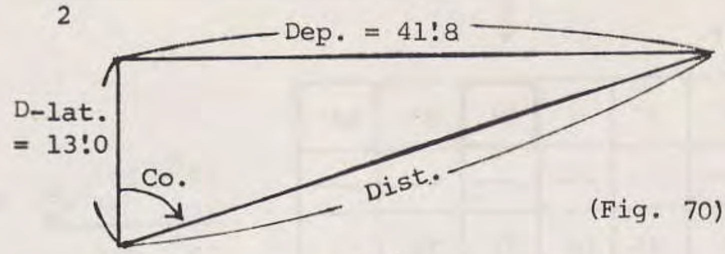
$Y = \frac{0.3}{1.2} = \frac{3}{12} = \frac{1}{4} = 0.25 \approx 0^\circ.3$

Co. = 73° − 0°.3 = 72°.7

Ans. Equivalent single course N $72^{\circ}.7$ E

Equivalent single distance = 43'.7

2



$$\text{Tan. Co.} = \frac{41.8}{13.0} = 3.215$$

(Fig. 70)

Natural Trigonometric Function

	Tan.								
o	0'	10'	20'	30'	40'	50'	60'		Difference
---	---	---	---	---	---	---	---	---	---
70	2.747	2.773	2.798	2.824	2.850	2.877	2.904	19	25-27
71	2.904	2.932	2.960	2.989	3.018	3.047	3.078	18	28-31
72	3.078	3.108	3.140	3.172	204	237	271	17	30-34
73	271	305	340	376	412	450	487	16	34-38
74	487	526	566	606	647	689	3.732	15	39-43
---	---	---	---	---	---	---	---	---	---
---	---	---	---	---	---	---	---	---	---

Cot.

From the tables, the course angle should be between 72°-40' and 72°-50', because the Tan. Co. = 3.215.

	40'	50'	Difference
72°	3.204	3.237	30-34 (33)
	---	---	---

$$\begin{array}{r} 3.237 \\ 3.215 \ \ominus \\ \hline 22 \end{array}$$

P.P.

Diff.	1'	2'	3'	4'	5'	6'	7'	8'	9'
---	---	---	---	---	---	---	---	---	---
30	3	6	9	12	15	18	21	24	27
31	3	6	9	12	16	19	22	25	28
32	3	6	10	13	16	19	22	26	29
33	3	7	10	13	17	20	23	26	30
34	3	7	10	14	17	20	24	27	31
35	4	7	11	14	18	21	25	28	32
---	---	---	---	---	---	---	---	---	---

$$
\begin{array}{r}
72^{\circ}-50' \\
7' \ominus \\
\hline
72^{\circ}-43' \\
\approx 72.^{\circ}7
\end{array}
$$

$$
\begin{array}{r}
3.237 \\
3.204 \ominus \\
\hline
33
\end{array}
\qquad
\begin{array}{r}
3.237 \\
3.215 \ominus \\
\hline
22
\end{array}
$$

$$\text{Cos. } 72^{\circ}-43' = \frac{13!0}{\text{Dist.}}$$

$$\text{Dist.} = \frac{13!0}{\text{Cos. } 72^{\circ}-43'} = \frac{13!0}{0.2971} = 43!75631$$

$$\approx 43!8$$

Ans: N 72°- 43'E (or N 72.°7E), 43!8

Attention: ≈ nearly equal to

Example 7

A ship has proceeded with the courses and distances as tabulated below.

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

Solve by using an electronic calculator

No	True Course	Distance
1	N 59°E	73 miles
2	S 33°E	50 miles
3	S 13°W	40 miles
4	N 87°W	35 miles
5	S 18°W	150 miles

Example 8

A ship has proceeded with the courses and distances as tabulated below.

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

Solve by using an electronic calculator and the Traverse Tables

No	True Course	Distance
1	N 64°W	75 miles
2	S 84°W	63 miles
3	S 2°E	55 miles

Example 9

A ship has proceeded with the courses and distances as tabulated below

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

Note: The current was running into S 74°.5 E for 39 nautical miles

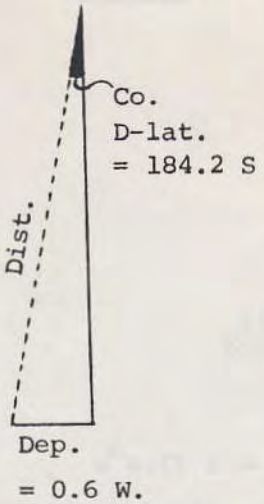
No.	True Course	Distance
1	N 44°E	126 miles
2	S 65°W	85 miles
3	S 25°E	68 miles
4	S 88°E	73 miles

Solution to Example 7

			D-lat.		Dep.	
No.	True Course	Dist.	N	S	E	W
1	N 59°E	73'	37.6		62.6	
2	S 33°E	50'		41.9	27.2	
3	S 13°W	40'		39.0		9.0
4	N 87°W	35'	1.8			35.0
5	S 18°W	150'		142.7		46.4
	Total		39.4 N	223.6S	89.8 E	90.4 W

$$\begin{array}{r} 223.6\ S \\ \underline{39.4\ N} \ominus \end{array} \qquad \begin{array}{r} 90.4\ W \\ \underline{89.8\ E} \ominus \end{array}$$

$$\text{D-lat.} = \underline{\underline{184.2\ S}} \qquad \text{Dep.} = \underline{\underline{0.6\ W}}$$



(Fig. 71)

$$\text{Tan. Co.} = \frac{0.6}{184.2} \div 0.0033$$

$$\therefore \text{Co.} = 11' \rightarrow S\ 11'\ W$$

$$\sin. 11' = \frac{0.6}{\text{Dist.}}$$

$$\therefore \text{Dist.} = \frac{0.6}{\sin. 11'} = \frac{0.6}{0.00320} = 187!5$$

Ans. Equivalent single course = S 11'W or 180°11'

Equivalent single distance = 187.5 nautical miles

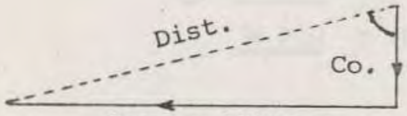
Solution to Example 8

1) By calculator and natural trigonometric functions.

			D-lat.		Dep.	
No.	True Course	Dist.	N	S	E	W
1.	N 64°W	75'	32.9			67.4
2.	S 84°W	63'		6.6		62.7
3.	S 2°E	55'		55.0	1.9	
Total			32.9 N	61.6S	1.9 E	130.1 W

$$\begin{array}{r} 61.6\ S \\ \underline{32.9\ N} \ominus \end{array} \qquad \begin{array}{r} 130.1\ W \\ \underline{1.9\ E} \ominus \end{array}$$

$$\text{D-lat.} = \underline{\underline{28.7\ S}} \qquad \text{Dep.} \quad \underline{\underline{128.2\ W}}$$



Dep. = 128'.2 W

(Fig. 72)

$$\text{D-lat.} = 28'.7\ S$$

$$\tan \text{Co.} = \frac{128'.2}{28'.7} = 4.467$$

$$\therefore \text{Co.} = S\ 77^{\circ}23'W$$

$$\sin 77^{\circ}23' = \frac{128'.2}{\text{Dist.}}$$

$$\therefore \text{Dist.} = \frac{128'.2}{\sin 77^{\circ}23'} = \frac{128'.2}{0.9759}$$

$$\doteqdot 131'.4$$

Ans. Equivalent single course $= S\ 77.4^{\circ}W$

Equivalent single distance = 131.4 nautical miles.

2) By Traverse Tables and Natural Trigonometric Functions.

By Natural Trigonometric Function Co. $\doteqdot 77^{\circ}23'$

Dep. = 128.2 W

(Fig. 73)

$$\text{D-lat.} = 28'.7\ S$$

$$\tan O = \frac{128'.7}{28'.7} = 4.467 \rightarrow O \doteqdot 77^{\circ}23'$$

Then you should check the Traverse Tables for 77° and 78° and, pick them up as follows:

From Traverse Table.

Co.	Dist.	D-lat.	Dep.
77°	132'.0	29'.7	128'.6
X (co.)	Y (Dist.)	28.7	128.2
78°	131.0	27.2	128.1
-1° $(77^{\circ}-78^{\circ})$	1' (132' - 131')	2'.5 (29'.7 - 27'.2)	0.5 (128'.6 - 128'.1)
α	β	1'.0 (29.7 - 28.7)	0'.4 (128.6 - 128.2)

From the above table, $X = 77^\circ - \alpha^\circ$

$Y = 132'.0 - \beta'$

$$-\frac{1}{\alpha} = \frac{2'.5}{1'.0} \quad \alpha = \frac{1'.0}{2'.5} \times (-1^\circ) = -0.4^\circ \qquad X = 77^\circ - (-0.4^\circ) = 77.4^\circ$$

$$\frac{1'}{\beta} = \frac{2'.5}{1.0} \quad \beta = \frac{1'.0}{2.5} \times 1' = 0'.4 \qquad Y = 132'.0 - 0'.4 = 131'.6$$

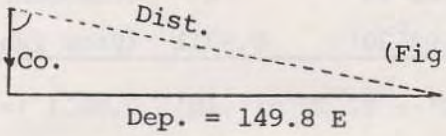
Solution to Example 9

No.	True Course	Dist	D-lat. N	D-lat. S	Dep. E	Dep. W
1	N 44°E	126'	90.6		87.5	
2	S 65°W	85'		35.9		77.0
3	S 25°E	68'		61.6	28.7	
4	S 88°E	73'		2.5	73.0	
5	S 74.5°E	39'		10.4	37.6	
Total			90.6 N	110.4 S	226.8 E	77.0 W

110.4 S   226.8 E
90.6 N (−)   77.0 W (−)

Balance: D-lat. = 19.8 S   Dep. = 149.8 E

D-lat. = 19.8S



(Fig. 74)

Dep. = 149.8 E

$$\tan.\ Co. = \frac{149'.8}{19'.8} = 7.566$$

	$82^\circ 30' \rightarrow$	7.596 (From Table)
	Co.(Unknown) $\leftarrow$	7.566 (Calculated by yourself)
	$82^\circ 20' \rightarrow$	7.429 (From Table)
$(82^\circ 30' - 82^\circ 20') \rightarrow 10'$		$0.167 \leftarrow (7.596 - 7.429)$
$(82^\circ 30' - \text{Co.}) \rightarrow X'$		$0.030 \leftarrow (7.596 - 7.566)$

$$\frac{10'}{X} = \frac{0.167}{0.030} \qquad \therefore X = 10' \; \frac{0.030}{0.167} = 1.8$$

$$82^\circ 30' - \text{Co.} = 1.8 \qquad \sin 82^\circ 28' = \frac{149.8}{\text{Dist.}}$$

$$\text{Co.} = 82^\circ 30' - 1.8' \qquad \text{Dist.} = \frac{149'.8}{\sin 82^\circ 28'}$$

$$= 82^\circ 28.2' \qquad = \frac{149.8}{0.9912} \doteqdot 151'.1$$

$$\doteqdot 82^\circ 28'$$

Ans. Equivalent single course — S $82^\circ 28'$E

Equivalent single distance — 151.1 nautical miles

How to obtain the value of $\sin 82^\circ 28'$

a) By "Proportional parts"

sin. $82^\circ 30'$	0.9914	(From Table)
sin. $82^\circ 28'$	X	(Unknown)
sin. $82^\circ 20'$	0.9911	(From Table)

$(82^\circ 30' - 82^\circ 20' =)$ 10' 0.0003 (= 0.9914 - 0.9911)

$(82^\circ 30' - 82^\circ 28' =)$ 2' Y (= 0.9914 - X)

$$\frac{10'}{2'} = \frac{0.0003}{Y} \qquad Y = 0.0003 \times \frac{2'}{10'} = 0.00006$$

$$\begin{aligned} Y &= 0.9914 - X \\ X &= 0.9914 - Y \\ &= 0.9914 - 0.00006 \\ &= 0.99134 \qquad \therefore \sin 82^\circ 28' = 0.99134 \end{aligned}$$

$$\text{Dist.} = \frac{\text{149!8}}{\sin 82^{\circ}28'} = \frac{149.8}{0.99134} = \text{151!1086} \ldots\ldots$$

$$\doteqdot \text{151!1}$$

b) By Tables of Natural Trigonometric Functions

$\sin 82^{\circ}28 = 0.99137$

$$\text{Dist.} = \frac{\text{149!8}}{\sin 82^{\circ}28'} = \frac{\text{149!8}}{0.99137} = 151.1040 \ldots. \doteqdot \text{151!1}$$

TABLE 31
Natural Trigonometric Functions

7° → ↓	sin	Diff 1'	csc	Diff 1'	tan	Diff 1'	cot	Diff 1'	sec	Diff 1'	cos	Diff 1'	← 172° ↓
---	---	---	---	---	---	---	---	---	---	---	---	---	---
		29		1696		29		1712		4		4	
30	0.13053	28	7.66130	1689	0.13165	30	7.59575	1703	1.00863	4	0.99144	3	30
31	.13081	29	.64441	1682	.13195	29	.57872	1696	.00867	4	.99141	4	29
32	.13110	29	.62759	1674	.13224	30	.56176	1689	.00871	4	.99137	4	28
33	.13139	29	.61085	1667	.13254	30	.54487	1681	.00875	3	.99133	4	27
34	.13168	29	.59418	1659	.13284	29	.52806	1674	.00878	4	.99129	4	26
---	---	---	---	---	---	---	---	---	---	---	---	---	---
↑ 97° →	cos	Diff 1'	sec	Diff 1'	cot	Diff 1'	tan	Diff 1'	csc	Diff 1'	sin	Diff 1'	82° ↑ ←

c) By Tables of Natural Trigonometric Functions which have a column for proportional parts (p.p)

P.P					°	sin.								Diff
6′	7′	8′	9′	Diff		0′	10′	20′	30′	40′	50′	60′		
				30	45	0.7071	0.7092	0.7112	0.7133	0.7153	0.7173	0.7193	44	21–20
					46	.7193	.7214	.7234	.7254	.7274	.7294	.7314	43	21–20
					47	.7314	.7333	.7353	.7373	.7392	.7412	.7431	42	20–19
								---	---	---	---	---	--	---
								---	---	---	---	---	--	---
4	4	5	5	6				---	---	---	---	---	--	---
					82	.9903	.9907	.9911	.9914	.9918	.9922	.9925	7	4–3
					83	.9925	.9929	.9932	.9936	.9939	.9942	.9945	6	4–3
3	4	4	5	5	84	.9945	.9948	.9951	.9954	.9957	.9959	.9962	5	3–2
					85	0.9962	0.9964	0.9967	0.9969	0.9971	0.9974	0.9976	4	3–2
2	3	3	4	4	86	.9976	.9978	.9980	.9981	.9983	.9985	.9986	3	2–1
2	2	(2)	3	3	87	.9986	.9988	.9989	.9990	.9992	.9993	.9994	2	2–1
1	1	2	2	2	88	.9994	.9995	.9996	0.9997	0.9997	0.9998	0.9998	1	1–0
1	1	1	1	1	89	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	0	1–0
6′	7′	8′	9′	Diff		60′	50′	40′	30′	20′	10′	0′	°	Diff.
						cos.								

1st step, sin $82^\circ 20'$ → 0.9911 (From Table)

sin $82^\circ 30'$ → 0.9914 − (From Table)

$$\begin{array}{r} 0.9914 \\ -\,0.9911 \\ \hline 0.0003 \end{array}$$

Balance 0.0003 → Diff. 3

2nd step, $82^\circ 28'$

$82^\circ 20'$ −

Balance 8'

3rd step, By Column of P.P

You could have the value (2) from Column Diff. 3 and P.P. 8'

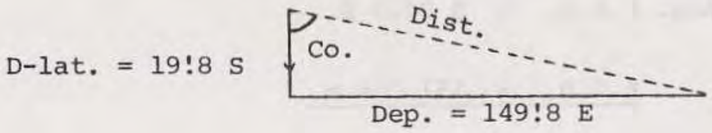
Then, $\sin 82^{\circ}28' = 0.9911 + 0.000\ (2) = 0.9913$

$$\text{Dist.} = \frac{149'.8}{\sin 82^{\circ}28'} = \frac{149'.8}{0.9913} = 151'.1146 \ldots \doteqdot 151'.1$$

3) By Traverse Tables and Tables of Natural Trigonometric Functions.

We have already calculated the equivalent single D-lat. and Dep. as follows:

D-lat. = 19'.8 S Dep. = 149'.8 E



(Fig. 75)

1st step. $\tan \text{Co.}\ \frac{149'.8}{19'.8} = 7.5657$

Using Natural Trigonometric Functions Table 31.

$7.5657 \rightarrow 82^{\circ}29'$

2nd step. Using Traverse Table

Co.	Dist.	D-lat.	Dep.
82° (From Table)	152' (From Table)	21'.2 (From Table)	150'.5 (From Table)
X (Unknown)	Y (Unknown)	19'.8 (calculated)	149'.8 (calculated)
83° (From Table)	150' (From Table)	18'.3 (From Table)	148'.9 (From Table)
-1° (= 82° - 83°) α (= 82° - X)	2 (= 152'-150') β (= 152'-Y)	2'.9 (21'.2-18'.3) 1'.4 = 21'.2-19'.8	1'.6 (= 150'.5-148'.9) 0'.7 (= 150'.5-149'.8)

$$\frac{-1^\circ}{\alpha} = \frac{2'\!.9}{1'\!.4} \qquad \frac{2'}{\beta} = \frac{2'\!.9}{1'\!.4}$$

$$\alpha = -1^\circ \times \frac{1'\!.4}{2'\!.9} = -0^\circ\!.52 \qquad \beta = \frac{1'\!.4}{2'\!.9} \times 2' = 0'\!.97$$

$$-0^\circ\!.52 = 82^\circ - X \qquad 0.97' = 152' - Y$$

$$X = 82^\circ + 0^\circ\!.52 \doteqdot 82^\circ\!.5 \qquad Y = 152' - 0'\!.97$$

$$= 151'\!.03 \doteqdot 151'\!.0$$

Ans. E.S.C. = S 82°.5 E

E.S.D. = 151.0 n.m.

Example 10

A ship has sailed with courses and distances as tabulated below.

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

Attention! In this case you have only "Compass course" and "Compass error", therefore you must compute "True course" first.

No.	Compass course	Compass error	Distance
1	N 56° E	3° E	73 miles
2	S 32° E	1° W.	50 miles
3	S 15° W	2° W	40 miles
4	West	3° E	35 miles
5	S 20° W	2° W	150 miles

Example 11

A ship has sailed with courses and distances as tabulated below.

Find: (1) Equivalent single course, and

(2) Equivalent single distance.

Attention! You must compute "Compass error" first, and calculate "True course".

No.	Compass Course	Deviation	Variation	Distance
1	N 70° W	1° E	5° E	75 miles
2	S 80° W	1° W	5° E	63 miles
3	S 5° E	2° W	5° E	55 miles

Example 12

A ship has sailed with courses and distances as tabulated below. Find Equivalent single course and distance.

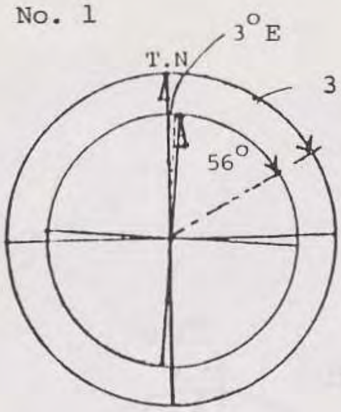
Note : Variation was 7^{o}W, and during this sailing, current ran to ESE and distance 39 nautical miles.

No.	Compass Course	Deviation	Distance
1	N E	6^{o} E	126 miles
2	S 77^{o} W	5^{o} W	85 miles
3	S S E	4^{o}- 30 E	68 miles
4	East	9^{o} E	73 miles

Solution to Example 10

No.	Comp. Co.	Computation	True Co.	Distance
1	N 56°E	$56^\circ + 3^\circ$	N 59°E	73 miles
2	S 32°E	$32^\circ + 1^\circ$	S 33°E	50 miles
3	S 15°W	$15^\circ - 2^\circ$	S 13°W	40 miles
4	West	$270^\circ + 3^\circ$	N 87°W	35 miles
5	S 20°W	$20^\circ - 2^\circ$	S 18°W	150 miles

No. 1



$3 + 56 = 59^\circ \rightarrow$ True Co. 59°

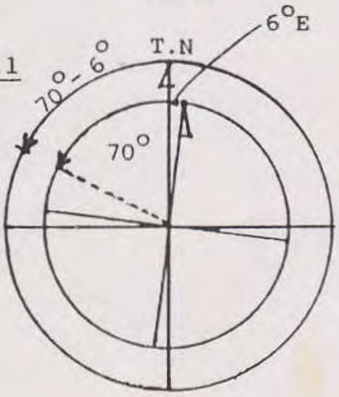
This answer is also the answer to the Solution to Example 7 on pages 60 and 61.

(Fig. 76)

Solution to Example 11

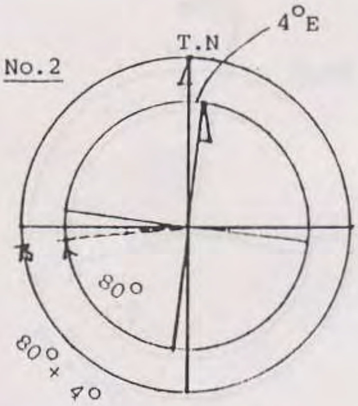
No.	Comp. Co.	Computation	Computation	True Co.	Distance
1	N 70°W	$1^\circ + 5^\circ = 6^\circ$E	$70^\circ - 6^\circ = 64^\circ$	N 64°W	75 miles
2	S 80°W	$-1^\circ + 5^\circ = 4^\circ$E	$80^\circ + 4^\circ = 84^\circ$	S 84°W	63 miles
3	S 5°E	$-2^\circ + 5^\circ = 3^\circ$E	$5^\circ + 3^\circ = 8^\circ$	S 2°E	55 miles

No.1



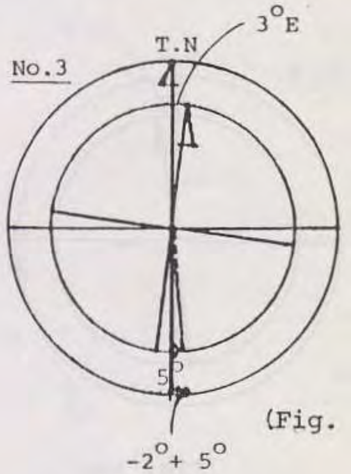
(Fig. 77)

No.2



(Fig. 78)

No.3



(Fig. 79)

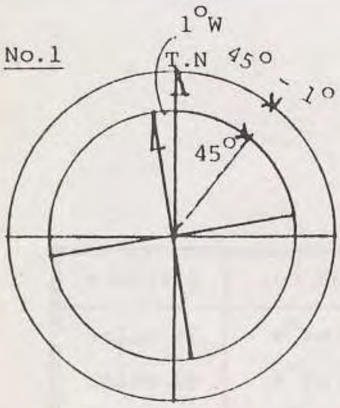
This answer is also the answer to the Solution to Example 8 on pages 61 and 62.

Solution to Example 12

No.	Comp. Co.	Var.	Dev.	Computation	Comp. error	Computation
1	N 45°E	7°W	6°E	$-7^\circ + 6^\circ = -1^\circ$	1°W	$45^\circ - 1^\circ = 44^\circ$
2	S 77°W	7°W	5°W	$-7^\circ + (-5^\circ) = -12^\circ$	12°W	$77^\circ - 12^\circ = 65^\circ$
3	S 22°.5 E	7°W	4°.5 E	$-7^\circ + 4^\circ.5 = -2^\circ.5$	2°.5 W	$22^\circ.5 + 2^\circ.5 = 25^\circ$
4	090°	7°W	9°E	$-7^\circ + 9^\circ = +2^\circ$	2°E	$90^\circ + 2^\circ = 92^\circ$
5	S67°.5 E(SSE)	7°W	NIL	$+7^\circ$		$67^\circ.5 + 7^\circ = 74^\circ.5$

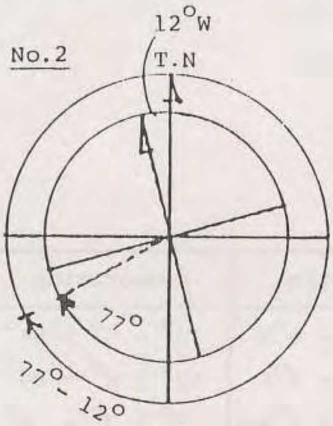
True Co.	Distance
N 44°E	126 miles
S 65°W	85 miles
S 25°E	68 miles
S 88°E	73 miles
S 74°.5 E	39 miles

No.1



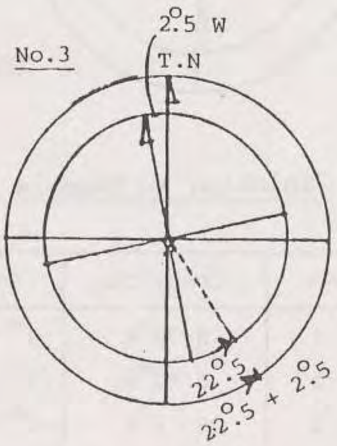
(Fig. 80)

No.2



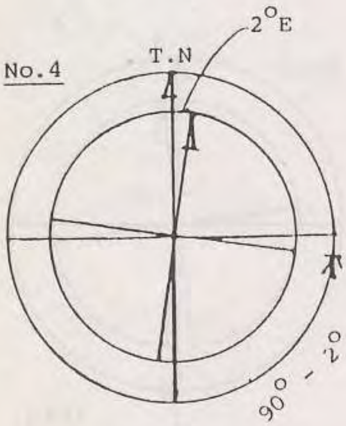
(Fig. 81)

No.3



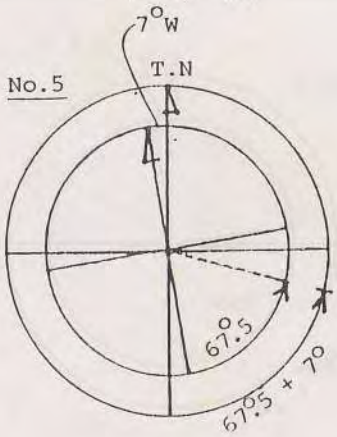
(Fig. 82)

No.4



(Fig. 83)

No.5



(Fig. 84)

This answer is also the answer to the Solution to Example 9 on page 63

3. Mercator Sailing and Chart

The chart used most popularly by navigators is the Mercator chart. The meridians on the earth become closer together as the latitude increases, but on the Mercator chart, the meridians are parallel, equidistant from each other and perpendicular to the parallels of latitude.

The main characteristic feature of this chart is that both the meridians and parallels are expanded by the same ratio with increased latitude. The ratio is derived from the formula

D - Long. = Dep. x sec lat.,

the expansion is equal to the secant of latitude, and the same in all directions.

The Mercator chart cannot include both the north pole and south pole, because the secant of 90° is infinity. Rhumb lines are shown as straight lines, the direction and distance can be measured directly on this chart.

The latitude scales are used for measuring distances, the expansion of the scale being the same as that of distances at the same latitude.

Great circles are shown as curved lines concave to the equator except meridians and the equator.

The plotting of positions by making use of latitude and longitude is the same as the plotting method of rectangular coordinates.

1. Meridional part

If the earth is a perfect sphere, the length of 1 minute of latitude should be equal to that of 1 minute of longitude. To express both the length of 1 minute of longitude on the Mercator chart, we suppose that the length of 1 minute of longitude is equal to 1 centimetre, then the length between the equator and the latitude 1 minute (= L_0^{cm}) is shown below:

$L_0^{cm} = 1^{cm} \times \sec 0'$ (because, D-Long. = Dep. x sec Lat.)

so between latitude 1 minute and 2 minutes, (L_1^{cm})

$$L_1^{cm} = 1^{cm} \times \sec 1'$$

Between latitude 2 minutes and 3 minutes (L_2^{cm})

$$L_2^{cm} = 1^{cm} \times \sec 2'$$

between latitude 3 minutes and 4 minutes, (L_3^{cm})

$$L_3^{cm} = 1^{cm} \times \sec 3'$$

between latitude (ℓ-1) minutes and ℓ minutes, $(L_{\ell-1}^{cm})$

$$L_{\ell-1}^{cm} = 1^{cm} \times \sec (\ell-1)'$$

So the total length (L_t) between the equator and latitude (ℓ) is shown as follows,

$$L_t = L_0^{cm} + L_1^{cm} + L_2^{cm} + L_3^{cm} + \ldots\ldots L_{\ell-1}^{cm}$$

$$= (1^{cm} \times \sec 0') + (1^{cm} \times \sec 1') + (1^{cm} \times \sec 2') +$$

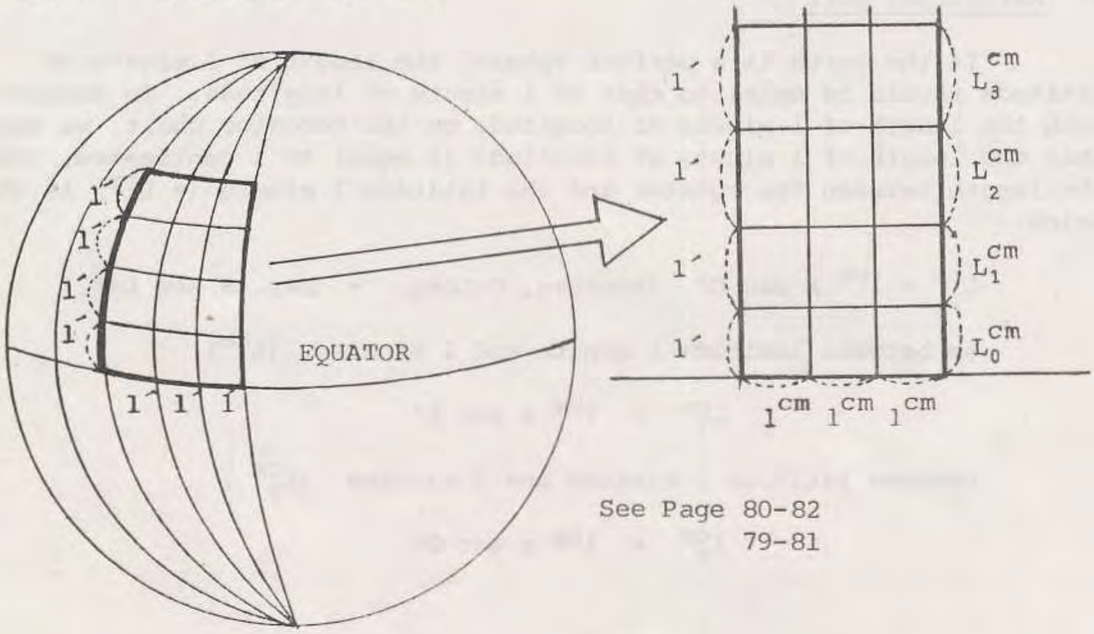
$$(1^{cm} \times \sec 3') + \ldots\ldots \{1^{cm} \times \sec (\ell-1)'\}.$$

The above-mentioned equations show that the length of 1 minute of each latitude is expanded as the latitude increases.

The length measured by the unit : 1 minute of difference of latitude = 1 minute of difference of longitude (D-Lat 1' = D-Long 1'), on the equator is called meridional part and the difference of meridional parts of two latitudes is called meridional difference of latitude (M.D.-Lat).

The formula of meridional part at latitude ℓ^o is shown below;

$$M\ell^o = 7915.704468 \times \log_{10} \tan (45^o + \ell^o/2) - 23.268932 \times \sin \ell^o - 0.0525 \times \sin^3\ell^o - 0.000213 \times \sin^5\ell^o.$$

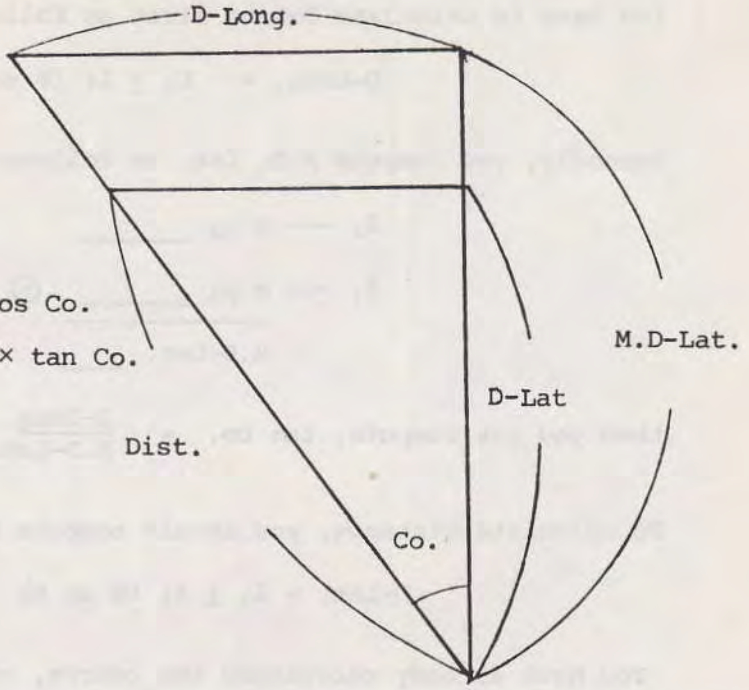


See Page 80-82
79-81

Fig. 85 Mercator chart.

2. Computation of Mercator sailing

a. How to compute the arrival latitude (ℓ_2) and longitude (L_2) with the given course, distance and latitude (ℓ_1), and longitude (L_1) of departure.



D-Lat. = Dist. × cos Co.

D-Long. = M.D-Lat. × tan Co.

Fig. 86 Difference of Longitude.

First you have to calculate the D-Lat., and the arrival latitude (ℓ_2), because $\ell_2 = \ell_1 \pm$ D-Lat. Secondly, you calculate M.D - Lat as follows:

$$\begin{array}{ll} \ell_2 \text{ --- } m.p_2 & (m.p_2 = \text{meridional part of } \ell_2) \\ \ell_1 \text{ --- } m.p_1 \ominus & (m.p_1 = \text{meridional part of } \ell_1) \\ \hline \text{M.D-Lat.} & \end{array}$$

You can find the meridional part from the table or the formula

$$M\ell^{\circ} = 7915.704468 \times \log_{10} \tan(45^{\circ} + \ell^{\circ}/2) - 23.268932 \times \sin \ell^{\circ} - 0.0525 \times \sin^3 \ell^{\circ} - 0.000213 \times \sin^5 \ell^{\circ}.$$

This M.D - Lat. gives you D-Long., because of D-Long. = M.D-Lat × tan Co.

Then you can have $L_1 = L_2 \pm$ D-Long.

b. How to compute the course and distance by the given two positions (latitude ℓ_1, longitude L_1) and (latitude ℓ_2, longitude L_2).

$$\tan \text{co.} = \frac{\text{D - Long.}}{\text{M.D. - Lat.}}$$

$$\text{Dist.} = \text{D - Lat.} \times \sec \text{co.}$$

You have to calculate D-Long first as follows:

$$\text{D-Long.} = L_2 \pm L_1 \text{ (E or W)}$$

Secondly, you compute M.D. Lat. as follows:

$$\ell_2 \text{ --- m.p}_2 \text{ \_\_\_\_\_\_}$$

$$\ell_1 \text{ --- m.p}_1 \text{ \_\_\_\_\_\_ } \ominus$$

$$\text{M.D-Lat. \_\_\_}$$

then you can compute, $\tan \text{co.} = \dfrac{\text{D-Long.}}{\text{M.D-Lat.}}$

To calculate distance, you should compute D-Lat. as follows:

$$\text{D-Lat.} = \ell_2 \pm \ell_1 \text{ (N or S)}$$

You have already calculated the course, so you can compute the distance by using the formula Dist. = D-Lat. × sec co..

Example 13

A ship has sailed from 34°- 12!0 N. 132°- 20!0 E. with the true course S 28°E, and the distance 658 nautical miles. Find the arrival latitude and longitude by Mercator sailing.

Solution

1) ℓ_2 D-Lat. = Dist. × cos co. = 658' × cos. 28° = 581!0 S

$= 9^\circ$- 41!0 S

$\ell_1 = 34^\circ$ - 12!0 N $\qquad \ell_2 = 24^\circ$- 31!0 N --- mp_2 = 1508!4

ℓ_1 = 34 - 12!0 N --- $mp_1 = \dfrac{2172.9}{664.5}$ $\ominus$

M.D-lat

D-Lat. $= \dfrac{9^\circ\text{- 41!0 S}}{24^\circ\text{- 31!0 N}}$ $\ominus$

ℓ_2

2) L_2 D-Long. = M.D-Lat × tan co. = 664!5 × tan 28° = 353!3 E

= 5°- 53!3 E

L_1 = 132°- 20!0 E

D-Long. = $\frac{5°- 53!3\ E}{138°- 13!3\ E}$ ⊕

Ans. = Arrival lat. 24° - 31!0 N

Arrival long. 138°- 13!3 E

Example 14

A ship has sailed from 50°- 30!0 N 27°- 15!0 W to 47°- 15!0 N 31°- 15!0 W. Find the true course and distance between the two positions by Mercator sailing.

Solution

1) Co.

L_2	31°- 15!0 W	
L_1	27°- 15!0 W	
D-Long. =	4°- 00.0 W	= 240!0 W

tan co = $\frac{\text{D-Long.}}{\text{M.D-Lat.}}$ = $\frac{240!0}{295!7}$ = 0.8116

ℓ_2	47°- 15!0	mp_2	3207!7
ℓ_1	50°- 30!0	mp_1	3503.4
		M.D.- Lat.	295!7

co. = S 39°W

2) Dist.

ℓ_2	47°- 15!0 N	
ℓ_1	50°- 30!0 N	
D-Lat	3°- 15!0 S	= 195!0 S

Dist. = D-Lat. × sec co.
= 195' × sec 39°
= 195' × 1.28676
= 250!9

Ans. Co. S 39°W, Dist. 250!9

Exercise 6 (See pages 108-111)

38. A ship has sailed from $37^{\circ}- 10'.0$ N, $165^{\circ}- 40'.0$ E with the true course S 65° W, and the distance 450' nautical miles. Find the arrival latitude and longitude by Mercator sailing.

(Ans. ℓ $33^{\circ}- 59'.8$ N, L $157^{\circ}- 21'.2$ E)

39. A ship has sailed from $45^{\circ}- 22'.0$ N, $170^{\circ}- 46'.0$ E to $40^{\circ}- 40'.0$ N, $173^{\circ}- 15'.0$ E. Find the course and distance between the two positions by Mercator sailing.

(Ans. S $21^{\circ}.1$ E, $302'.3$)

40. A ship left the position $40^{\circ}- 20'.0$ N, $179^{\circ}- 10'.0$ E at 10:00 with the true course 065° and at the speed of 10 nautical miles per hour. What time did this ship reach the longitude 180°? What was the latitude at that time? Calculate by Mercator sailing.

(Ans. at $14^{h}- 12^{m}$, lat. = $40^{\circ}- 37'.8$ N)

41. A ship has left the location $15^{\circ}- 42'.0$ N, $178^{\circ}- 56'.0$ E with the true course 145° and at the speed of 20 nautical miles per hour. How long did it take to get to the equator and what was the longitude at that time on the equator? Calculate by Mercator sailing.

(Ans. $57^{h}- 30^{m}$, long. = $\dfrac{170^{\circ}- 00'.4 \text{ W}}{\text{Attention}}$)

42. A ship left the position $27^{\circ}- 30'.0$ N, $174^{\circ}- 15'.0$ E. with the true course $115^{\circ}$and at the speed of 10 nautical miles per hour for the longitude 180° at 12:00 on 11 Oct.. When did this ship reach the longitude 180° and what was the latitude at that time? Calculate by Mercator sailing.

(Ans. at $22^{h}- 18^{m}- 36^{s}$ on 12 Oct. lat. = $25^{\circ}- 05'.0$ N.)

Example 15

If the length of interval of longitudes for each degree is one centimetre, how many centimetres should the length of interval of latitudes be for each degree from 40^o, 41^o, 42^o and 43^o? Use the table of meridional parts from NAVIGATION TABLES, TD/LN/52.

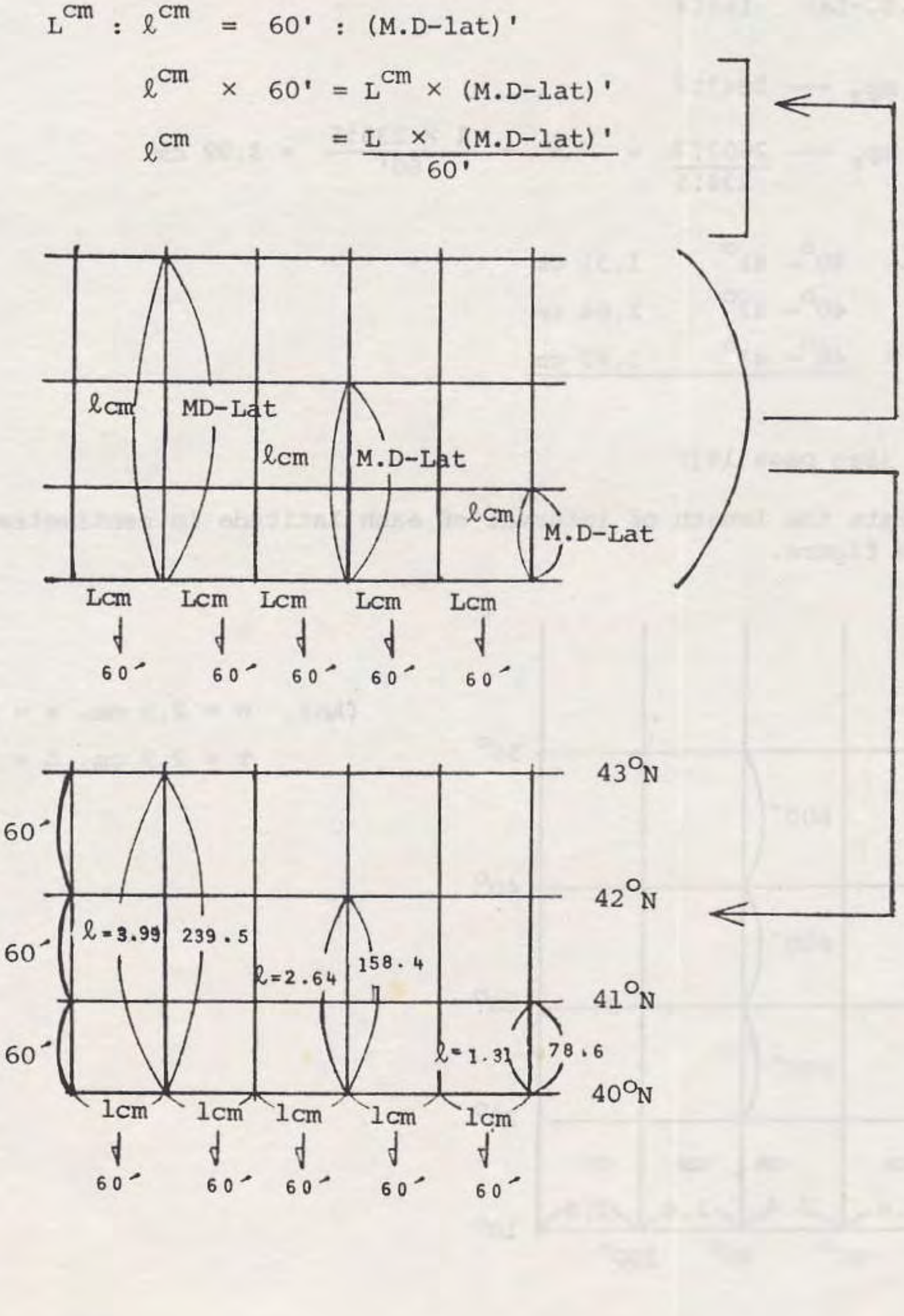
Solution

According to theory of the Mercator chart, we have the formula,

$$L^{cm} : \ell^{cm} = 60' : (M.D\text{-}lat)'$$

$$\ell^{cm} \times 60' = L^{cm} \times (M.D\text{-}lat)'$$

$$\ell^{cm} = \frac{L \times (M.D\text{-}lat)'}{60'}$$



41° mp_1 --- 2686'.3

40° mp_0 --- 2607'.7 - $\ell = \dfrac{1 \times 78'.6}{60'} = 1.31$ cm

M.D.-Lat 78'.6

42° mp_2 --- 2766'.1

40 mp_0 --- 2607'.7 - $\ell = \dfrac{1 \times 158'.4}{60'} = 2.64$ cm

M.D.-Lat 158'.4

43° mp_3 --- 2847'.2

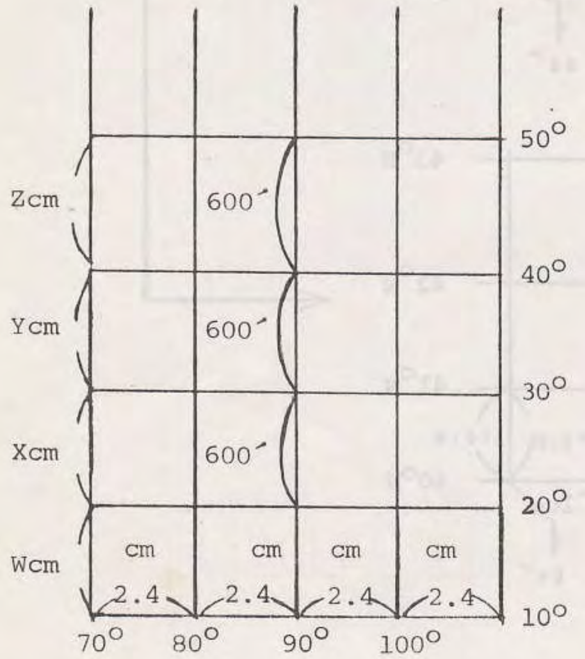
40° mp_0 --- 2607'.7 - $\ell = \dfrac{1 \times 239'.5}{60'} = 3.99$ cm

239'.5

Ans.	40°- 41°	1.31 cm
	40°- 42°	2.64 cm
	40°- 43°	3.99 cm

Exercise 7 (See page 101)

43. Calculate the length of interval of each latitude in centimetres on the figure.



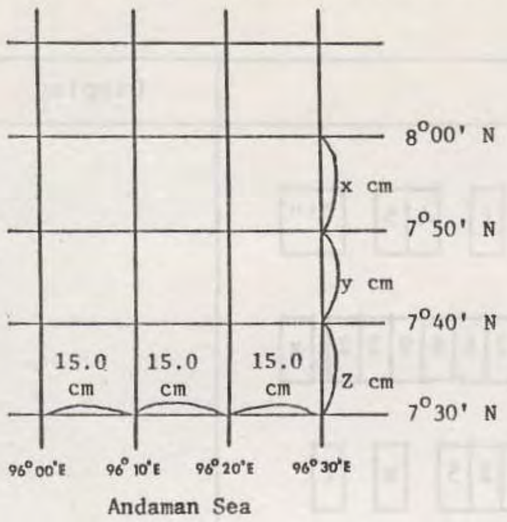
(Ans. W = 2.5 cm, x = 2.6 cm
Y = 2.9 cm, Z = 3.4 cm)

Example 16

If the length of interval of longitudes for each ten minutes is fifteen centimetres, how many centimetres should the length of interval of latitudes be for each from 8°00' N, 7°50' N, 7°40' N and 7°30' N.

Solution

By scientific calculator CASIO fx-4000 P.



Formula;

$$M = 7915.704468 \times \log.\tan\left(45^\circ + \frac{\ell^\circ}{2}\right) - 23.268932 \times \sin \ell^\circ - 0.0525 \times \sin^3 \ell^\circ - 0.000213 \times \sin^5 \ell.$$

1. 8°00' N

Operation Keystrokes	Display
7 9 1 5 . 7 0 4 4 6 8 [x] [log] [tan] [(] 4 5 [° ' "] [+]	
8 [° ' "] [÷] 2 [)] [-] 2 3 . 2 6 8 9 3 2 [x] [sin]	
8 [° ' "] [-] 0 . 0 5 2 5 [x] [(] [sin] 8 [° ' "] [)]	
[x^y] 3 [EXE]	478.328735
[-] 0 . 0 0 0 2 1 3 [x] [(] [Sin] 8 [° ' "] [)] [x^y] 5	
[EXE]	478.328735

2. 7°50' N

Operation Keystrokes	Display
7 9 1 5 . 7 0 4 4 6 8 x log tan (4 5 °'"	
+ 7 °'" 5 0 °'" ÷ 2) - 2 3 . 2 6 8 9 3 2 x	
Sin 7 °'" 5 0 °'" - 0 . 0 5 2 5 x (	
Sin 7 °'" 5 0 °'") x^y 3 EXE	468.2995588
- 0 . 0 0 0 2 1 3 x (Sin 7 °'" 5 0 °'"	
) x^y 5 EXE	468.2995588

3. 7°40' N

Operation Keystrokes	Display
7 9 1 5 . 7 0 4 4 6 8 x log tan (4 5 °'"	
+ 7 °'" 4 0 °'" ÷ 2) − 2 3 . 2 6 8 9	
3 2 x Sin 7 °'" 4 0 °'" − 0 . 0 5 2 5	
x (Sin 7 °'" 4 0 °'") x^y 3 EXE	458.2744486
− 0 . 0 0 0 2 1 3 x (Sin 7 °'" 4 0 °'"	
) x^y 5 EXE	458.2744486

4. 7°30' N

Operation Keystrokes	Display
7 9 1 5 . 7 0 4 4 6 8 x log tan (4 5 °'"	
+ 7 °'" 30 °'" ÷ 2) − 2 3 . 2 6 8 9	
3 2 x Sin 7 °'" 3 0 °'" − 0 . 0 5 2 5	
x (Sin 7 °'" 3 0 °'") x^y 3 EXE	448.2533154
− 0 . 0 0 0 2 1 3 x (Sin 7 °'" 3 0 °'"	
) x^y 5 EXE	448.2533154

M (8°00' N) = 478.3287350

M (7°50' N) = 468.2995588 ⊖

M.D. lat. = 10.0291762

$$X = \frac{15cm \times 10.0291762}{10'} = 15.0437643 \doteqdot 15.044 \text{ cm}$$

M (7°50' N) = 468.2995580

M (7°40' N) = 458.2744486 ⊖

M.D. lat. = 10.0251094

$$Y = \frac{15cm \times 10'.0251094}{10'} = 15.0376641 \doteqdot 15.038 \text{ cm}$$

M (7°40' N) = 458.2744486

M (7°30' N) = 448.2533154 ⊖

M.D. lat. = 10.0211332

$$Z = \frac{15cm \times 10'.0211332}{10'} = 15.0316998 \doteqdot 15.032 \text{ cm}$$

Ans. X = 15.044 cm

Y = 15.038 cm

Z = 15.032 cm

Solution by Programmable Computer

A computer is an electronic calculator designed for the solution of mathematical problems. A computer has a programming capability, therefore, it can be employed to do the navigational calculation work much faster than the old conventional method.

There are various types of computer. Some of them, portable computers, are introduced here:

1. Portable computer; Model PC-1350, SHARP CORPORATION Co., Ltd.
2. Scientific Calculator; Model fx-7000 G, (Graphic), CASIO Computer Co., Ltd.
3. Scientific Calculator; Model fx-4000 P, CASIO Computer Co., Ltd.

The programming of the formula to calculate Meridianal Difference of Latitude is as follows:

1. Model : PC-1350 (SHARP)

Programming List

Line No.	Statement
600 :	Meridianal Part
610 :	"M" INPUT "Lat 1="; L
620 :	L = DEG L
630 :	INPUT "Lat 2="; N
640 :	N = DEG N
650 :	INPUT "cm="; X
660 :	INPUT "Y="; Y
670 :	M = 7915.704468 * LOG TAN (45+L/2) - 23.268932 * SIN L - 0.0525 * SIN L ∧ 3 - 0.000213 * SIN L ∧ 5)

Ref. Y = 10,60 or 600

Line No.	Statement
680 :	P = (M - (7915.704468 ✱LOG TAN (45 + N/2) - 23.268932 ✱SIN N - 0.0525 ✱SIN N ∧ 3 - 0.000213 ✱SIN N ∧ 5)) ✱X/Y
690 :	PRINT "M 1="; M
700 :	PRINT "Meridianal Part="; P
710 :	Goto 610
720 :	END

Ref. Y = 10,60 or 600

or by Subroutine function:

Line No.	Statement
800 :	"N" INPUT "Lat 1="; M, "Lat 2="; N, "cm="; X, "Y="; Y
810 :	M = DEG M, N = DEG N
820 :	L = M : Gosub 870
830 :	B = D : L = N : Gosub 870
840 :	C = D
850 :	PRINT B-C
860 :	END
870 :	D = X/Y ✱(7915.704468 ✱LOG TAN (45 + L/2) - 23.268932 ✱SIN L - 0.0525 ✱SIN L∧3 - 0.000213 ✱SIN L ∧ 5)
880 :	RETURN

Ref. Y = 10,60 or 600

Calculation

1. Example 15 (page 79)

No.	Operation Keystrokes	Display
1.	[R][U][N] [6][1][0] [ENTER] or [DEF] [M]	RUN 610 Lat 1 = -
2.	[4][3] [ENTER]	Lat 1 = 43 Lat 2 = -
3.	[4][2] [ENTER]	Lat 2 = 42 cm = -
4.	[1] [ENTER]	Lat 1 = 43 Lat 2 = 42 cm = 1 Y = -
5.	[6][0] [ENTER]	Lat 2 = 42 cm = 1 Y = 60 M 1 = 2847.209241
6.	[ENTER]	Y = 60 M 1 = 2847.209241 Meridianal Part = 1.351382057

No.	Operation Keystrokes	Display
7.	ENTER	Lat 1 = -
8.	4 2 ENTER	Lat 1 = 42 Lat 2 = -
9.	4 1 ENTER	057 Lat 1 = 42 Lat 2 = 41 cm = -
10.	1 ENTER	Lat 1 = 42 Lat 2 = 41 cm = 1 Y = -
11.	6 0 ENTER	Lat 2 = 41 cm = 1 Y = 60 M 1 = 2766.126318
12.	ENTER	Y = 60 M 1 = 2766.126318 Meridianal Part = 1.330151434
13.	ENTER	M 1 = 2766.126318 Meridianal Part = 1.330151434 Lat 1 = -

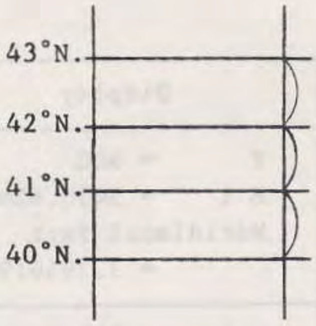
No.	Operation Keystrokes	Display
14.	4 1 ENTER	Lat 1 = 41 Lat 2 = -
15.	4 0 ENTER	434 Lat 1 = 41 Lat 2 = 40 cm = -
16.	1 ENTER	cm = 1 Y = -
17.	6 0 ENTER	Lat 2 = 40 cm = 1 Y = 60 M 1 = 2686.317232
18.	ENTER	Y = 60 M 1 = 2686.317232 Meridianal Part = 1.309966448

Ans. 43° - 42° ⟶ 1.351382057 ≑ 1.35 cm.
42° - 41° ⟶ 1.330151434 ≑ 1.33 cm.
41° - 40° ⟶ 1.309966448 ≑ 1.31 cm.

or

40° - 43° ⟶ 3.99 (= 1.31 + 1.33 + 1.35) cm.
40° - 42° ⟶ 2.64 (= 1.31 + 1.33) cm.
40° - 41° ⟶ 1.31 (= 1.31) cm.

Ref.



1.351382057 cm. ≈ 1.35 cm. (= 3.99-2.64)

1.330151434 cm. ≈ 1.33 cm. (= 2.64-1.31)

1.309966448 cm. ≈ 1.31 cm. (= 1.31-0)

Ans. 40°-41°, 1.31 cm. 41°-42°, 1.33 cm. 42°-43°, 1.35 cm.

or 40°-41°, 1.31 cm. 40°-42°, 2.64 cm. 40°-43°, 3.99 cm.

2. Exercise 7 (page 80)

No.	Operation Keystrokes	Display
1.	[RUN] [6][1][0] [ENTER]	RUN 610 Lat 1 = -
2.	[5][0] [ENTER]	RUN 610 Lat 1 = 50 Lat 2 = -
3.	[4][0] [ENTER]	RUN 610 Lat 1 = 50 Lat 2 = 40 cm = -
4.	[2][.][4] [ENTER]	Lat 1 = 50 Lat 2 = 40 cm = 2.4 Y = -
5.	[6][0][0] [ENTER]	Lat 2 = 40 cm = 2.4 Y = 600 M 1 = 3456.624176

No.	Operation Keystrokes	Display
6.	ENTER	Y = 600 M 1 = 3456.624176 Meridianal Part = 3.395619724
7.	ENTER	M 1 = 3456.624176 Meridianal Part = 3.395619724 Lat 1 = -
8.	4 0 ENTER	Meridianal Part = 3.395619724 Lat 1 = 40 Lat 2 = -
9.	3 0 ENTER	724 Lat 1 = 40 Lat 2 = 30 cm = -
10.	2 . 4 ENTER	Lat 1 = 40 Lat 2 = 30 cm = 2.4 Y = -
11.	6 0 0 ENTER	Lat 2 = 30 cm = 2.4 Y = 600 M 1 = 2607.719245
12.	ENTER	Y = 600 M 1 = 2607.719245 Meridianal Part = 2.923939425

No.	Operation Keystrokes	Display
13.	ENTER	M 1 = 2607.719245 Meridianal Part = 2.923939425 Lat 1 = -
14.	4 0 ENTER	Meridianal Part = 2.923939425 Lat 1 = 40 Lat 2 = -
15.	3 0 ENTER	425 Lat 1 = 40 Lat 2 = 30 cm =
16.	2 . 4 ENTER	Lat 1 = 40 Lat 2 = 30 cm = 2.4 Y = -
17.	6 0 0 ENTER	Lat 2 = 30 cm = 2.4 Y = 600 M 1 = 2607.719245
18.	ENTER	Y = 600 M 1 = 2607.719245 Meridianal Part = 2.923939425
19.	ENTER	M 1 = 2607.719245 Meridianal Part = 2.923939425 Lat 1 = -

No.	Operation Keystrokes	Display
20.	3 0 ENTER	Meridianal Part = 2.923939425 Lat 1 = 30 Lat 2 = -
21.	2 0 ENTER	425 Lat 1 = 20 Lat 2 = 30 cm = -
22.	2 . 4 ENTER	Lat 1 = 30 Lat 2 = 20 cm = 2.4 Y = -
23.	6 0 0 ENTER	Lat 2 = 20 cm = 2.4 Y = 600 M 1 = 1876.734389
24.	ENTER	Y = 600 M 1 = 1876.734389 Meridianal Part = 2.63822352
25.	ENTER	M 1 = 1876.734389 Meridianal Part = 2.63822352 Lat 1 = -
26.	2 0 ENTER	Meridianal Part = 2.63822352 Lat 1 = 20 Lat 2 = -

No.	Operation Keystrokes	Display
27.	[1] [0] [ENTER]	52 Lat 1 = 20 Lat 2 = 10 cm = -
28.	[2] [.] [4] [ENTER]	Lat 1 = 20 Lat 2 = 10 cm = 2.4 Y = -
29.	[6] [0] [0] [ENTER]	Lat 2 = 10 cm = 2.4 Y = 600 M 1 = 1217.178509
30.	[ENTER]	Y = 600 M 1 = 1217.178509 Meridianal Part = 2.472599249

Ans. Z (50°-40°) = 3.395619724 ≑ 3.4 cm.
Y (40°-30°) = 2.923939425 ≑ 2.9 cm.
X (30°-20°) = 2.63822352 ≑ 2.6 cm.
W (20°-10°) = 2.472599249 ≑ 2.5 cm.

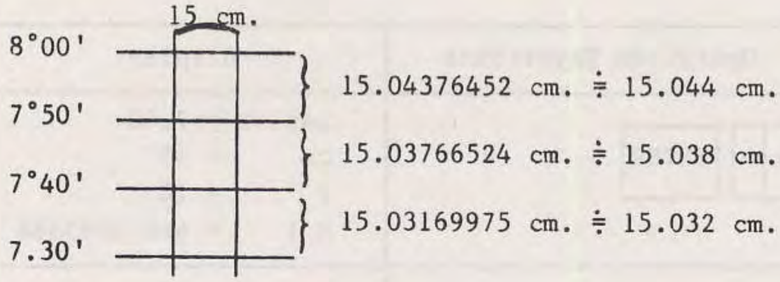
3. Example 16 (Page 81)

No.	Operation Keystrokes	Display
1.	[RUN] [6] [1] [0] [ENTER]	RUN 610 Lat 1 = -
2.	[8] [ENTER]	RUN 610 Lat 1 = 8 Lat 2 = -

No.	Operation Keystrokes	Display
3.	7 . 5 0 ENTER	RUN 610 Lat 1 = 8 Lat 2 = 7.50 cm =
4.	1 5 ENTER	Lat 1 = 8 Lat 2 = 7.50 cm = 15 Y = -
5.	1 0 ENTER	Lat 2 = 7.50 cm = 15 Y = 10 M 1 = 478.3287351
6.	ENTER	Y = 10 M 1 = 478.3287351 Meridianal Part = 15.04376452
7.	ENTER	M 1 = 478.3287351 Meridianal Part = 15.04376452 Lat 1 = -
8.	7 . 5 0 ENTER	Meridianal Part = 15.04376452 Lat 1 = 7.50 Lat 2 = -
9.	7 . 4 0	452 Lat 1 = 7.50 Lat 2 = 7.40 cm = -
10.	1 5 ENTER	Lat 1 = 7.50 Lat 2 = 7.40 cm = 15 Y = -

No.	Operation Keystrokes	Display
11.	1 0 ENTER	Lat 2 = 7.40 cm = 15 Y = 10 M 1 = 468.2995588
12.	ENTER	Y = 10 M 1 = 468.2995588 Meridianal Part = 15.03766524
13.	ENTER	M 1 = 468.2995588 Meridianal Part = 15.03766524 Lat 1 = -
14.	7 . 4 0 ENTER	Meridianal Part = 15.03766524 Lat 1 = 7.40 Lat 2 = -
15.	7 . 3 0 ENTER	524 Lat 1 = 7.40 Lat 2 = 7.30 cm = -
16.	1 5 ENTER	Lat 1 = 7.40 Lat 2 = 7.30 cm = 15 Y = -
17.	1 0 ENTER	Lat 2 = 7.30 cm = 15 Y = 10 M 1 = 458.2744486
18.	ENTER	Y = 10 M 1 = 458.2744486 Meridianal Part = 15.03169975

Ref.



Ans. 7°30' - 7°40', 15.032 cm.
7°40' - 7°50', 15.038 cm.
7°50' - 8°00', 15.044 cm.

2. Model : fx-7000 G (CASIO)

Programming List

Statement
Lbℓ 1 : "LAT = K°":?⟶ K : "LAT = L°":?⟶ L : "X = CM":?⟶ X : " Y = NM":?⟶ Y : (7915.7 x (log tan (45°+ K°÷2) - log tan (45°+L°÷2)) - 23.3 x (sin K°-sin L°))x X ÷ Y ◢ Goto 1

<u>Note</u> :

$$M = 7915.704468 \times \log_{10}\tan(45° + K°/2) - 23.368932 \times \sin K° - 0.0525 \times \sin^3 K° - 0.000213 \times \sin^5 K°$$
$$\doteqdot 7915.7 \times \log_{10}\tan(45° + K°/2) - 23.3 \times \sin K°$$

$$M(K° \rightarrow L°) = \{7915.7 \times \log_{10}\tan(45° + K°/2) - 23.3 \times \sin K°\}$$
$$-\{7915.7 \times \log_{10}\tan(45° + L°/2) - 23.3 \times \sin L°\}$$
$$= \{(7915.7 \times (\log_{10}\tan(45° + L° \div 2) - \log_{10}\tan(45° + L° \div 2)) - 23.3 \times (\sin K° - \sin L°)\}$$

$$P = (K° \rightarrow L°) \times X \div Y$$
$$= [7915.7 \times (\log_{10}\tan(45° + K° \div 2) - \log\tan(45° + L° \div 2)) - 23.3 \times (\sin K° - \sin L°)] \times X \div Y$$

Calculation

1. Example 15 (page 79)

No.	Operation Keystrokes	Display
	Lat. 43° - Lat. 40°	
1.	MODE 1 PROG 1 EXE	LAT = K°?
2.	4 3 °'" EXE	43° LAT = L°?
3.	4 0 °'" EXE	40° X = CM?
4.	1 EXE	1 Y = NM?
5.	6 0 EXE	60 3.991522673 - Disp -
	Lat. 42° - Lat. 40°	
6.	EXE	LAT = K°?
7.	4 2 °'" EXE	42° LAT = L°?
8.	4 0 °'" EXE	40° X = CM?

No.	Operation Keystrokes	Display
9.	[1] [EXE]	1 Y = NM?
10.	[6] [0] [EXE]	60 2.640132583 - Disp -
	Lat. 41° - Lat. 40°	
11.	[EXE]	LAT = K°?
12.	[4] [1] [° ' "] [EXE]	41° LAT = L°?
13.	[4] [0] [° ' "] [EXE]	40° X = CM?
14.	[1] [EXE]	1 Y = NM?
15.	[6] [0] [EXE]	60° 1.309973569 - Disp -

Ans. 43° - 40° ⟶ 3.991522673 ≑ 3.99 cm.
42° - 40° ⟶ 2.640132583 ≑ 2.64 cm.
41° - 40° ⟶ 1.309973569 ≑ 1.31 cm.

Calculation

1. Example 15 (page 79)

No.	Operation Keystrokes	Display
	Lat. 43° - Lat. 40°	
1.	MODE 1 PROG 1 EXE	LAT = K°?
2.	4 3 °'" EXE	43° LAT = L°?
3.	4 0 °'" EXE	40° X = CM?
4.	1 EXE	1 Y = NM?
5.	6 0 EXE	60 3.991522673 - Disp -
	Lat. 42° - Lat. 40°	
6.	EXE	LAT = K°?
7.	4 2 °'" EXE	42° LAT = L°?
8.	4 0 °'" EXE	40° X = CM?

No.	Operation Keystrokes	Display
9.	[1] [EXE]	1 Y = NM?
10.	[6][0] [EXE]	60 2.640132583 - Disp -
	Lat. 41° - Lat. 40°	
11.	[EXE]	LAT = K°?
12.	[4][1] [°'"] [EXE]	41° LAT = L°?
13.	[4][0] [°'"] [EXE]	40° X = CM?
14.	[1] [EXE]	1 Y = NM?
15.	[6][0] [EXE]	60° 1.309973569 - Disp -

Ans. 43° - 40° ⟶ 3.991522673 ≑ 3.99 cm.
42° - 40° ⟶ 2.640132583 ≑ 2.64 cm.
41° - 40° ⟶ 1.309973569 ≑ 1.31 cm.

2. Exercise 7 (page 80)

No.	Operation Keystrokes	Display
1.	MODE 1 Prog 1 EXE	LAT = K°?
2.	5 0 °' " EXE	50° LAT = L°?
3.	4 0 °' " EXE	40° X = CM?
4.	2 . 4 EXE	2.4 Y = NM?
5.	6 0 0 EXE	600 3.395641242 - Disp -
7.	EXE	LAT = K°?
8.	4 0 °' " EXE	40° LAT = L°?
9.	3 0 °' " EXE	30° X = CM?
10.	2 . 4 EXE	2.4 Y = NM?

No.	Operation Keystrokes	Display
11.	[6][0][0] [EXE]	2.923949613 - Disp -
12.	[EXE]	LAT = K°?
13.	[3][0] [° ' "] [EXE]	30° LAT = L°?
14.	[2][0] [° ' "] [EXE]	20° X = CM?
15.	[2][.][4] [EXE]	2.4 Y = NM?
16.	[6][0][0] [EXE]	600 2.638220260 - Disp -

3. Example 16 (page 81)

No.	Operation Keystrokes	Display
1.	[MODE] [1] [Prog] [1] [EXE]	LAT = K°?
2.	[8] [° ' "] [EXE]	8° LAT = L°?

No.	Operation Keystrokes	Display
3.	7 °'" 5 0 °'" EXE	7° 50° Y = CM?
4.	1 5 EXE	15 Y = NM?
5.	1 0 EXE	10 15.04363451 - Disp -
6.	EXE	LAT = K°?
7.	7 °'" 5 0 °'" EXE	7° 50° LAT = L°?
8.	7 °'" 4 0 °'" EXE	7° 40° X = CM?
9.	1 5 EXE	15 Y = NM?
10.	1 0 EXE	10 15.03753477 - Disp -
11.	EXE	LAT = K°?

No.	Operation Keystrokes	Display
12.	[7] [° ' "] [4][0] [° ' "] [EXE]	7° 40° LAT = L°?
13.	[7] [° ' "] [3][0] [° ' "] [EXE]	7° 30° X = CM?
14.	[1][5] [EXE]	15 Y = NM?
15.	[1][0] [EXE]	10 15.03156872 - Disp -

Ans. 8° - 7°50' ⟶ 15.04363450 ≑ 15.04
7°50' - 7°40' ⟶ 15.03753477 ≑ 15.04
7°40' - 7°30' ⟶ 15.03156872 ≑ 15.03

3. Model : fx-4000 p (CASIO)

Programming List

Statement
Lbℓ 1 : "LAT = K°":? ⟶ K : "LAT = L°":? ⟶ L : "X = CM":? ⟶ X : "Y = NM":? ⟶ Y : (7915.7 x (log tan (45°+ K°÷ 2)-log tan (45°+ L°÷ 2))- 23.3 x (sin K°- sin L°)) x X ÷ Y ◢ Goto 1

Calculation

1. Example 15 (page 79)

No.	Operation Keystrokes	Display
1.	MODE 1 Prog 1 EXE	LAT = K°?
2.	4 3 °'" EXE	LAT = L°?
3.	4 0 °'" EXE	X = CM?
4.	1 EXE	Y = NM?
5.	6 0 EXE	3.991522673
6.	EXE	LAT = K°?
7.	4 2 °'" EXE	LAT = L°?
8.	4 0 °'" EXE	X = CM?
9.	1 EXE	Y = NM?

No.	Operation Keystrokes	Display
10.	[6] [0] [EXE]	2.640132583
11.	[EXE]	LAT = K°?
12.	[4] [1] [° ' "] [EXE]	LAT = L°?
13.	[4] [0] [° ' "] [EXE]	X = CM?
14.	[1] [EXE]	Y = NM?
15.	[6] [0] [EXE]	1.309973568

Ans. 43° - 40° ⟶ 3.991522673 ≑ 3.99 cm.
42° - 40° ⟶ 2.640132583 ≑ 2.64 cm.
41° - 40° ⟶ 1.309973568 ≑ 1.31 cm.

2. Exercise 7 (page 80)

No.	Operation Keystrokes	Display
1.	[MODE] [1] [Prog] [1] [EXE]	LAT = K°?
2.	[5] [0] [° ' "] [EXE]	LAT = L°?

No.	Operation Keystrokes	Display
3.	4 0 °'" EXE	X = CM?
4.	2 . 4 EXE	Y = NM?
5.	6 0 0 EXE	3.395641242
6.	EXE	LAT = K°?
7.	4 0 °'" EXE	LAT = L°?
8.	3 0 °'" EXE	X = CM?
9.	2 . 4 EXE	Y = NM?
10.	6 0 0 EXE	2.923949613
11.	EXE	LAT = K°?
12.	3 0 °'" EXE	LAT = L°?

No.	Operation Keystrokes	Display
13.	[2] [0] [° ' "] [EXE]	X = CM?
14.	[2] [.] [4] [EXE]	Y = NM?
15.	[6] [0] [0] [EXE]	2.63822026
16.	[EXE]	LAT = K°?
17.	[2] [0] [° ' "] [EXE]	LAT = L°?
18.	[1] [0] [° ' "] [EXE]	X = CM?
19.	[2] [.] [4] [EXE]	Y = NM?
20.	[6] [0] [0] [EXE]	2.472584226

Ans. Z (50° ⟶ 40°) = 3.395641242 ≑ 3.4 cm.
Y (40° ⟶ 30°) = 2.923949613 ≑ 2.9 cm.
X (30° ⟶ 20°) = 2.638220260 ≑ 2.6 cm.
W (20° ⟶ 10°) = 2.472584226 ≑ 2.5 cm.

3. Example 16 (page 81)

No.	Operation Keystrokes	Display
1.	MODE 1 Prog 1 EXE	LAT = K°?
2.	8 °'" EXE	LAT = L°?
3.	7 °'" 5 0 °'" EXE	X = CM?
4.	1 5 EXE	Y = NM?
5.	1 0 EXE	15.04363454
6.	EXE	LAT = K°?
7.	7 °'" 5 0 °'" EXE	LAT = L°?
8.	7 °'" 4 0 °'" EXE	X = CM?
9.	1 5 EXE	Y = NM?

No.	Operation Keystrokes	Display
10.	[1] [0] [EXE]	15.037534690
11.	[EXE]	LAT = K°?
12.	[7] [° ' "] [4] [0] [° ' "] [EXE]	LAT = L°?
13.	[7] [° ' "] [3] [0] [° ' "] [EXE]	X = CM?
14.	[1] [5] [EXE]	Y = NM?
15.	[1] [0] [EXE]	15.03156876

Ans. X = 15.043634540 ≑ 15.044 cm.
Y = 15.037534690 ≑ 15.038 cm.
Z = 15.031568760 ≑ 15.032 cm.

When a trainee intends to produce a nautical chart of a fishing-ground by making use of Mercator Projection, first he should fix the appropriate value of the longitudinal interval length in centimetres.

Then he must calculate the value of latitudinal interval length of the fishing ground in centimetres according to the value of longitudinal interval length in centimetres using the formula of Meridianal Difference of Latitude (M.D.L.);

$$\text{M.D.L. (at latitude } A^\circ) = 7915.704468 \times \log_{10} \tan (45^\circ + A^\circ/2) - 23.2689 \times \sin A^\circ - 0.0525 \times \sin^3 A^\circ - 0.000213 \times \sin^5 A$$

$$\doteqdot \boxed{7915.7 \times \log_{10} \tan (45^\circ + A^\circ/2) - 23.3 \times \sin A^\circ}$$

To draw latitudinal line K° and L° on paper, he calculates M.D.L. between latitudes K° and L° as follows;

$$\begin{aligned}\text{M.D.L.} &= (7915.7 \times \log_{10}\tan(45° + K°/2) - 23.3 \times \sin K°) \\ &\quad - (7915.7 \times \log_{10}\tan(45° + L°/2) - 23.3 \times \sin L°) \\ &= 7915.7 \times (\log_{10}\tan(45° + K°/2) - \log_{10}\tan \\ &\quad (45° + L°/2)) - 23.3 \times (\sin K° - \sin L°)\end{aligned}$$

If he fixed longitudinal interval length as X centimetres and latitudinal interval as Y nautical miles, the intervals of latitude in centimetres are given by the following formula:

$$\text{M.D.L. in centimetres} = \boxed{\left[7915.7 \times (\log_{10}\tan(45° + K°/2) - \log_{10}\tan(45° + L°/2) - 23.3 \times (\sin K° - \sin L°)\right] \times X \div Y}$$

Here is a case study of a fishing-ground nautical chart:

In the South China Sea at latitude 07°45' N., longitude 111°40' E., lies the Rifleman Bank found by a British gunboat. "Bombay Castle", the shallowest part of the bank, has a depth of 3 metres and breaks in all but the finest weather.

The SEAFDEC Training Department sent a training vessel there to conduct shipboard training in bottom longline fishing on 2 Oct, 1984 and 27 Nov. 1987.

Before carrying out the training, the features of the bank and an area in the vicinity of the bank were examined on a nautical chart and the "Sailing Directions for the South China Sea and Gulf of Thailand" were read to ensure the safety of the boat.

A fishing-ground nautical chart was then produced and all the dangers such as "Bombay Castle", "Johnson Patch", "Kingston Shoal", etc. were marked on it.

The Rifleman Bank lies between latitudes 7°20' N. and 8°00' N. and longitudes 111°20' E. and 112°00' E. (see Fig. 87-7)

To produce the fishing ground nautical chart of the bank, 5 longitudinal and latitudinal lines were drawn on paper as shown in Fig. 87-1.

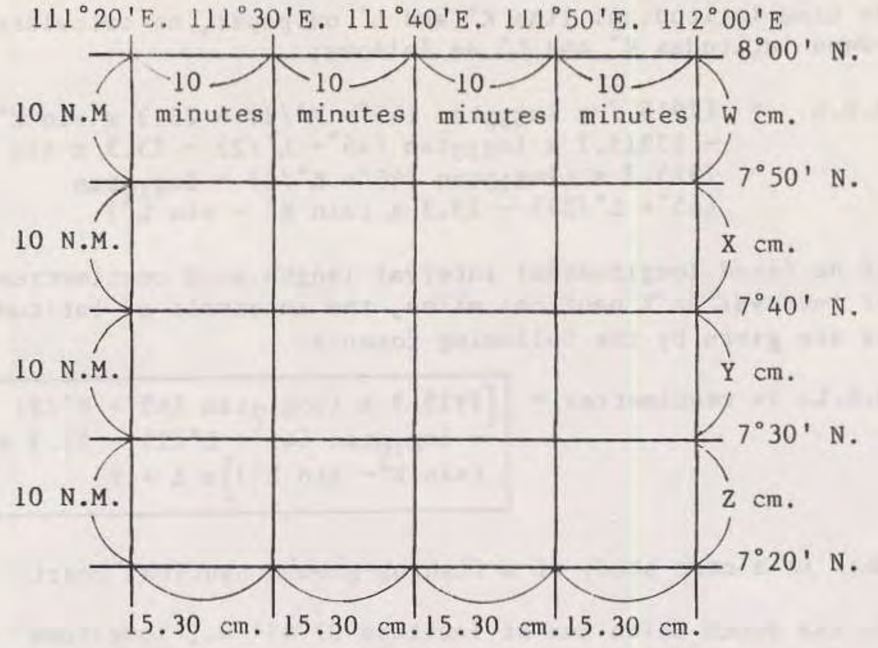


Fig. 87-1. Latitudinal and longitudinal lines and their interval length.

If the value of longitudinal interval length was fixed as 15.30 cm. by the trainee, the value of latitudinal interval length W, X, Y and Z (see Fig. 87-1) should be calculated as shown below:

Calculation by personal computer : Model fx-4000 (CASIO)

No.	Operation Keystrokes	Display
1.	MODE 1 Prog 1 EXE	LAT = K°
2.	8 ° ' " EXE	LAT = L°?

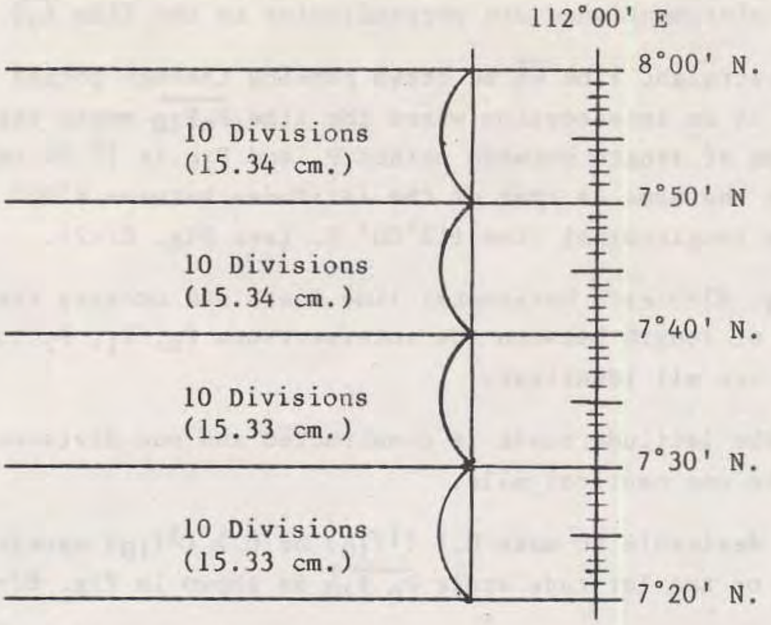
No.	Operation Keystrokes	Display
3.	[7] [° ' "] [5][0] [° ' "] [EXE]	X = CM?
4.	[1][5][.][3] [EXE]	Y = NM?
5.	[1][0] [EXE]	15.34450724
6.	[EXE]	LAT = K°?
7.	[7] [° ' "] [5][0] [° ' "] [EXE]	LAT = L°?
8.	[7] [° ' "] [4][0] [° ' "] [EXE]	X = CM?
9.	[1][5][.][3] [EXE]	Y = NM?
10.	[1][0] [EXE]	15.33828538
11.	[EXE]	LAT = K°?

No.	Operation Keystrokes	Display
12.	7 °'" 4 0 °'"	LAT = L°?
13.	7 °'" 3 0 °'"	X = CM?
14.	1 5 . 3 EXE	Y = NM?
15.	1 0 EXE	15.33220014
16.	EXE	LAT = K°?
17.	7 °'" 3 0 °'" EXE	LAT = L°?
18.	7 °'" 2 0 °'" EXE	X = CM?
19.	1 5 . 3 EXE	Y = NM?
20.	1 0 EXE	15.32625089

Latitudinal interval length

W = 15.344507240 ≈ 15.34 cm. (Length between 8° and 7°50')
X = 15.338285380 ≈ 15.34 cm. (Length between 7°50' and 7°40')
Y = 15.332200140 ≈ 15.33 cm. (Length between 7°40' and 7°30')
Z = 15.326250890 ≈ 15.33 cm. (Length between 7°30' and 7°20')

Next, a latitude scale should be drawn on the longitudinal line 112°00' E (see Fig. 87-1) to measure the value of latitudes and distance on the chart (see Fig. 87-2)



One division is equivalent to the one nautical mile Latitude Scale.

Fig. 87-2. Latitude scale on 112°00' E., longitudinal line.

Here is an explanation of how to construct a latitude scale with ten divisions-one division shows 0.1 nautical mile (n.m.) using a piece of section paper (see Figs. 87-3 and 87-4).

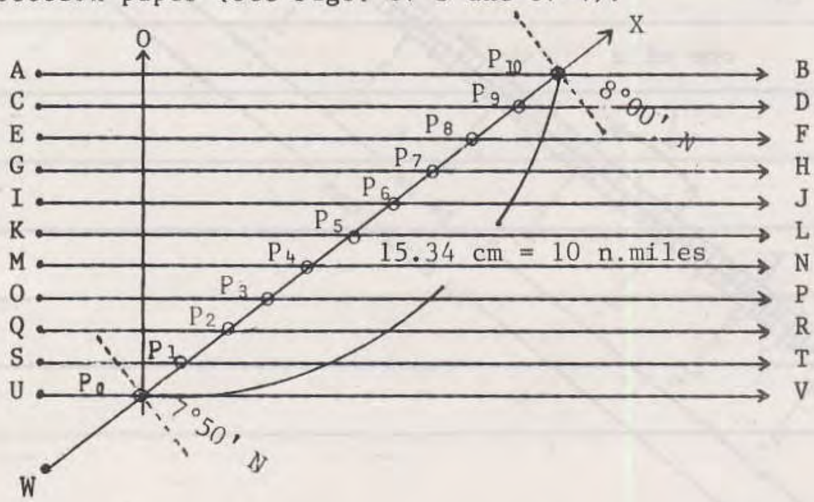


Fig. 87-3. To construct a latitude scale with 10 divisions on a piece of section paper.

In Fig. 87-3 there are 10 divisions between the line $\overrightarrow{AB}$ and $\overrightarrow{UV}$ on the line $\overrightarrow{P_oP_{10}}$ and the lines $\overrightarrow{AB}$, $\overrightarrow{CD}$, $\overrightarrow{EF}$--- and $\overrightarrow{UV}$ are parallel and, the lines aforementioned are perpendicular to the line $\overrightarrow{P_oQ}$.

Let a straight line $\overrightarrow{WX}$ be drawn passing through points P_o and P_{10}. Point P_o is an intersection where the line $\overrightarrow{P_oP_{10}}$ meets the line $\overrightarrow{UV}$ and the value of length between points P_o and P_{10} is 15.34 centimetres which is the same as that of the latitudes between 8°00' N. and 7°50' N. on the longitudinal line 112°00' E. (see Fig. 87-2).

In Fig. 87-3 each horizontal line meets and crosses the line $\overrightarrow{WX}$. The value of length between the intersections P_o, P_1, P_2,...P_{10} on the line $\overrightarrow{WX}$ are all identical.

Thus the latitude scale is constructed and one division of it is equivalent to one nautical mile.

It is desirable to make 0.1 ($^1/_{10}$) or 0.2 ($^2/_{10}$) nautical mile divisions on the latitude scale $\overrightarrow{P_o\ P_{10}}$ as shown in Fig. 87-4.

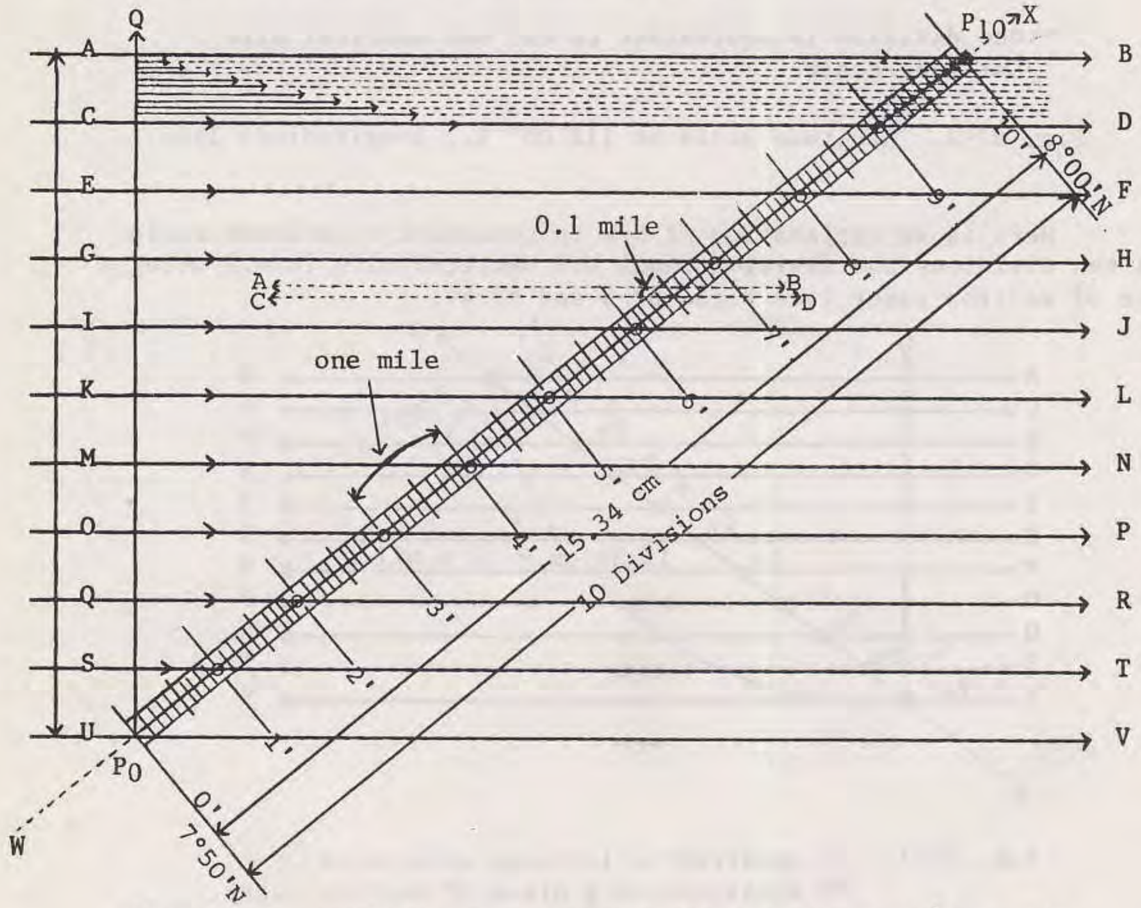


Fig. 87-4. Latitude scale shows both one mile and $^1/_{10}$ mile.

In Fig. 87-4, nine horizontal dotted lines are shown in the upper part of the figure, running parallel between the line $\overrightarrow{AB}$ and $\overrightarrow{CD}$.

Each horizontally dotted line meets and crosses the line $\overrightarrow{P_0P_{10}}$ (the latitude scale). On the line $\overrightarrow{P_0\,P_{10}}$ there are 9 intersections and, an interval length between 2 intersections shows 0.1 ($^1/_{10}$) nautical mile.

The same method can be applied to construct latitude scales between the latitudes 7°50' N. and 7°40' N., 7°40' N. and 7°30' N., and 7°30' N. and 7°20' N., also longitudinal scales between the longitudes 111°20' E. and 111°30' E., and 111°30' E. and 111°40' E., 111°40' E. and 111°50' E. and, 111°50' E. and 112°00' E. (see Fig. 87-5).

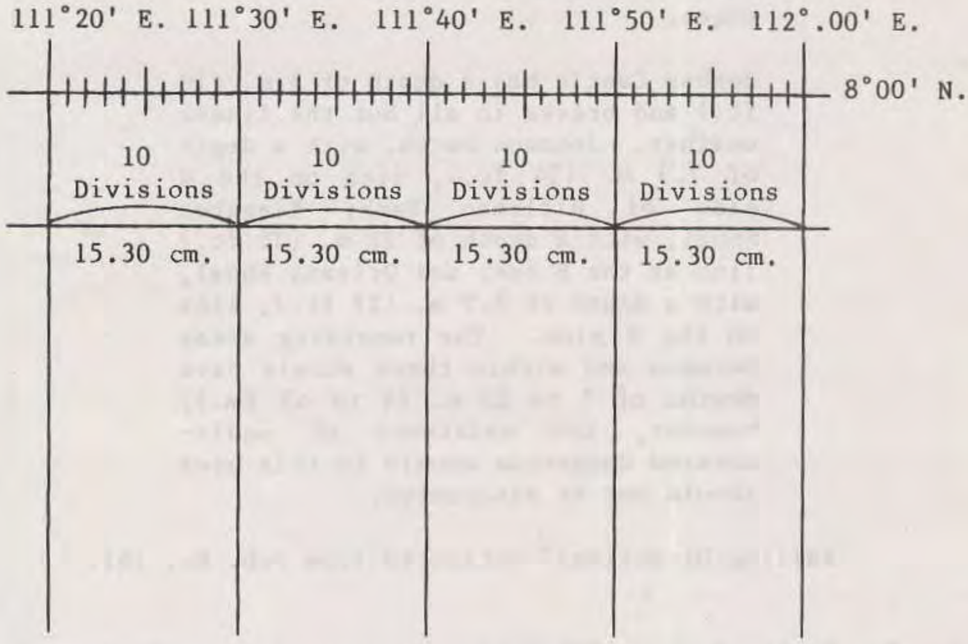


Fig. 87-5. Longitude Scale.

After drawing latitudinal and longitudinal lines with latitude scale and longitude scale of 10-divisions on a piece of paper, the positions of dangers such as castles, patches, shoals, etc., and an outline of the Rifleman Bank and, any other relevant chart data should be transferred from published nautical charts, Sailing Directions and data obtained by the vessel's surveys onto a piece of paper to make a fishing-ground nautical chart.

Ref. Data and information on the Rifleman Bank are given in;

1. "Sailing Directions (Enroute) for the South China Sea and Gulf of Thailand", Pub. No. 161 — The Defense Mapping Agency, Hydrographic/Topographic Center, U.S.A.

> Rifleman Bank lies about 70 miles W of Amboyna Cay with its N end, BOMBAY CASTLE, in 7°56' N., 111°42' E. The bank extends about 28 miles S from Bombay Castle and has a maximum breadth of 15 miles with many shallow patches of sand and coral around its edges.
>
> Bombay Castle has a depth of 3 m. (10 ft.) and breaks in all but the finest weather. Johnson Patch. with a depth of 7.3 m. (24 ft.), lies on the W side of Rifleman Bank; Kingston Shoal, with a depth of 11 m. (36 ft.) lies at the S end; and Orleana Shoal, with a depth of 8.2 m. (27 ft.), lies on the E side. The remaining areas between and within these shoals have depths of 7 to 82 m. (4 to 45 fm.); however, the existence of undiscovered dangerous shoals in this area should not be discounted.

"Sailing Directions" extracted from Pub. No. 161.

2. Nautical chart "China Sea Southern Portion-Western Sheet:, published at the Admiralty under the Superintendence of Captain Sir Frederick J. Evance. R.N., London. Pub. No. 2660 A.

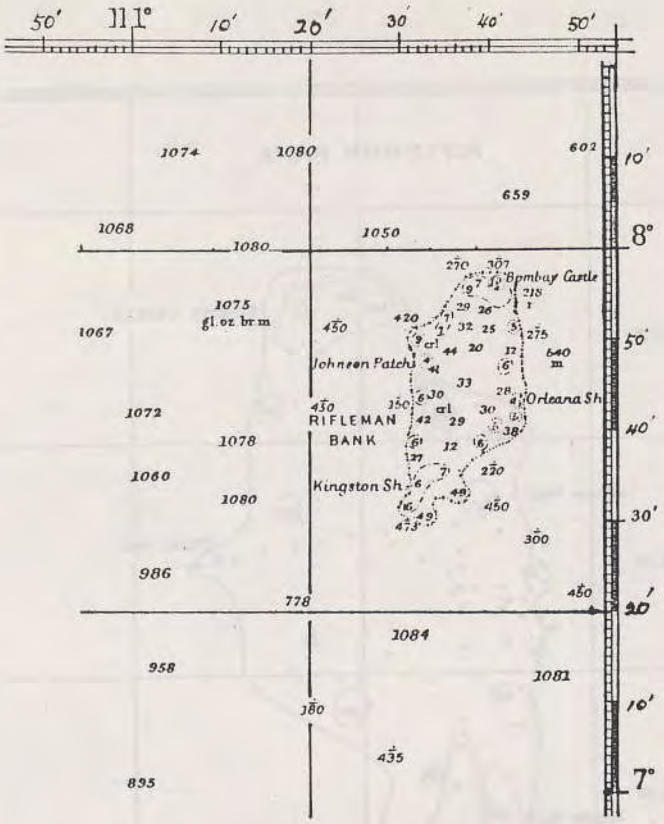


Fig. 87-6. The Rifleman Bank

Extract from the nautical chart "China Sea-South Portion-Western Sheet" Pub. No. 2660 A, London.

After transferring the data onto the fishing-ground nautical chart, the training boat's fishing activities might be carried out safely by checking the vessel's location on it. Fig. 87-7 shows an example of a fishing-ground nautical chart.

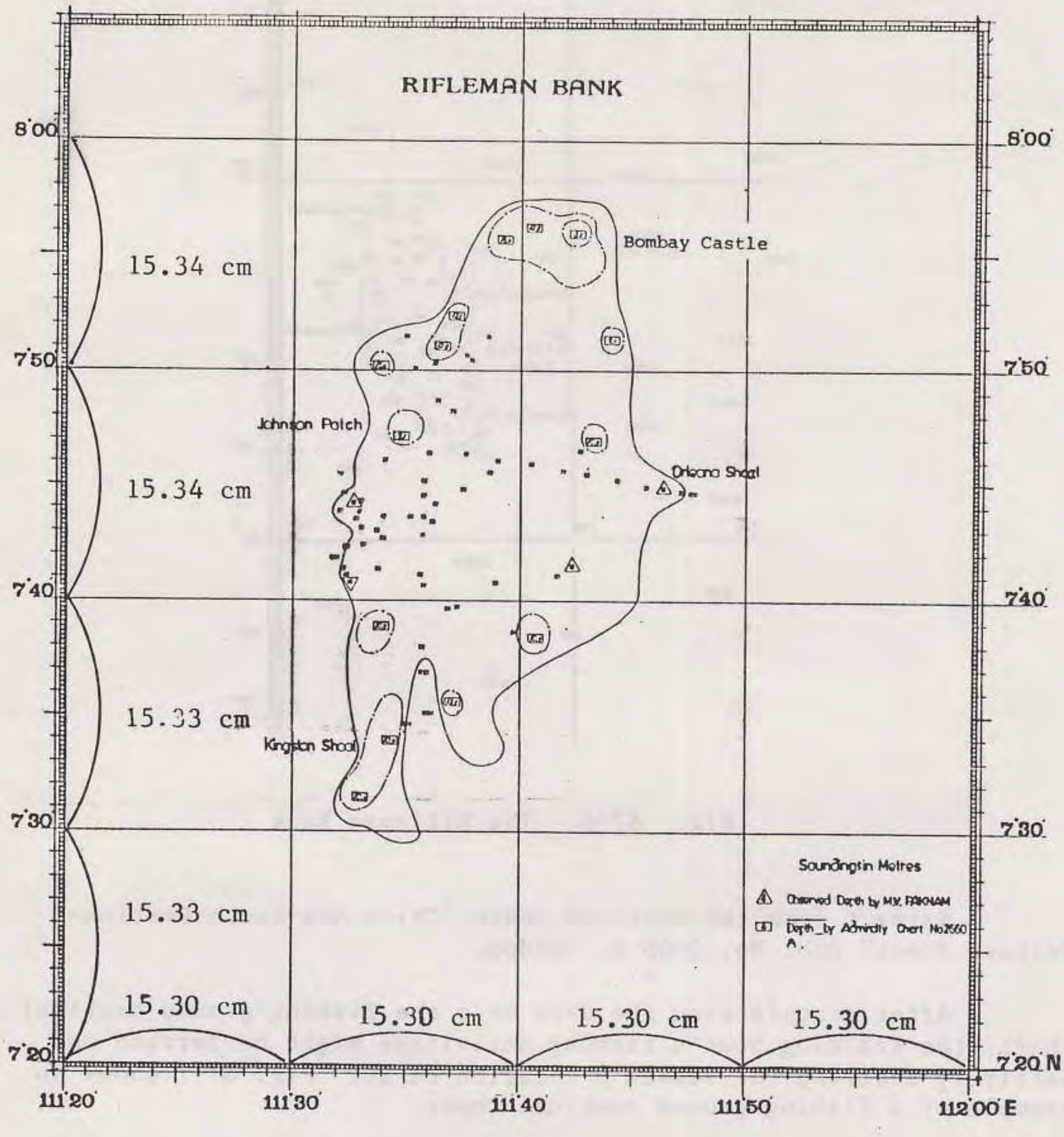


Fig. 87-7. Fishing-ground chart of the Rifleman Bank in the South China Sea.

Prepared by Mr. Somboon Siriraksophon, graduate of the 1986-1987 Regular Courses of the SEAFDEC Training Department.

Position

1. Lines of position

Navigators make use of lines of position to fix or estimate a ship's location at sea. This line is one on a point of which the vessel can be presumed to be located. Lines of position are mostly valuable and accurate, but they can be in error, because of some imperfection in navigational instruments and devices used for obtaining them, such as index error, or compass error.

A line of position is a straight line, an arc of a circle and a part of some other curve such as a hyperbola, and an isobath.

When a navigator draws a position line on the nautical chart, an appropriate lable should be placed on the plotted line of position at the time it is drawn (Fig. 91, 94, 95) to avoid misunderstanding or confusion.

2. Bearings

A bearing is the horizontal direction of one terrestrial point from another. It can be expressed as the angular difference between a reference direction and the given direction.

In navigation, north is used as the reference direction, and angles are measured clockwise through 360°.

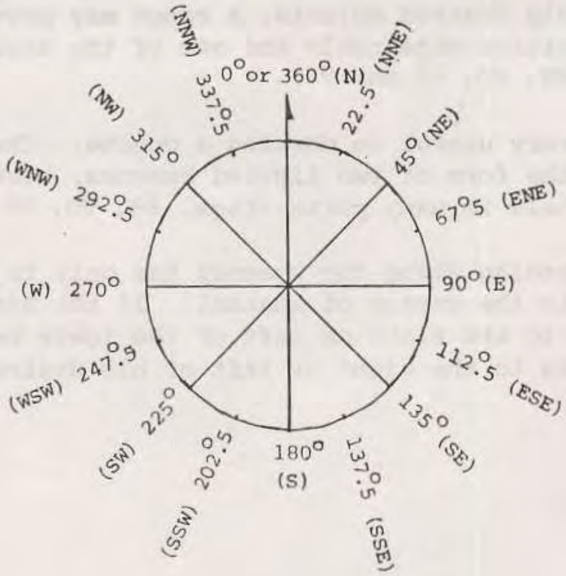


Fig. 88 · Bearings or Directions

It is customary to express all bearings in three digits. Thus north is 000^{o} or 360^{o}, then a direction 7^{o} to the right side of north is 007^{o}, and east is 090^{o}, northeast is 045^{o}, northwest is 315^{o}, southeast is 135^{o}, southwest is 225^{o}, south is 180^{o}, and west is 270^{o} (Fig. 88).

When plotting, true north is used as the reference direction. A bearing measured from this reference is called a true bearing, a magnetic bearing, or a compass bearing, which results from using magnetic north or compass north, respectively, can also be used as a reference direction.

This is similar to the designation of courses. A bearing line extending in the direction of an observed bearing of a charted object is one of the most widely used lines of position.

If one knows that an identified landmark has a bearing from his ship, the ship can only be on the line at which such a bearing might be observed, for at any other point the bearing would be different.

This line extends outward from the landmark, along the reciprocal of the observed bearing. Thus, if a lighthouse is east (090^{o}) of a ship, that ship is west (270^{o}) of the lighthouse. If a radio beacon bears 110^{o}, the observer (ship) must be on a line extending $110^{o} + 180^{o} = 290^{o}$ from the radio beacon. Bearings are generally obtained by magnetic compass, gyrocompass etc.

Another type of bearing is also obtainable by eye without using a compass. When two objects appear directly in a line, for example one beacon behind another beacon, they are said to be "in range" and together they constitute a range (Figs. 89, 90, 95 and 96).

For accurately charted objects, a range may provide the most accurate line of position obtainable and one of the easiest methods of observation (Figs. 89, 90, 95 and 96).

A range is very useful in marking a course. Therefore, artificial ranges, usually in the form of two lighted beacons, have been installed in a line with channels in many ports (Figs. 89, 90, 95 and 96).

A ship proceeding along the channel has only to keep the beacons in range to remain in the center of channel. If the higher beacon appears to "open out" (move to the right or left of the lower beacon), the navigator can tell if he is to the right or left of his desired track.

The line defined by the range is called a "range line" or "leading line" (Fig. 90-1, 90-2). In this case, one does not need to know the numerical value of the bearing represented by these lines.

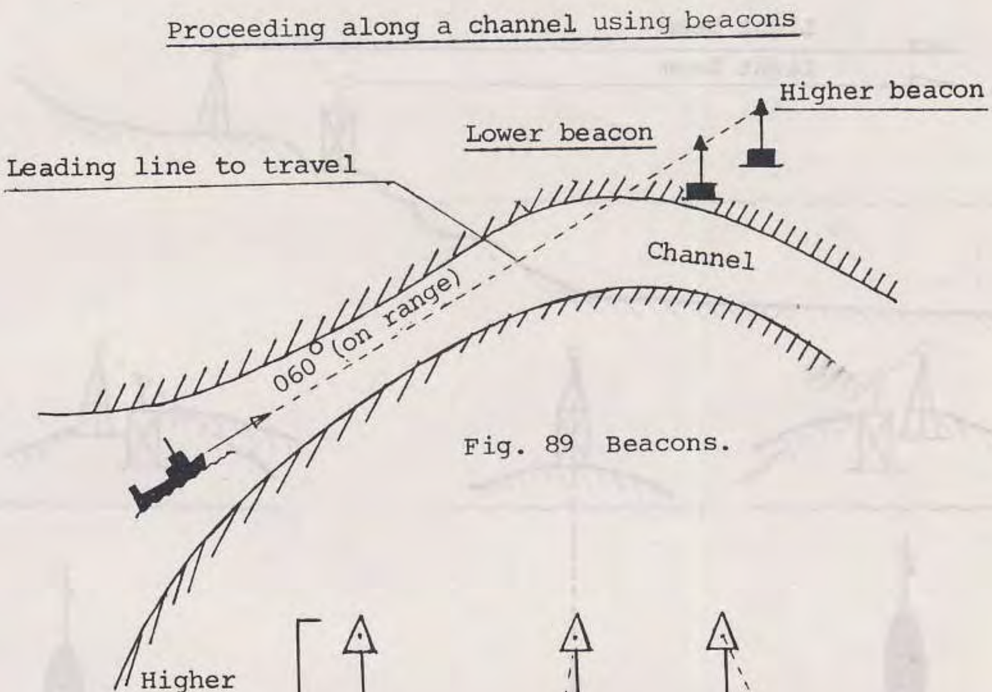


Fig. 89 Beacons.

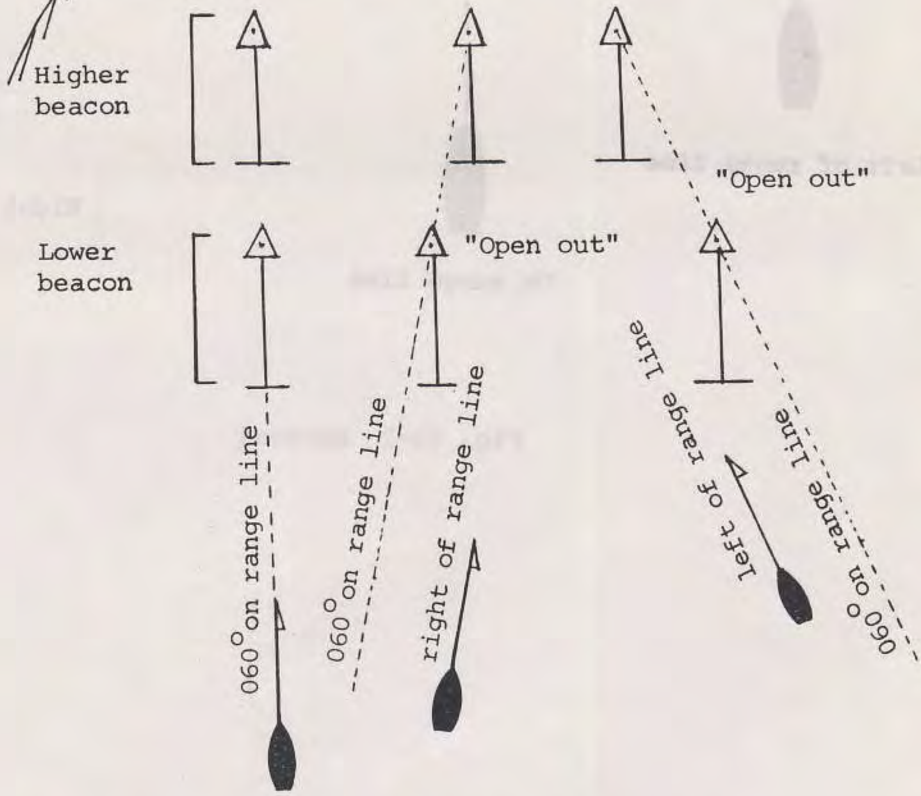


Fig. 90-1 Beacons.

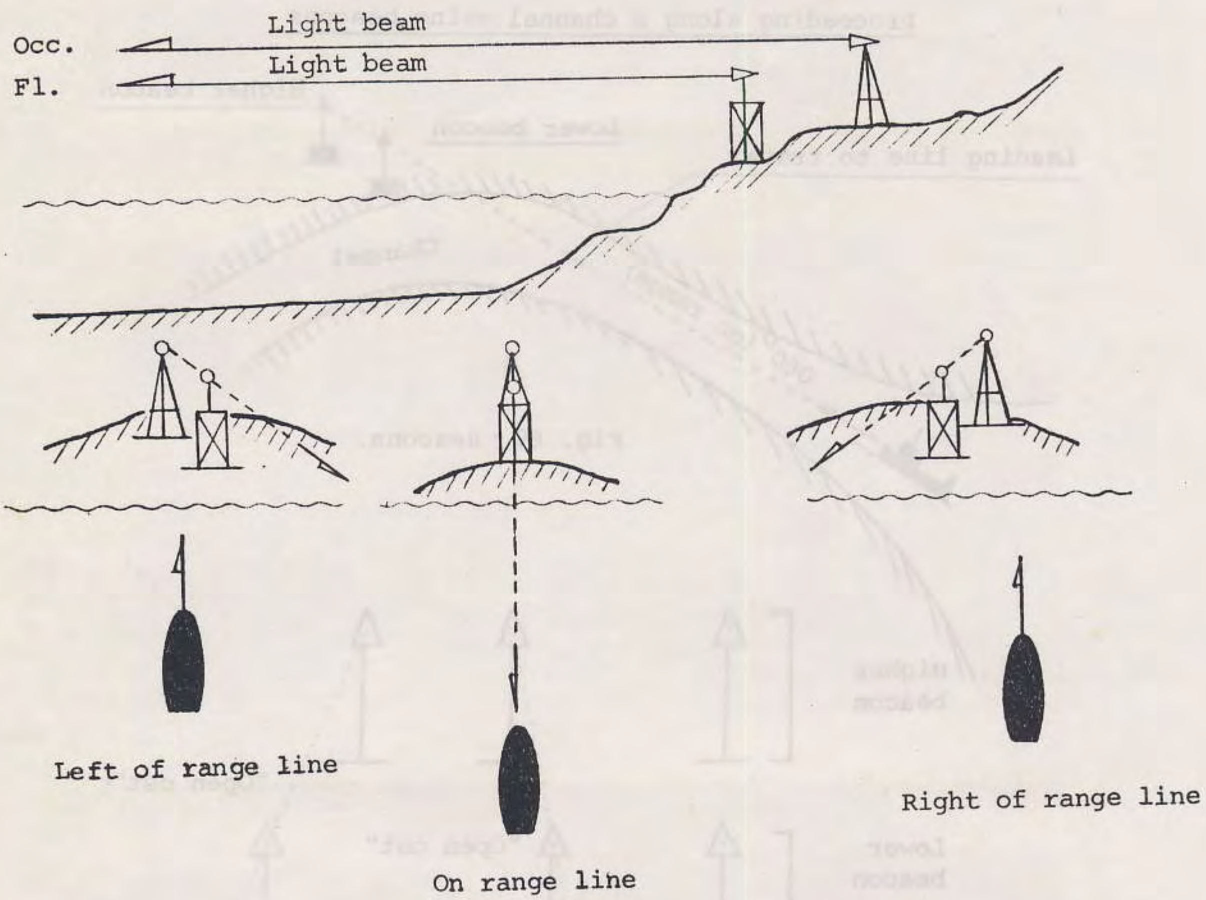


Fig. 90-2 Beacons

Fig. 90-3

Left of range line

The entrance to the port, Songkhla (16 Oct. 1985)

View from M.V. PAKNAM

Fig. 90-4

Right of range line

The entrance to the port, Songkhla (16 Oct. 1985)

View from M.V. PAKNAM

Fig. 90-5

On range line

The entrance to the port, Songkhla (16 Oct. 1985)

View from M.V. PAKNAM

3. Distance

If a ship is known to be a certain distance from an identified point on the chart, it must be somewhere on the circumference of a circle which has that point as the center and the distance as the radius. A single distance arc is labeled with the time above the line (Figs. 94 and 95).

Distances are obtainable by radar, range finder, vertical-sextant angles etc.

If vertical sextant angles are used, the measurement should be from the top of the object to the visible sea horizon, if it is available.

4. The fix - Simaltaneous observation

A line of position represents a series of possible positions, but not a single position, if two simultaneous, nonparallel lines of position are available, the only position that satisfies the requirements of being on both lines at the same time is the intersection of the two lines. This point is one form of fix (Figs. 91, 92, 93, 96 and 97).

Examples of several types of fix by landmarks are given in the illustrations. In Figures 91, 92 and 93 a fix at 08:30 is obtained from two bearing lines. The fix of Figure 94 is obtained by two distance circles. The fix, Figure 95, is obtained by a range and a distance. In Figure 95 (right-hand), a bearing and distance of a single object are used.

A small circle (0) on the crossed lines to show position, is used to indicate the fix at the intersection of the lines of position. The time of the fix is the time at which the lines of position were observed. Navigators should pay careful attention to the selection of objects to provide a fix. It is essential, for instance, that the objects be identified. The angle between lines of position is important. The ideal angle is 90°.

If the angle is small, a slight error in measuring or plotting either line results in a relatively large error in the indicated position. In the case of a bearing line, nearby objects are preferable to those at a considerable distance. Error resulting from an angular error increases with distance.

The type of object should be carefully selected. Lighthouses, spires, flagpoles and so on, are good objects because the point of observation is well defined.

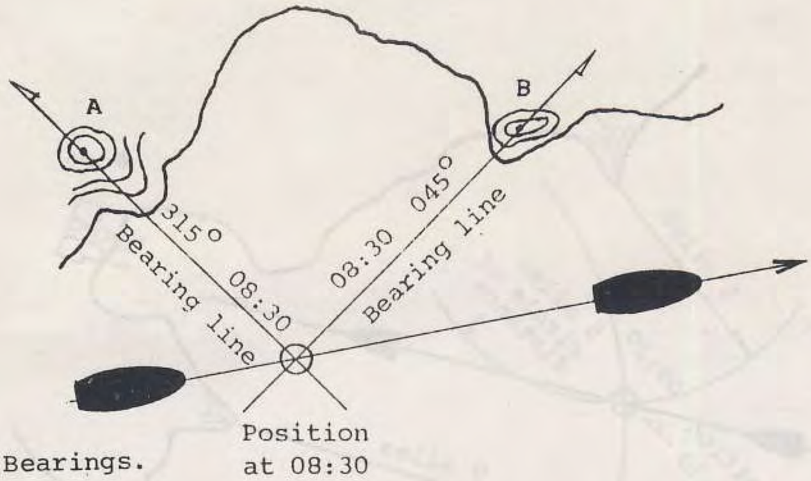


Fig. 91 Bearings.

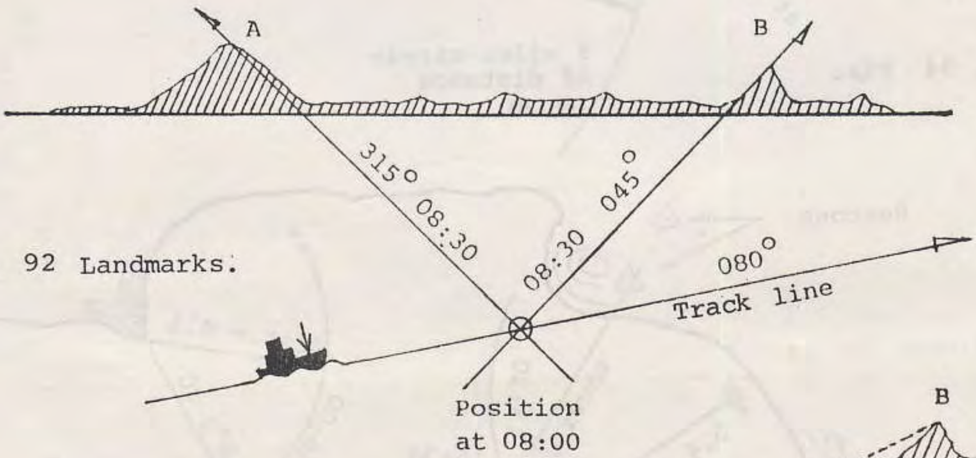


Fig. 92 Landmarks.

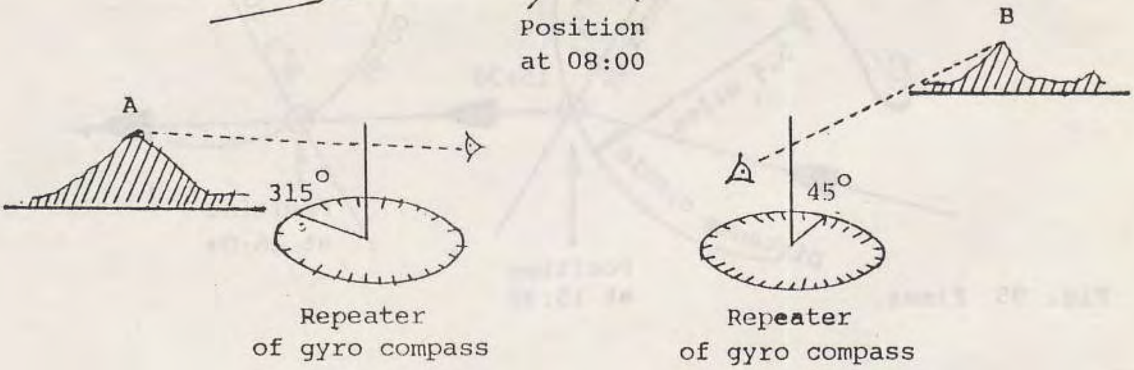


Fig. 93 Compass.

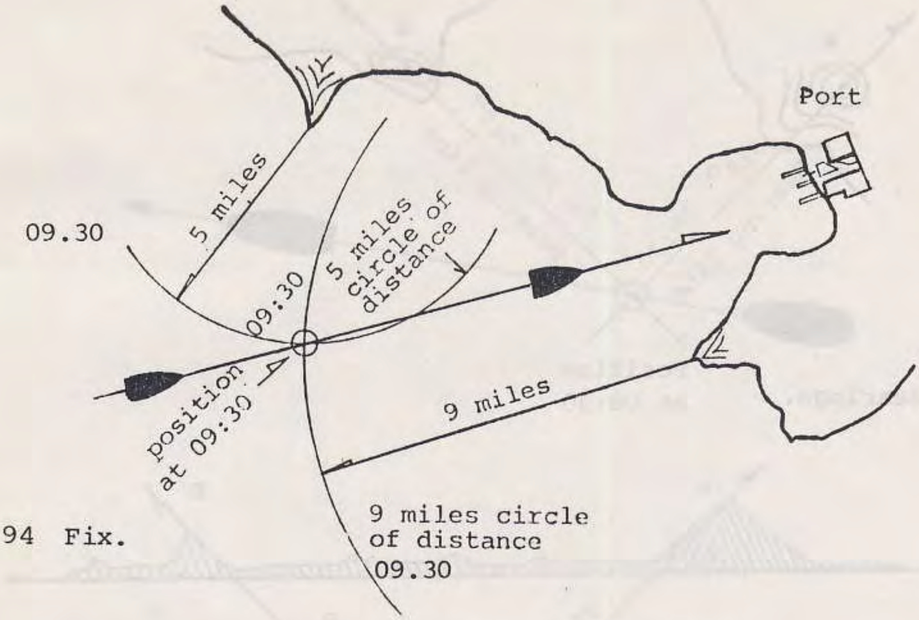


Fig. 94 Fix.

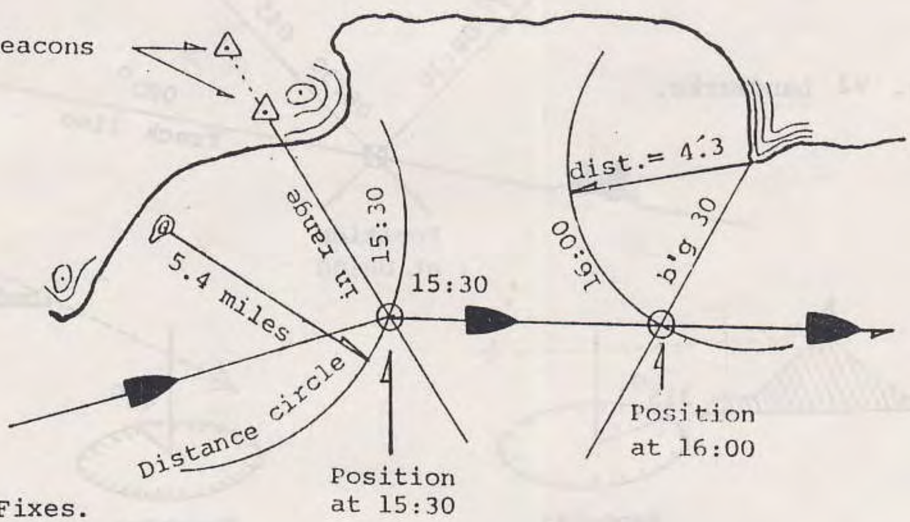


Fig. 95 Fixes.

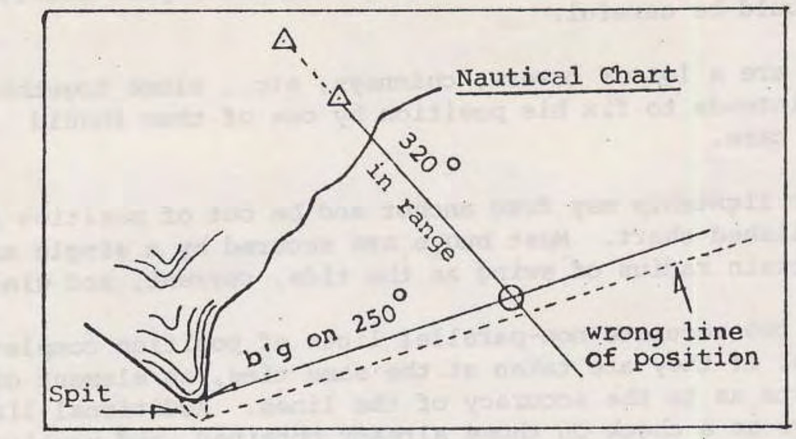


Fig. 96 Spit.

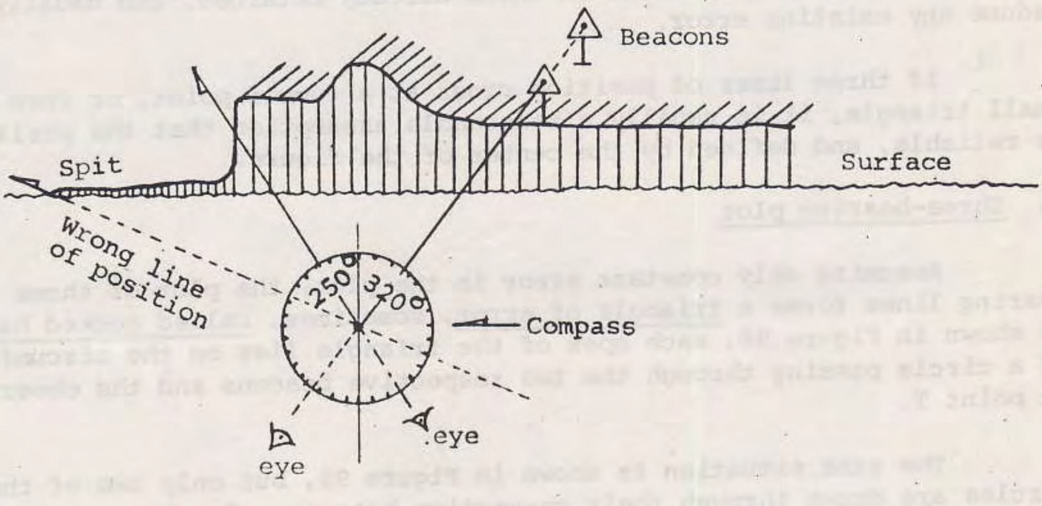


Fig. 97 Spit.

A large building, the nearest mountain, a point of land, etc., may leave some doubt as to the exact point used for the observation.

If a tangent bearing line is used (Figs. 96 and 97), there is a possibility that a low spit may extend seaward from the part observed, so navigators should be careful.

If there are a lot of towers, chimneys, etc., close together, a navigator who intends to fix his position by one of them should identify it with care.

A buoy or lightship may drag anchor and be out of position as shown on the published chart. Most buoys are secured by a single anchor and so have a certain radius of swing as the tide, current, and wind change.

Although two accurate non-parallel lines of position completely define a position, if they are taken at the same time, an element of doubt always exists as to the accuracy of the lines. Additional lines of position can serve as a check on those already obtained, and usually reduce any existing error.

If three lines of position cross at a common point, or form a small triangle, it is usually a reasonable assumption that the position is reliable, and defined by the center of the figure.

5. Three-bearing plot

Assuming only constant error in the plot, the plot of three bearing lines forms a triangle of error, sometimes, called cocked hat. As shown in Figure 98, each apex of the triangle lies on the circumference of a circle passing through the two respective beacons and the observer at point T.

The same situation is shown in Figure 99, but only two of the circles are drawn through their respective beacons. For the set of angular differences established by the differences of the bearing observations, the observer can be located only at the intersection of the two circles at point T.

Be careful that point T is not inside the triangle in this instance. If all error is due to constant error and the bearing spread, that is, the angular difference between the extreme left and right beacons, is less than 180°, point T is always outside the triangle (Figs. 98 and 100).

If all error is due to constant error, and the bearing spread is greater than 180°, point T is always inside the triangle, as illustrated in Figures 102 and 103.

When a bearing spread is greater than 180°, and assuming only constant error, the fixed position in a three-bearing plot forming a triangle of error is the geometric center of the triangle as shown in Figure 102.

The geometric center is located at the intersection of the bisectors of the three interior angles as illustrated in Figure 103.

When the bearing spread is less than 180°, the direction of either the extremely left-hand or right-hand object from the observer's approximate position is used as the reference direction. When the bearing spread is greater than 180°, the direction of any one of the three objects from the observers position can be used as the reference direction.

If the bearing spread is less than 180° and the plotted bearing line extended through the extreme left-hand or right-hand object lies to the right of the intersection of the other two plotted bearing lines, the error of the compass is east, otherwise the error is west (Fig. 100).

If the bearing spread is greater than 180° and the plotted bearing line through one object lies to the right of the intersection of the other two plotted bearing lines, the error is east, otherwise the error is west (Fig. 101).

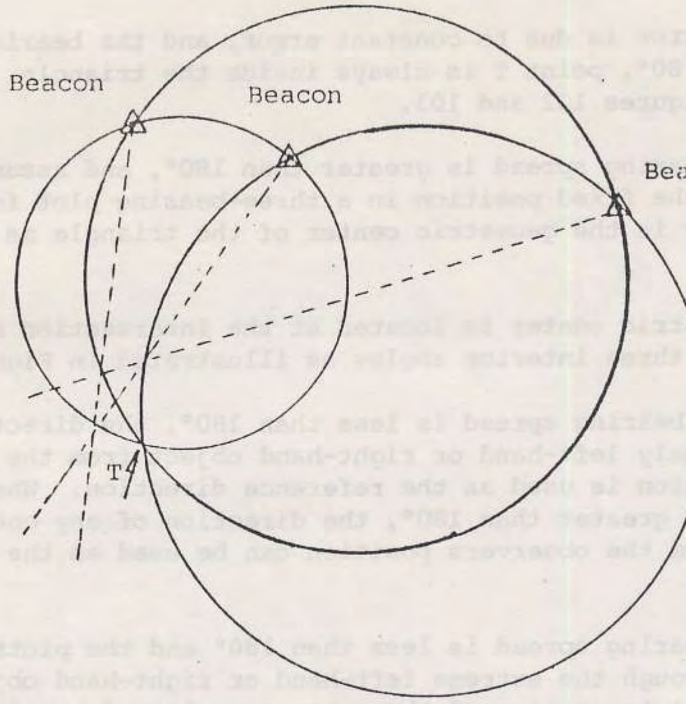
6. <u>The fix-nonsimultaneous observations</u>

For fully accurate results, observation made to fix the position of a moving ship should be made simultaneously, or nearly so. On a slow-moving vessel, relatively little error is introduced by making serveral observations in quick succession.

A wise precaution is to observe the objects more infront or astern first, since these are least affected by the motion of the observer.

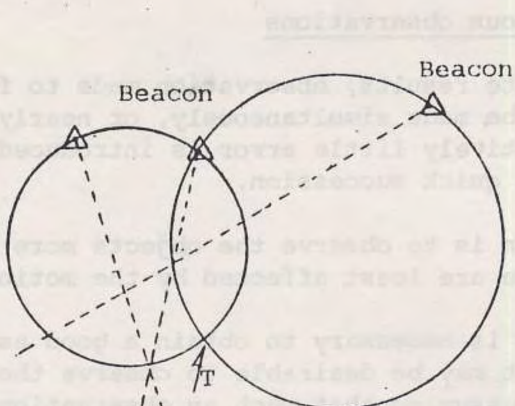
However, when it is necessary to obtain a good estimate of the speed being made good, it may be desirable to observe the most rapidly changing bearing first, assuming that such an observation can be better coordinated with the time mark.

Sometimes it is not possible or necessary to make simultaneous or nearly simultaneous observations.



Triangle of error

(Fig. 98)



Three bearing plot

(Fig. 99)

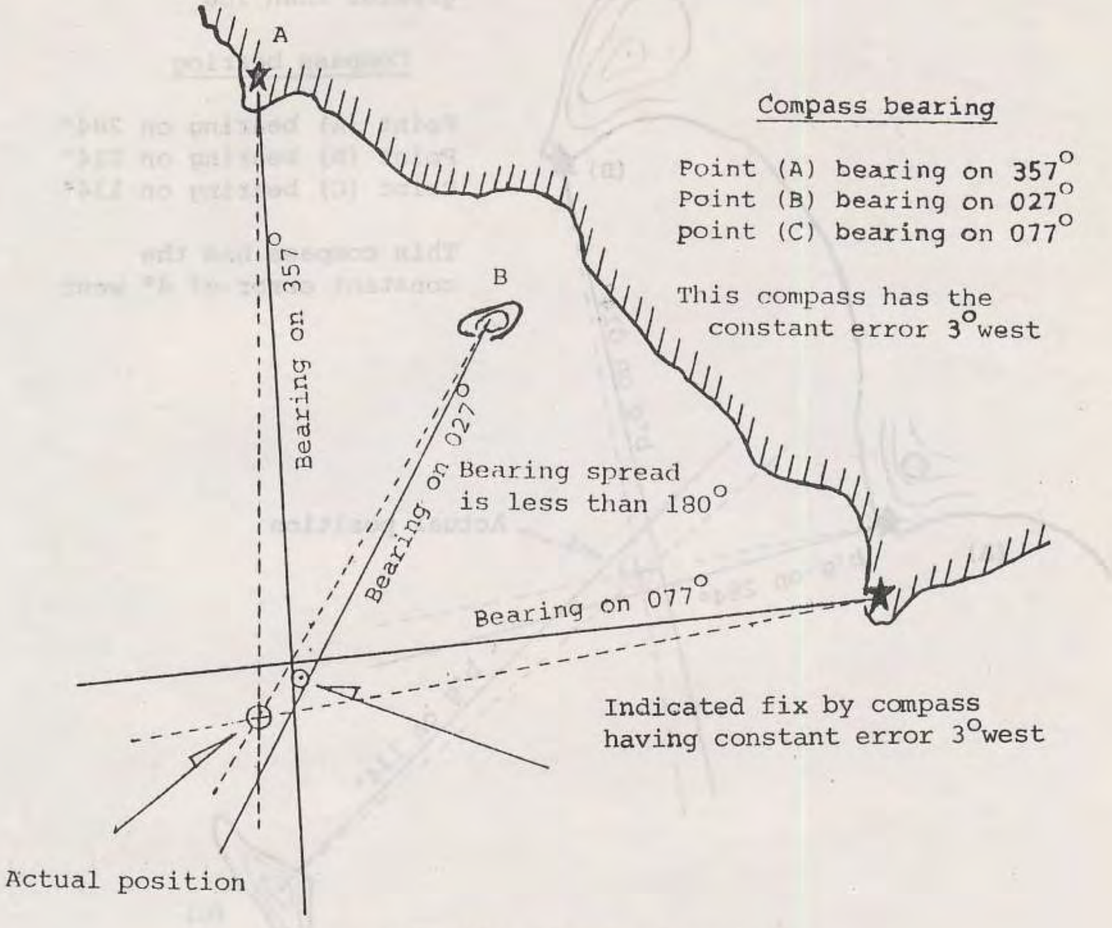


Fig. 100 Error of Position.

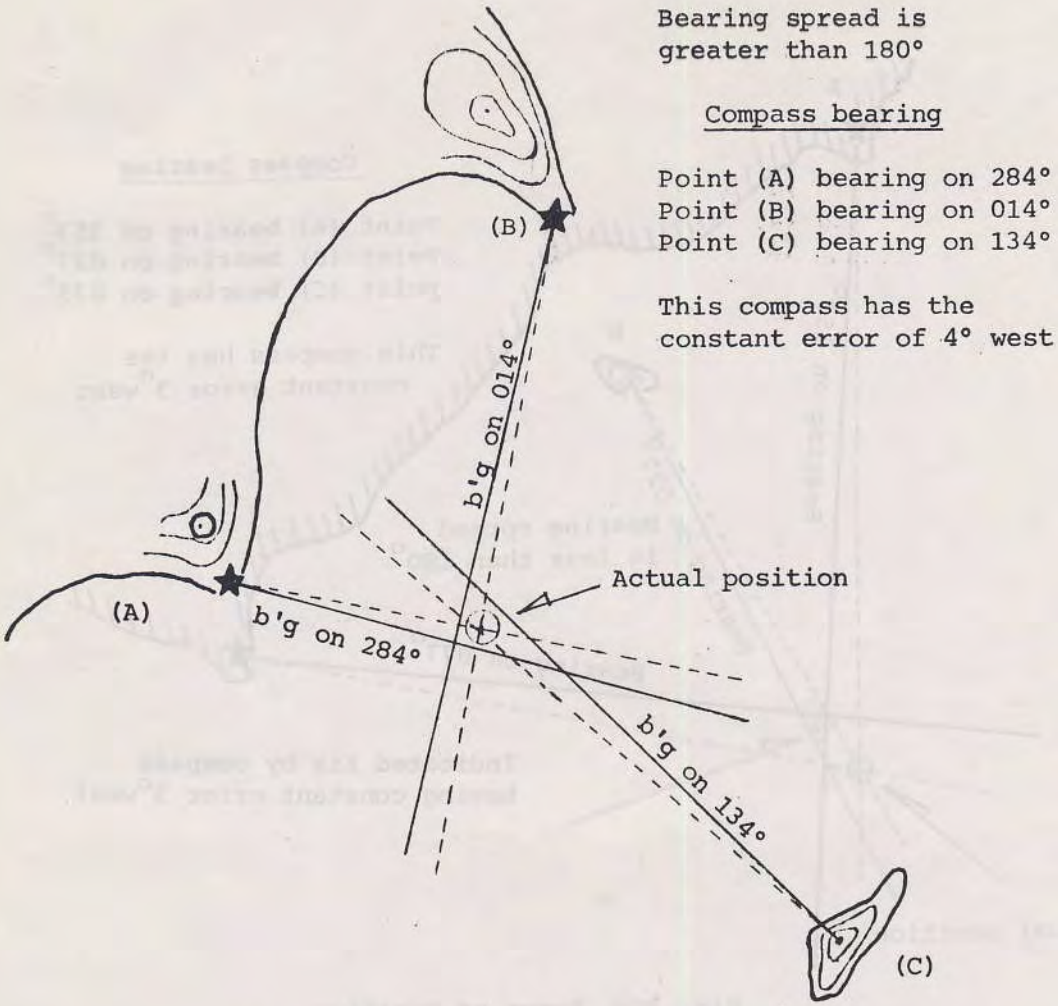


Fig. 101 Triangle of error and position.

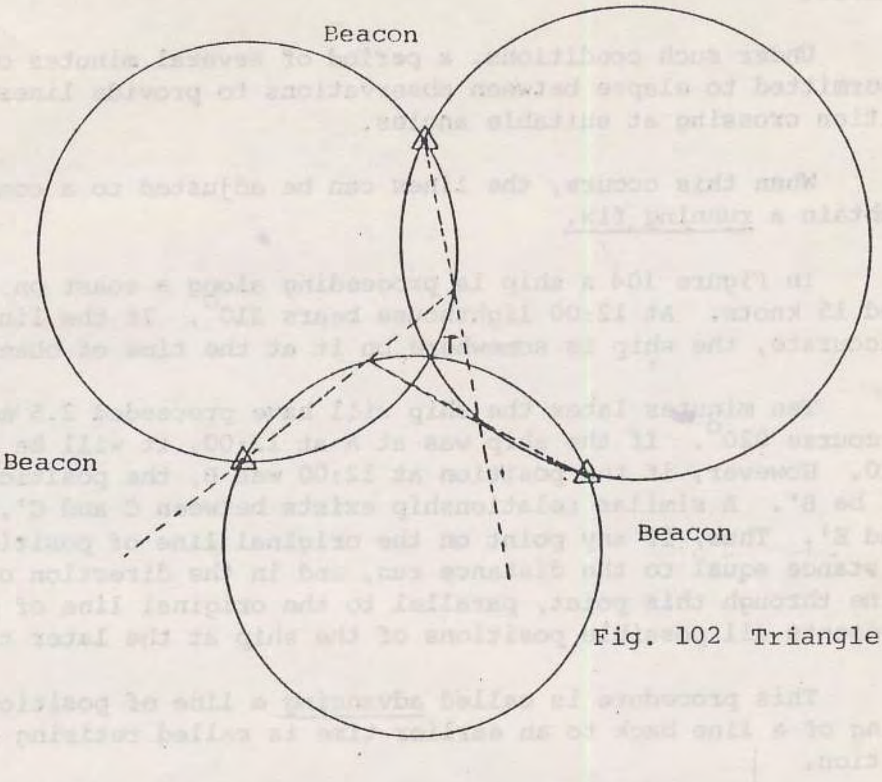


Fig. 102 Triangle of error.

Fig. 103 Triangle of error.

Such a situation may arise, for instance, when a single object is available for observation, or when all available objects are on nearly the same or reciprocal bearings, and there is no means of determining distance.

Under such conditions, a period of several minutes or more may be permitted to elapse between observations to provide lines of position crossing at suitable angles.

When this occurs, the lines can be adjusted to a common time to obtain a running fix.

In Figure 104 a ship is proceeding along a coast on course 020° and speed 15 knots. At 12:00 lighthouse bears 310°. If the line of position is accurate, the ship is somewhere on it at the time of observation.

Ten minutes later the ship will have proceeded 2.5 miles on its course 020°. If the ship was at A at 12:00, it will be at A' at 12:10. However, if the position at 12:00 was B, the position at 12:10 will be B'. A similar relationship exists between C and C', D and D', E and E'. Thus, if any point on the original line of position is moved a distance equal to the distance run, and in the direction of the motion, a line through this point, parallel to the original line of position, represents all possible positions of the ship at the later time.

This procedure is called advancing a line of position. The moving of a line back to an earlier time is called retiring a line of position.

Navigators can use the method of advancing a line of position and another bearing line of the same landmark to fix position (Figs. 105 and 106).

A position is obtainable by using the first bearing line, advancing first bearing line of position and second bearing line of another landmark when a ship is proceeding along the coast (Fig. 107).

In simultaneous observations or running fix, the idea of advancing a line of position is very important.

A fix obtained by means of lines of position taken at different times and adjusted to a common time is called a running fix.

In piloting, common practice is to advance earlier lines to the time of the last observation.

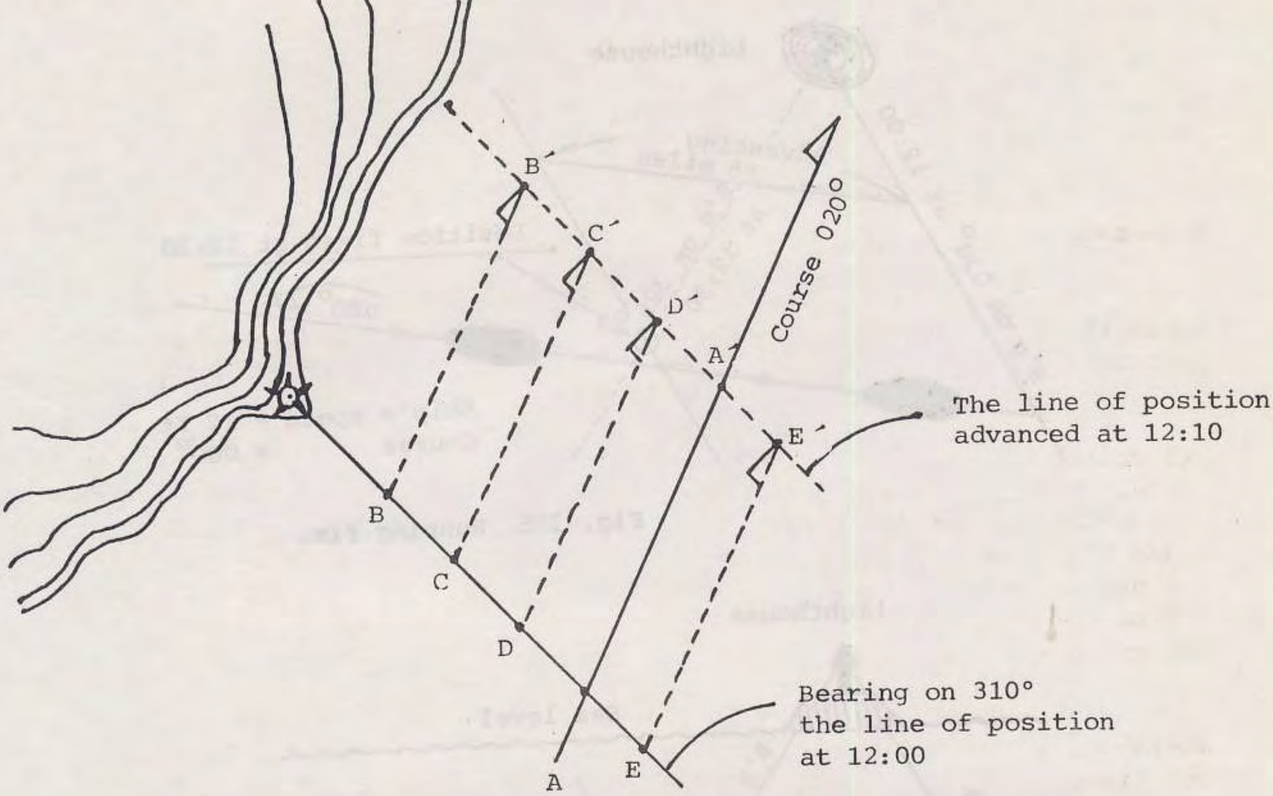


Fig. 104 Advancing a line of position

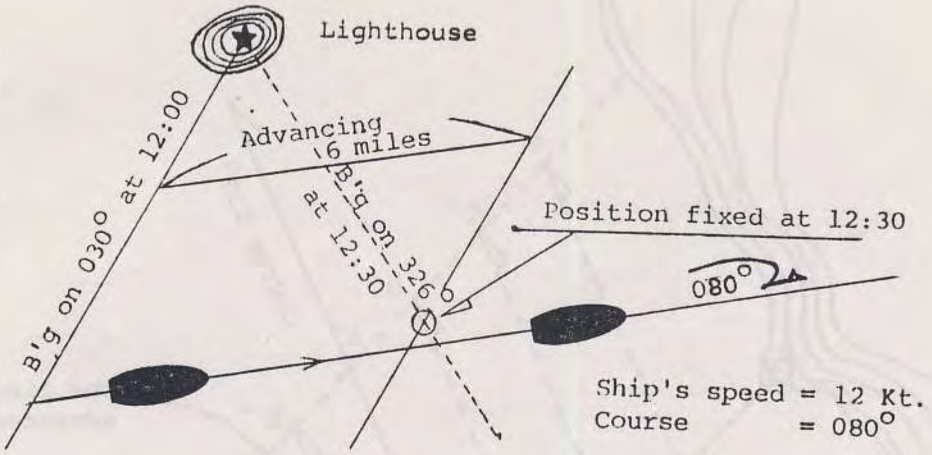


Fig. 105 Running fix.

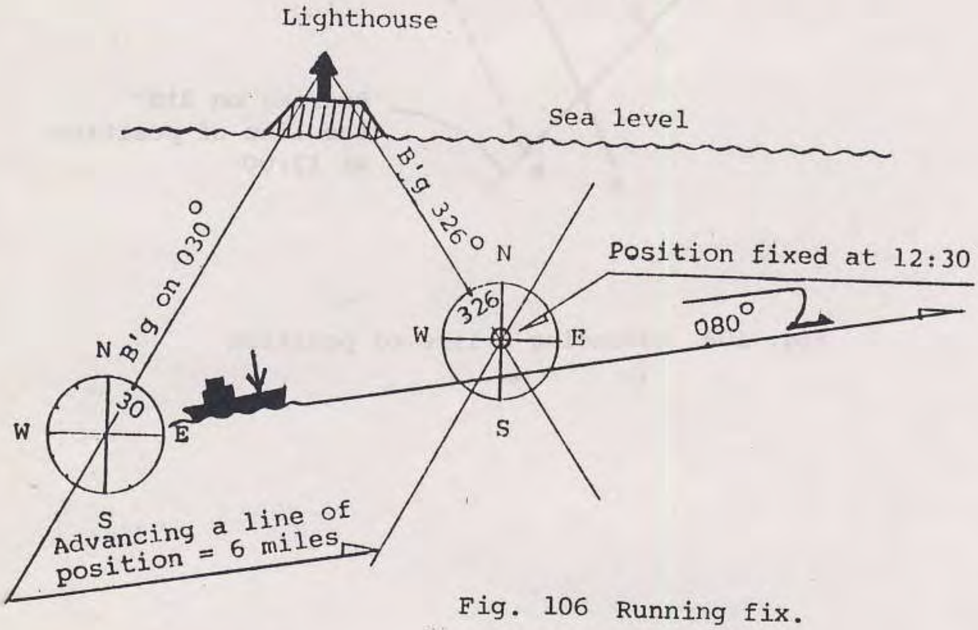


Fig. 106 Running fix.

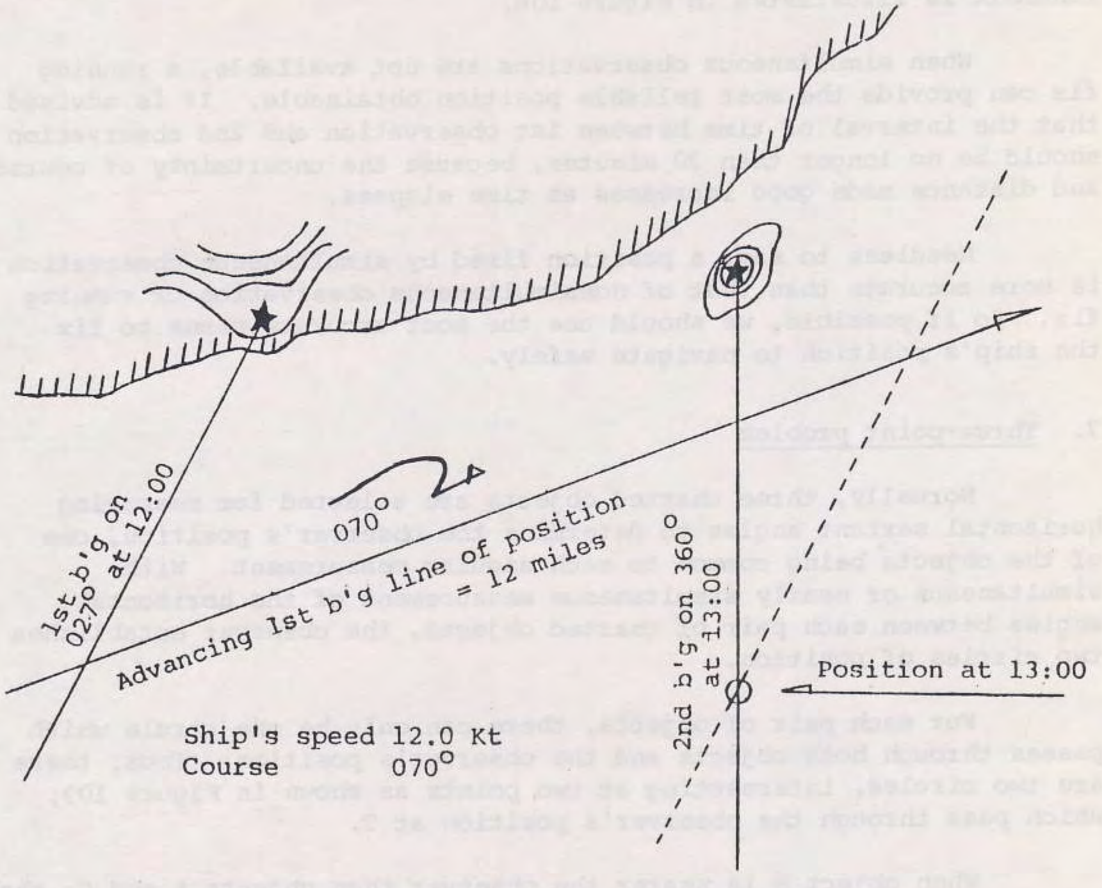


Fig. 107 Running fix.

Figure 105 shows a running fix obtained from two bearings of the same object. The lines of position are not only straight lines such as direction of an object but also circles of position. A running fix by two circles of position by making use of the same landmark is illustrated in Figure 108.

When simultaneous observations are not available, a running fix can provide the most reliable position obtainable. It is advised that the interval of time between 1st observation and 2nd observation should be no longer than 30 minutes, because the uncertainty of course and distance made good increases as time elapses.

Needless to say, a position fixed by simultaneous observation is more accurate than that of nonsimultaneous observation or running fix. So if possible, we should use the most accurate means to fix the ship's position to navigate safely.

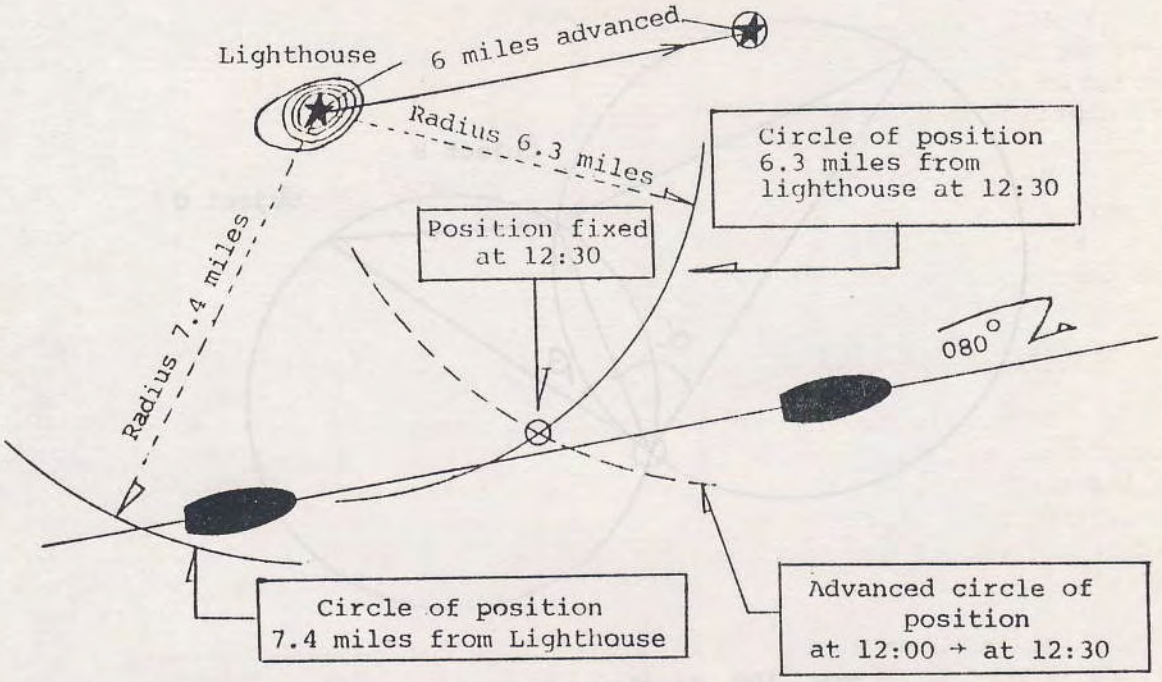
7. Three-point problem

Normally, three charted objects are selected for measuring horizontal sextant angles to determine the observer's position, one of the objects being common to each angular measurement. With simultaneous or nearly simultaneous measurement of the horizontal angles between each pair of charted objects, the observer establishes two circles of position.

For each pair of objects, there can only be one circle which passes through both objects and the observer's position. Thus, there are two circles, intersecting at two points as shown in Figure 109, which pass through the observer's position at T.

When object B is nearer the observer than objects A and C, the position fix is accurate. The solution of what is known as the three-point problem is effected by placing the hairlines of the arms of a plastic three-arm protractor over the three observed objects on the chart as shown in Figures 110 and 111.

With the arms so placed, the center of the protractor disc is over the observer's position on the chart at the time of measurement.



Ship's speed = 12.0 Kt
Course = 080°

Fig. 108 Running fix.

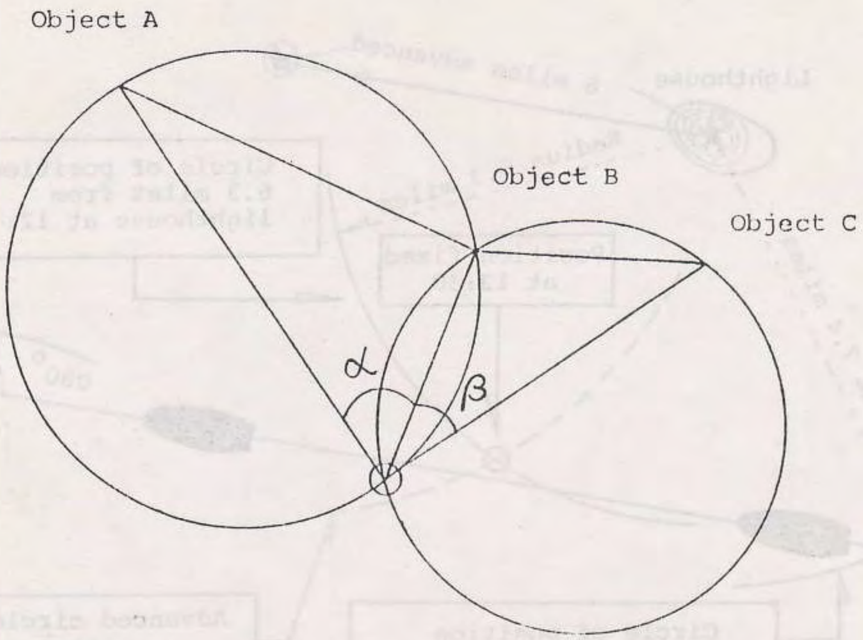


Fig. 109 Circles of position.

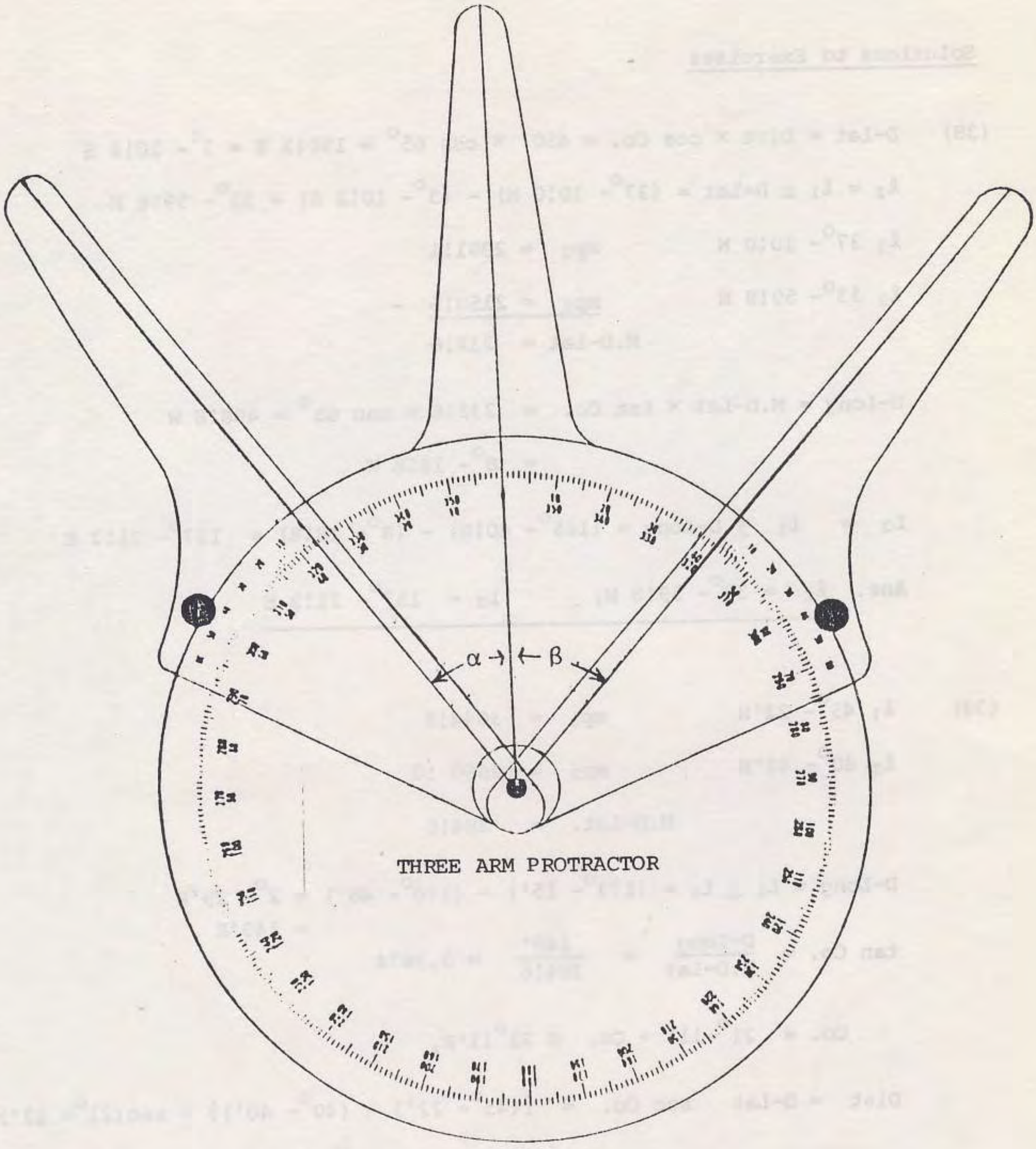


Fig. 110 Three-arm protractor.

See p. 224

Solutions to Exercises

(38) D-Lat = Dist × cos Co. = 450' × cos 65° = 190!2 S = 3°- 10!2 S

$\ell_2 = \ell_1 \pm$ D-Lat = (37°- 10!0 N) - (3°- 10!2 S) = 33°- 59!8 N

ℓ_1 37°- 10!0 N　　mp_1 = 2391!1

ℓ_2 33°- 59!8 N　　mp_2 = 2158!5 -

M.D-Lat = 232!6

D-Long = M.D-Lat × tan Co. = 232!6 × tan 65° = 498!8 W

= 8°- 18!8 W

L_2 = L_1 ± D-Long = (165°- 40!0) - (8°- 18!8) = 157°- 21!2 E

Ans. ℓ_2 = 33°- 59!8 N,　　L_2 = 157°- 21!2 E

(39) ℓ_1 45°- 22'N　　mp_1 = 3044!6

ℓ_2 40°- 40'N　　mp_2 = 2660 !0

M.D-Lat. = 384!6

D-Long = $L_2 \pm L_1$ = (173°- 15') - (170°- 46') = 2°- 29'E

= 149'E

$$\tan Co. = \frac{\text{D-Long}}{\text{M.D-Lat}} = \frac{149'}{384!6} = 0.3874$$

Co. = 21°-11' → Co. S 21°11'E,

Dist = D-Lat sec Co. = {(45°- 22') - (40°- 40')} × sec(21°- 11')

= 282' × sec (21°- 11')

= 302!3

Ans. Co. = S 21°.1 E, Dist. = 302!3

(40) D-Long = $180^\circ - (179^\circ - 10') = 50'E$

$\tan Co. = \dfrac{\text{D-Long}}{\text{M.D.-Lat}}$ $\quad \text{M.D-Lat} = \dfrac{\text{D-Long}}{\tan Co.} = \dfrac{50'}{\tan 65^\circ}$

$= \dfrac{50'}{2.1445} = 23!3\ N$

$\ell_1 \quad 40^\circ - 20!0\ N \qquad mp_1 = 2633!8$

M.D-Lat = $\underline{23!3}$ +

$\dfrac{\ell_2}{\text{D-Lat}} \quad \dfrac{40^\circ - 37.8\ N}{17.8\ N} \leftarrow mp_2 = 2657!1$

Dist. = D-Lat × sec Co. = $17!8 \times \sec 65^\circ = 17!8 \times 2.3662 = 42!1$

$T = (10^h 00^m) + \dfrac{42!1}{10!0} = (10^h 00^m) + 4.2^h = (10^h 00^m) + (4^2\ 12^m)$

$= 14^h 12^m$

Ans. $= \underline{14^h 12^m}$

(41) $\ell_1 \quad 15^\circ - 42!0\ N \qquad mp_1 = 947!7$
$\ell_2 \quad 00^\circ - 00!0 \qquad mp_2 = 000!0$

D-Lat 942'N M.D-lat = 947!7

Dist. = D-Lat × sec Co. = $942' \times \sec 35^\circ$. $(155^\circ = S\ 35^\circ E)$

$= 942' \times 1.22077 = 1149!97 \doteqdot 1150'$

$T = \dfrac{1150}{20} = 57^h 30^m$

$\tan Co. = \dfrac{\text{D-Long}}{\text{M.D-Lat}}$ $\quad$ D-Long = M.D-Lat × tan Co.

$= 947!7 \times \tan 35^\circ$

$= 947.7 \times 0.70021$

$= 663!6\ E = 11^\circ - 03!6\ E$

$L_2 = L_1 \pm \text{D-Long} = (178^\circ - 56!0) + (11^\circ - 03!6) = 189^\circ - 59!6\ E$

(42) L_1 $174^{\circ}-15'E$

L_2 $180^{\circ}-00'$

D-Long $5^{\circ}-45' = 345'E$

$\tan Co. = \frac{D\text{-}Long}{M.D\text{-}Lat}$

$M.D\text{-}Lat = \frac{D\text{-}Long}{\tan Co.}$

$= \frac{345'}{\tan 65^{\circ}}$

$= \frac{345}{2.14451} = 160'.9$

$mp_1 - mp_2 = M.D\text{-}Lat$

$mp_2 = (mp_1) - (M.D\text{-}Lat)$

$= (1706'.5) - (160'.9)$ $(\ell_1 = 27^{\circ}-30\ N)$

$= 1545'.6$ $mp_1 = 1706.5)$

ℓ_1 $27^{\circ}-30'.0\ N$ $mp_1 = 1706'.5$

ℓ_2 $25^{\circ}-05'.0\ N$ $mp_2 = 1545'.6$

D-Lat $2^{\circ}\ 25'.0\ S$ $M.D\text{-}Lat = 160'.9$

$= 145'S$

Dist. = D-Lat × sec Co. = 145' × sec 65 = 145' × 2.36620 = 343.1

$T = \frac{343.1}{10} = 34^{h}.31 = 34^{h}18^{m}36^{s}$

$$
\begin{array}{r}
10^{m}-11^{d}-12^{h}-00^{m}-00^{s} \\
34^{h}-18^{m}-36^{s}\ + \\
\hline
10^{m}-11^{d}-46^{h}-18^{m}-36^{s} \\
24\ - \\
\hline
10^{m}-12^{d}-22^{h}-18^{m}-36^{s}
\end{array}
$$

Ans. Oct. 12, $22^{h}-18^{m}-36^{s}$

$\ell_2 = 25^{\circ}-05'N$

By the NAVIGATION TABLE (Meridional parts)

(43)

20°	mp_{20}	1217.2	$\left.\begin{matrix}20^\circ\\10^\circ\end{matrix}\right\}\ \ell_{10-20} = \dfrac{2.4 \times 618.2}{600} = 2.470 \doteqdot 2.5$ cm
10°	mp_{10}	599.0	
	M.D-Lat	618.2	
30°	mp_{30}	1876.7	$\left.\begin{matrix}30^\circ\\10^\circ\end{matrix}\right\}\ \ell_{10-30} = \dfrac{2.4 \times 1277.7}{600} = 5.110 = 5.1$ cm
10°	mp_{10}	599.0	
	M.D-Lat	1277.7	
40°	mp_{40}	2607.7	$\left.\begin{matrix}40^\circ\\10^\circ\end{matrix}\right\}\ \ell_{10-40} = \dfrac{2.4 \times 2008.7}{600} = 8.030 \doteqdot 8.0$ cm
10°	mp_{10}	599.0	
	M.D-Lat	2008.7	
50°	mp_{50}	3456.6	$\left.\begin{matrix}50^\circ\\10^\circ\end{matrix}\right\}\ \ell_{10-50} = \dfrac{2.4 \times 2857.6}{600} = 11.430 \doteqdot 11.4$ cm
10°	mp_{10}	599.0	
	M.D-Lat	2857.6	

By the Formula

$mp_{10} = 599.0286969$ $\qquad mp_{20} = 1217.178509$

$mp_{30} = 1876.734389$ $\qquad mp_{40} = 2607.719245$

Ans. <u>W = 2.5 cm, X = 2.6 cm, Y = 2.9 cm, Z = 3.4 cm</u>

How to Fix the Ship's Position Using Landmarks

Methods of fixing positions

1. Cross bearing
2. Fix by a bearing and angle
3. Fix by a transit (in range) and angle or bearing
4. Fix by horizontal sextant angles
5. Fix by horizontal distance of two landmarks
6. Fix by a bearing and a distance circle from a landmark
7. Running fix
8. Fix by doubling the angle on the bow
9. Fix by four-point method
10. Fix by three bearings of a landmark and distance run
11. Making use of the Depth of water and Isobathic Data

Cross bearing

There or more landmarks can give one the bearing lines to fix a position the ship's position lies at the intersection of these bearing lines. In this case, landmarks should be clearly visible. The three bearing lines generally make a triangle of error or "Cocked hat" as shown in Figure 111.

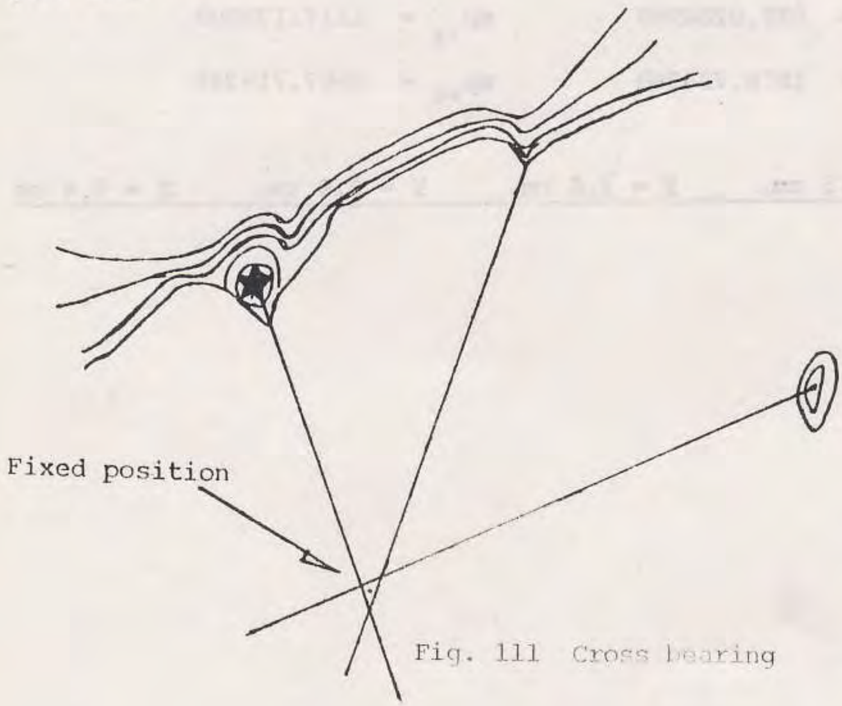


Fig. 111 Cross bearing

The causes of triangles of error are as follows:

1. The actual positions of landmarks are different from the position drawn on the nautical chart.

2. The compass errors calculated are not correct and true.

3. The method of observation to obtain a bearing line is not correct, and the compass direction readings to a landmark are wrong.

4. The drawing of position lines on the chart is not accurate.

When you have a very small error triangle, you can fix position in the center of the error triangle, but the probability of the ship's position being inside the error triangle is one to four. Therefore when you have a large-sized error triangle, you should observe the same landmarks or different landmarks again to obtain more correct bearing lines (a smaller error triangle).

If you are sailing near a dangerous area you should select the nearest of three intersections, as shown in Figure 112.

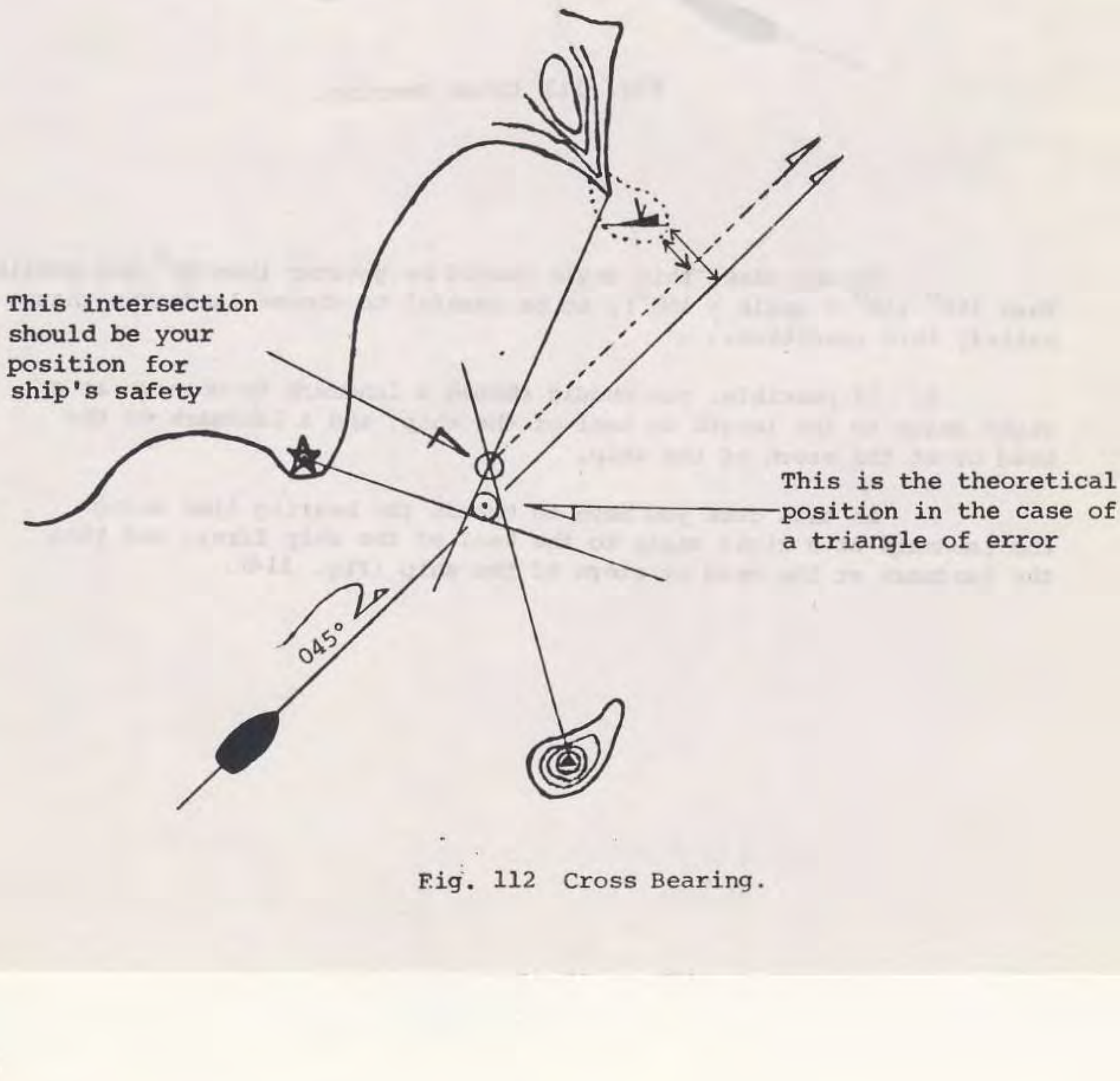


Fig. 112 Cross Bearing.

Important procedures to obtain bearing lines

1. You should choose two clearly visible landmarks whose bearing lines make a right angle or a near right angle at the intersection, as illustrated in Figure No. 113.

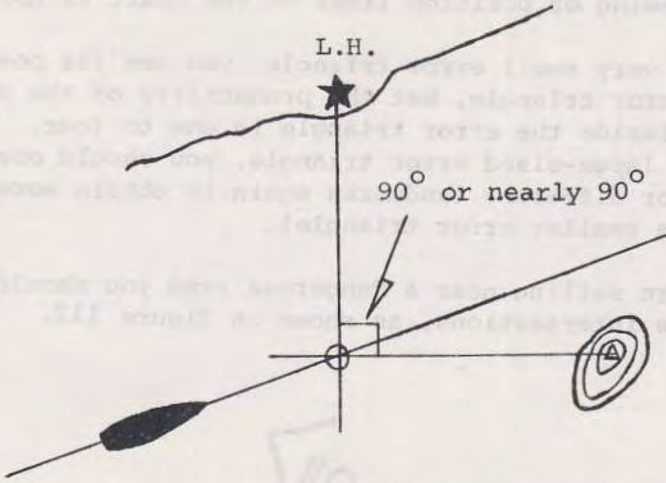


Fig. 113 Cross Bearing.

In any case, this angle should be greater than 30° and smaller than 150° (30° < angle < 150°), so be careful to choose landmarks that satisfy this condition.

2. If possible, you should choose a landmark to observe at a right angle to the length or keel of the ship, and a landmark at the head or at the stern of the ship.

In this case you have to obtain the bearing line using the landmark at a right angle to the keel of the ship first, and then the landmark at the head or stern of the ship (Fig. 114).

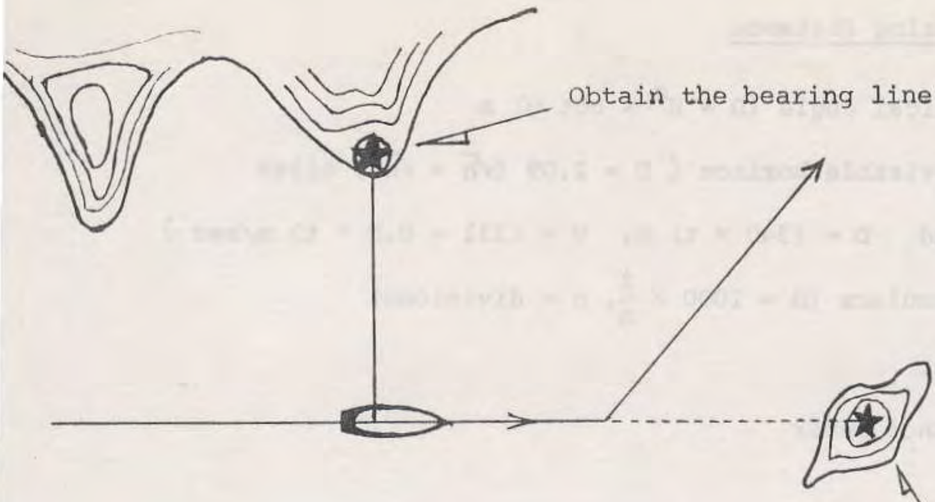


Fig. 114 Cross Bearing.

3. Select obvious landmarks, such as a lighthouse, a tall tower or a clear-cut mountain peak.

4. Observe the direction (bearing) of the nearest landmark first, and then the furthest landmark second.

5. You should already know the correct value of deviation or compass error, and when you observe the landmarks to obtain bearing lines your compass card should be horizontal.

6. After fixing your position on the chart, you have to circle the position and write the time fixed and log-distance sailed above the circled position.

The Methods of Measuring Distance

1. Distance by vertical angle ($D = H^m \times \cot \alpha$) m
2. **Distance to the visible horizon { $D = 2.09\ (\sqrt{h} + \sqrt{H})$ } miles**
3. Distance by sound $D = (340 \times t)$ m, $V = (331 + 0.5 \times t)$ m/sec }
4. Distance by binoculars ($D = 1000 \times \frac{\ell}{n}$, n = divisions)
5. Distance by radar
6. Theodolite distance meter

1. Distance by vertical angle

(A) In the case of a short distance from an observer to a landmark.

(1) In Figure 115, the distance between the ship and the lighthouse is ($b = a \times \cot A$). The position of the observer's eye is not on the sea surface, so $b \neq D$ in Figure 115, however, in ordinary circumstances, the error (b-D) is negligible because the resulting error is very small. Hence, if one knows the height of a lighthouse, from a marine chart, one can calculate the distance between a ship and the lighthouse by the measured vertical angle A

(2) In Figure 116,

H = Height of lighthouse (obtainable from charts or published Lighthouse Listings

h = Height of observer's eye above the water level $\overline{AE}$ (obtainable by measurement)

α = Measured vertical angle

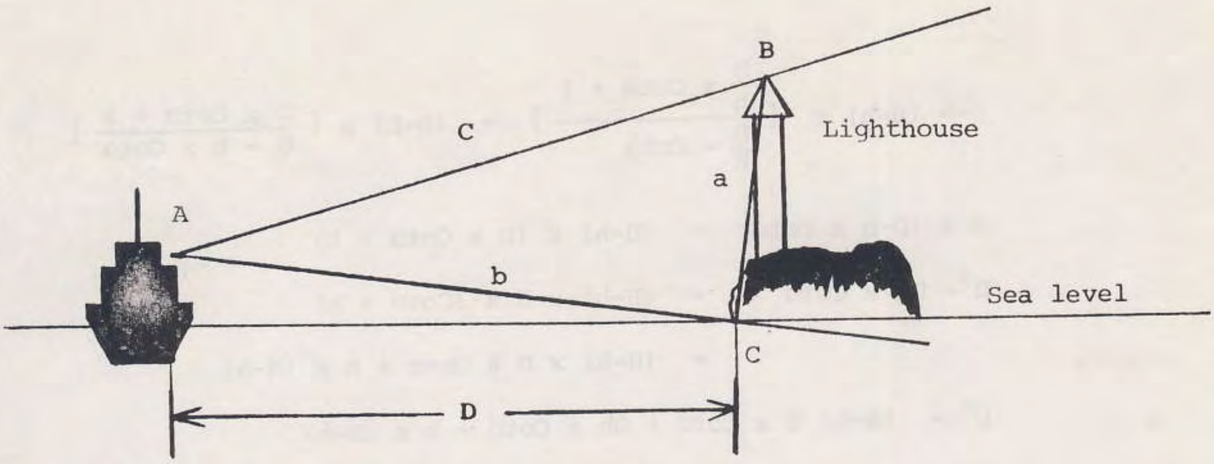
D = Distance between the lighthouse and the observer, $\overline{OE}$

$$\tan(\alpha - \theta) = \frac{\overline{BE}}{D} \rightarrow D = \frac{\overline{BE}}{\tan(\alpha-\theta)} = \overline{BE} \times \cot(\alpha-\theta),$$

$\overline{BE} = H-h$ then, $D = (H-h) \times \text{Cot}\ (\alpha-\theta)$

$$D = (H-h) \times \text{Cot}\ (\alpha-\theta) = (H-h) \times \frac{\text{Cot}\alpha \times \text{Cot}\theta + 1}{\text{Cot}\theta - \text{Cot}\alpha}$$

and $\text{Cot}\ \theta = \frac{D}{h}$, therefore



$\tan A = \frac{a}{b}$ $\qquad$ $b = a \times \text{Cot } A$

Fig. 115 Height of Lighthouse.

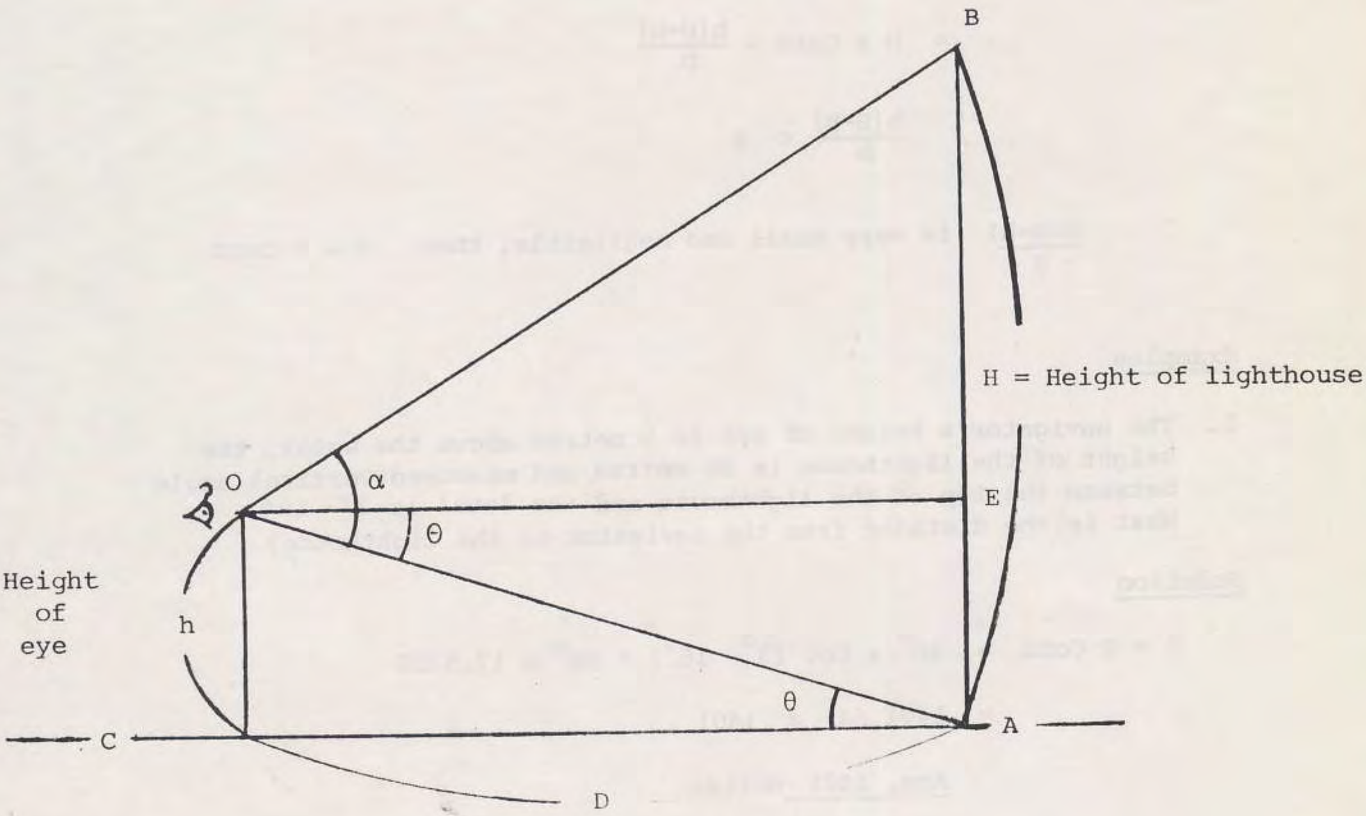


Fig. 116 Height of Lighthouse.

$$D = (H-h) \times \left\{ \frac{\frac{D}{h} \times \text{Cot}\alpha + 1}{\frac{D}{h} - \text{Cot}\alpha} \right\} = (H-h) \times \left\{ \frac{D \times \text{Cot}\alpha + h}{D - h \times \text{Cot}\alpha} \right\}$$

$$D \times (D-h \times \text{Cot}\alpha) = (H-h) \times (D \times \text{Cot}\alpha + h)$$

$$D^2 - Dh \times \text{Cot}\alpha = (H-h) \times D \times (\text{Cot}\alpha + h)$$

$$= (H-h) \times D \times \text{Cot}\alpha + h \times (H-h)$$

$$D^2 = (H-h)\, D \times \text{Cot}\alpha + Dh \times \text{Cot}\alpha + h \times (H-h)$$

$$= H \times D \times \text{Cot}\alpha - hD \times \text{Cot}\alpha + Dh + \text{Cot}\alpha + hH - h^2$$

$$= H \times D \times \text{Cot}\alpha + hH - h^2$$

$$\text{so,} \quad D = \frac{H \times D \times \text{Cot}\alpha + hH - h^2}{D}$$

$$= H \times \text{Cot}\alpha + \frac{hH-h^2}{D}$$

$$= H \times \text{Cot}\alpha + \frac{h(H-h)}{D}$$

$$\frac{h(H-h)}{D} < h$$

$\frac{h(H-h)}{D}$ is very small and negligible, then $D \approx H\ \text{Cot}\alpha$

Examples

1. The navigator's height of eye is 5 metres above the water, the height of the lighthouse is 80 metres and measured vertical angle between the top of the lighthouse and sea level is $3^{\circ}- 16'$. What is the distance from the navigator to the lighthouse?

Solution

$$D = H\ \text{Cot}\alpha = 80^{m} \times \text{Cot}\ (3^{\circ} - 16') = 80^{m} \times 17.5205$$

$$= 1401.64 \approx 1401$$

Ans. 1401 metres

Ref. $\frac{5(80-5)}{1401} \approx 0.27$ metre

$= 27$ cm ← negligible value

2. A navigator's height of eye is 5 metres above the water, the height of the lighthouse is 80 metres and measured vertical angle between the top of a lighthouse and sea level is $10^{\circ} - 36'$. What is the distance from the navigator to the lighthouse?

Solution

$$D = H \operatorname{Cot}\alpha$$

$$= 80^{m} \times \operatorname{Cot}\,(10^{\circ} - 36') = 80^{m} \times 5.34345$$

$$= 427.476^{m} \approx 427\text{m}$$

Ans. 427 metres

Ref. $\frac{h(H-h)}{D} = \frac{5(80-5)}{427} \approx 0.88$ metre $=$ 88cm

In comparison with 427m, the value 88cm is negligible.

2. Distance to the visible horizon with improvements in lighting power and lighting apparatus, the visible distance of light has increased. But the visible distance of light from a lighthouse is limited by the scattering and absorption of light in the air, luminous intensity of a light, and the curvature of the earth's surface.

1) The geographical range of a light means the geometric distance depending on its height from water level, regardless of the luminous intensity of the light.

According to Nathaniel Bowditch, The luminous range means the maximum distance at which a light can be seen under existing visibility conditions.

The luminous range is determined from the known nominal luminous range, called the nominal range, and the existing visibility conditions.

The nominal range is the maximum distance at which a light can be seen in clear weather as defined by the International Visibility Code (meteorological visibility of 10 nautical miles).

The geographical range sometimes printed on charts or tabulated in light lists is the maximum distance at which the curvature of the earth permits a light to be seen from a height of eye of 15 feet above the water when the elevation of the light is taken above the height datum of the largest scale chart of the locality.

(1) The geographical range of a light depends on the height of the light and the observer's height of eye above water level, as shown in Figure 117.

$D = D_1 + D_2$ (The geographical range of the light)

D_1 = Geographical range of the light regardless of h (observer's height of eye)

D_2 = Geographical range of observer's height of eye

H = Height of the light

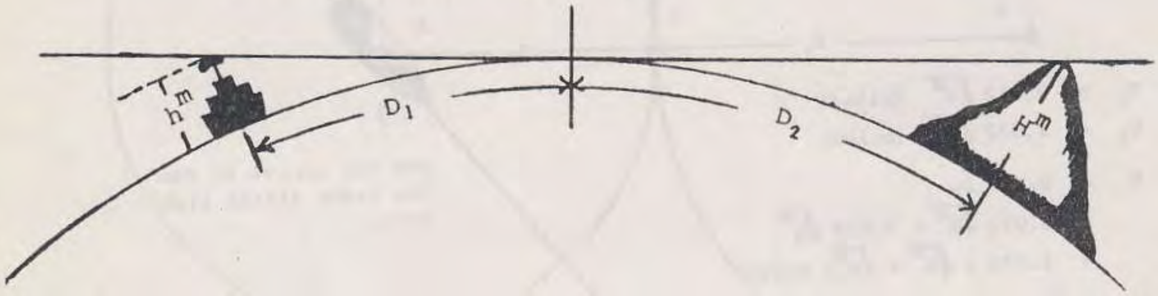
h = Observer's height of eye above the water

$D_1 = 2.072 \sqrt{H^m}$ (miles), $D_2 = 2.072 \sqrt{h^m}$ (miles)

$$D = D_1 + D_2 = 2.072 \sqrt{H^m} + 2.072 \sqrt{h^m}$$
$$= 2.072(\sqrt{H^m} + \sqrt{h^m}) \text{ miles}$$

Ref. Radar horizon (D miles)

$$D = 2.22 \ (\sqrt{H^m} + \sqrt{h^m}) \text{ miles}$$



$$D_1 = 2.22 \sqrt{h^m}$$

$$D_2 = 2.22 \sqrt{H^m}$$

$$D = D_1 + D_2 = 2.22 \sqrt{h^m} + 2.22 \sqrt{H^m}$$
$$= 2.22 \ (\sqrt{h^m} + \sqrt{H^m}) \text{ miles}$$

Fig. 118 Distance from the top of the island.

(2) Luminous range is calculated by the following empirical formula:

$$I = \frac{Ed^2}{T^d}$$

d	=	Luminous range
E	=	0.67 candle/square miles
I	=	luminous intensity (cd)
T	=	Coefficient of propagation of light in the air

weather		T
Thin fog	=	0.007 - 0.027
Haze	=	0.027 - 0.16
Light haze	=	0.16 - 0.48
Clear	=	0.48 - 0.70
Very clear	=	0.70 - 0.87
Exceptionally clear	=	0.87

According to Nathaniel Bowditch, the luminous range is the maximum distance at which a light can be seen under existing visibility conditions. This luminous range takes no account of the elevation of the light, the observer's height of eye, the curvature of the earth, or interference from background lighting.

Note: 1. Vertical angle should be measured from water level to the position where light exists in the lighthouse

2. The height of the mountain on which the lighthouse stands should be accurately indicated on the marine chart

(B) The case of a landmark beyond the visible horizon (Fig. 119)

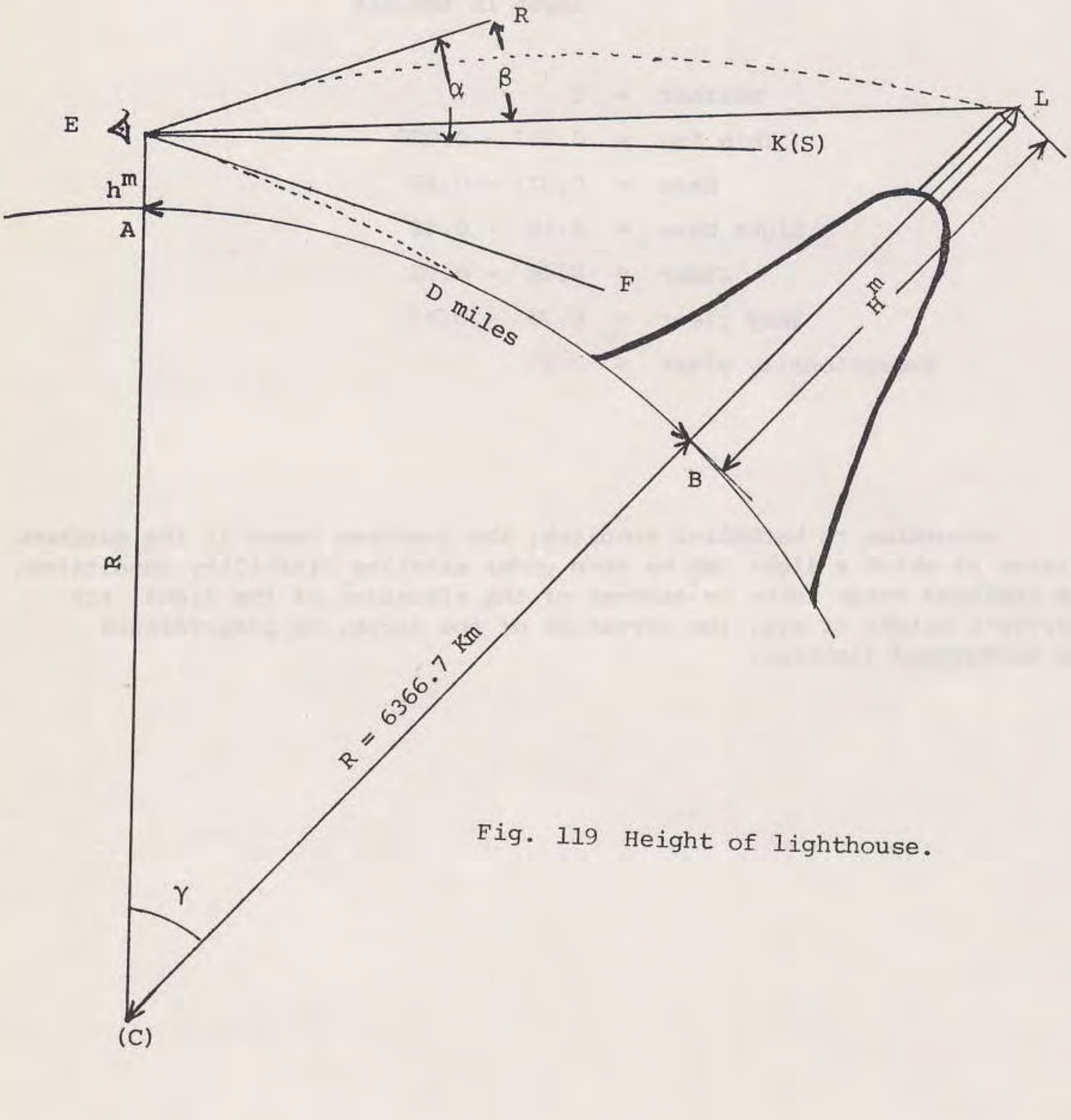


Fig. 119 Height of lighthouse.

$$\frac{R + H}{R + h} - \frac{R + h}{R + h} = \frac{\cos(\alpha - \beta)}{\cos(\alpha - \beta + \gamma)} - \frac{\cos(\alpha - \beta + \gamma)}{\cos(\alpha - \beta + \gamma)}$$

$$\downarrow$$

$$\frac{R + H - (R + h)}{R + h} = \frac{\cos(\alpha - \beta) - \cos(\alpha - \beta + \gamma)}{\cos(\alpha - \beta + \gamma)}$$

$$\downarrow$$

$$\frac{R + H - R - h}{R + h} = \frac{2 \sin \frac{1}{2} \{(\alpha - \beta) + (\alpha - \beta + \gamma)\} \sin \frac{1}{2} \{(\alpha - \beta + \gamma) - (\alpha - \beta\}}{\cos(\alpha - \beta + \gamma)}$$

$$\downarrow$$

$$\frac{H - h}{R + h} = \frac{2 \sin \frac{1}{2} (\alpha - \beta + \alpha - \beta + \gamma) \sin \frac{1}{2} (\alpha - \beta + \gamma - \alpha + \beta)}{\cos(\alpha - \beta + \gamma)}$$

$$= \frac{2 \sin \frac{1}{2} (2\alpha - 2\beta + \gamma) \sin \frac{1}{2} \gamma}{\cos(\alpha - \beta + \gamma)}$$

$$= \frac{2 \sin \frac{1}{2}\ 2 (\alpha - \beta + \frac{1}{2}\gamma) \sin \frac{1}{2} \gamma}{\cos(\alpha - \beta + \gamma)}$$

$$= \frac{2 \sin (\alpha - \beta + \frac{1}{2}\gamma) \sin \frac{1}{2} \gamma}{\cos(\alpha - \beta + \gamma)} \quad \text{---------} (2)$$

In comparison of R with h, h is negligible, so $R + h \doteqdot R$. Also the value of α and γ is quite small, then

$$\frac{H - h}{R} = \gamma \left(\frac{\gamma}{2} + \alpha - \beta \right) \quad \text{-----------} (3)$$

$$\downarrow$$

$$(H - h) = R.\gamma \left(\frac{\gamma}{2} + \alpha - \beta \right)$$

$$\downarrow$$

$$(H - h) = D \left(\frac{\gamma}{2} + \alpha - \beta \right) = D \left(\frac{\gamma}{2} + \alpha - \frac{\gamma}{13} \right)$$

$$= D \left(\frac{13\gamma}{26} + \alpha - \frac{2\gamma}{26} \right)$$

$$= D \left(\frac{11}{26} + \alpha \right) = D \left(\frac{11}{26} \cdot \frac{D}{R} + \alpha \right)$$

$$= \frac{11D^2}{26R} + D\alpha \qquad \text{because } \beta = \frac{1}{13} \cdot \gamma, \quad R.\gamma = D \ \downarrow \ (\gamma = \frac{D}{R})$$

$$\therefore H - h = \frac{11D^2}{26R} + D\alpha$$

$$= \frac{11 \times (1852 \times D)^2}{26 \times 6366740} + (1852\ D)\ \frac{\alpha}{3438}$$

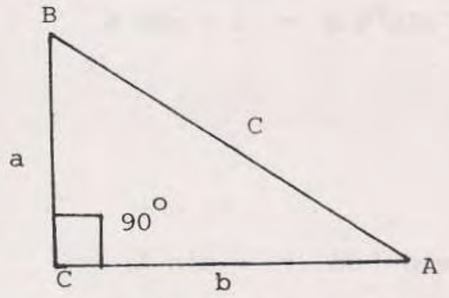
$$= \frac{11 \times (1852^2 \times D^2)}{26 \times 6366740} + \frac{1852 \times D \times \alpha}{3438}$$

$$= 0.2275\ D^2 + 0.5387\ D\alpha \quad \ldots\ldots (4)$$

$1 \text{ radian} = \frac{360^\circ}{2\pi}$
$= 57.295009^\circ$
$= 3437'.7468$
$\approx 3438'$

$$\therefore H-h = 0.2275\ D^2 + 0.5387\ D\alpha$$

Trigonometric function



(Fig. 120)

$\angle A = A \quad \angle B = B \quad \angle C = 90^\circ$

$\overline{BC} = a, \quad \overline{AC} = b, \quad \overline{AB} = c$

Formulae

1. $\sin A = \frac{a}{c}$ 2. $\sin B = \frac{b}{c}$ 3. $\cos A = \frac{b}{c}$
4. $\cos B = \frac{a}{c}$ 5. $\tan A = \frac{a}{b}$ 6. $\tan B = \frac{b}{a}$
7. $\cot A = \frac{b}{a}$ 8. $\cot B = \frac{a}{b}$ 9. $\sec A = \frac{c}{b}$
10. $\sec B = \frac{c}{a}$ 11. $\operatorname{cosec} A = \frac{c}{a}$ 12. $\operatorname{cosec} B = \frac{c}{b}$

In general,

$\angle A + \angle B = 90^\circ, \quad \angle A = 90^\circ - \angle B, \quad \angle B = 90^\circ - \angle A$

so, $\sin A = \cos B \qquad \sec A = \operatorname{cosec} B, \ \ldots\ldots\ldots\ldots$

13. $\sin^2 A + \cos^2 A = 1$

14. $\sin A = \cos A \tan A = \frac{\cos A}{\cot A} = \sqrt{1-\cos^2 A}$

$= 2 \sin \tfrac{1}{2} A \cos \tfrac{1}{2} A = \frac{1}{\operatorname{cosec} A}$

15. $\cos A = \dfrac{\sin A}{\tan A} = \sin A \cot A = \sqrt{1-\sin^2 A} = \cos^2 \tfrac{1}{2} A - \sin^2 \tfrac{1}{2} A$

$= \dfrac{1}{\sec A}$

16. $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{1}{\cot A} = \dfrac{\sin A}{\sqrt{1-\sin^2 A}} = \dfrac{\sin 2A}{1 + \cos 2A}$

17. $\cot A = \dfrac{1}{\tan A}$, 18. $\sec A = \dfrac{1}{\cos A}$, 19. $\operatorname{cosec} A = \dfrac{1}{\sin A}$

20. $\sin 2A = 2 \sin A \cos A$, 21. $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

$= \cos^2 A - \sin^2 A$

22. $2 \cos^2 \tfrac{1}{2} A = 1 + \cos A$, 23. $2 \sin^2 \tfrac{1}{2} A = 1 - \cos A$

24. $\tan (A + 45^\circ) = \dfrac{1 + \tan A}{1 - \tan A}$,

25. $\tan (A - 45^\circ) = \dfrac{\tan A - 1}{\tan A + 1}$

26. $\sin (90^\circ - A) = \cos A$, 27. $\sin (-A) = - \sin A$,

28. $\sin (90^\circ + A) = \cos A$, 29. $\cos (90^\circ - A) = \sin A$,

30. $\cos (90^\circ + A) = -\sin A$, 31. $\cos (-A) = \cos A$,

32. $\tan (90^\circ - A) = \cot A$, 33. $\tan (90^\circ - A) = \cot A$

34. $\tan (-A) = -\tan A$, 35. $\tan (90^\circ + A) = -\cot A$

36. $\tan (90^\circ - A) = \cot A$, 37. $\cot (90^\circ - A) = \tan A$,

38. $\cot (-A) = -\cot A$, 39. $\cot (90^\circ + A) = -\tan A$

40. $\sec (90^\circ - A) = \operatorname{cosec} A$, 41. $\sec (-A) = \sec A$

42. $\sec (90^\circ + A) = -\operatorname{cosec} A$, 43. $\operatorname{cosec} (90^\circ - A) = \sec A$

44. $\operatorname{cosec} (-A) = -\operatorname{cosec} A$, 45. $\operatorname{cosec} (90^\circ + A) = \sec A$

46. $\sin (180^\circ - A) = \sin A$, 47. $\sin (180^\circ + A) = -\sin A$

48. $\cos (180^\circ - A) = -\cos A$, 49. $\cos (180^\circ + A) = -\cos A$

50. $\tan (180^\circ - A) = -\tan A$, 51. $\tan (180^\circ + A) = \tan A$

52. $\cot (180^\circ - A) = -\cot A$, 53. $\cot (180^\circ + A) = \cot A$

52. $\sec(180^\circ - A) = -\sec A$, 53. $\sec(180^\circ + A) = -\sec A$

54. $\mathrm{cosec}\,(180^\circ - A) = \mathrm{cosec}\,A$, 55. $\mathrm{cosec}\,(180^\circ + A) = -\mathrm{cosec}\,A$

56. $\sin(270^\circ - A) = -\cos A$, 57. $\sin(270^\circ + A) = -\cos A$

58. $\cos(270^\circ - A) = -\sin A$, 59. $\cos(270^\circ + A) = \sin A$

60. $\tan(270^\circ - A) = \cot A$, 61. $\tan(270^\circ + A) = -\cot A$

62. $\cot(270^\circ - A) = \tan A$, 63. $\cot(270^\circ + A) = -\tan A$

64. $\sec(270^\circ - A) = -\mathrm{cosec}\,A$, 65. $\sec(270^\circ + A) = \mathrm{cosec}\,A$

66. $\mathrm{cosec}\,(270^\circ - A) = -\sec A$, 67. $\mathrm{cosec}\,(270^\circ + A) = -\sec A$

68. $1 + \tan^2\theta = \sec^2\theta$ 69. $\sin(\theta \pm \psi) = \sin\theta\cos\psi \pm \cos\theta\sin\psi$

70. $\cos(\theta \pm \psi) = \cos\theta\cos\psi \mp \sin\theta\sin\psi$

71. $\tan(\theta \pm \psi) = \dfrac{(\tan\theta \pm \tan\psi)}{(1 \mp \tan\theta\tan\psi)}$

72. $\sin\theta \pm \sin\psi = 2\sin\frac{1}{2}(\theta \pm \psi)\cos\frac{1}{2}(\theta \mp \psi)$

73. $\cos\theta + \cos\psi = 2\cos\frac{1}{2}(\theta + \psi)\cos\frac{1}{2}(\theta - \psi)$

74. $\cos\theta - \cos\psi = 2\sin\frac{1}{2}(\theta + \psi)\sin\frac{1}{2}(\psi - \theta)$

75. $\sin\theta\cos\psi = \frac{1}{2}\{\sin(\theta + \psi) + \sin(\theta - \psi)\}$

76. $\cos\theta\sin\psi = \frac{1}{2}\{\sin(\theta + \psi) - \sin(\theta - \psi)\}$

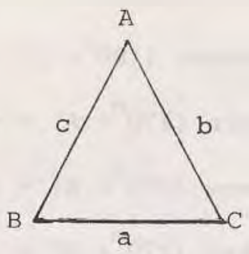
77. $\cos\theta\cos\psi = \frac{1}{2}\{\cos(\theta + \psi) + \cos(\theta - \psi)\}$

78. $\sin\theta\sin\psi = \frac{1}{2}\{\cos(\theta - \psi) - \cos(\theta + \psi)\}$

79. $\sin 2\theta = 2\sin\theta\cos\theta$, 80. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

81. $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, 82. $\cos 3\theta = 4\cos^3\theta - \cos\theta$

83. $\sin\frac{1}{2} = \pm\sqrt{\frac{1}{2}(1-\cos\theta)}$, 84. $\cos\frac{1}{2}\theta = \pm\sqrt{\frac{1}{2}(1+\cos\theta)}$

(Fig. 121) 

$\angle A = A$, $\angle B = B$, $\angle C = C$

$\overline{BC} = a$, $\overline{AC} = b$ $\overline{AB} = c$

85. $a^2 = b^2 + c^2 - 2bc \cos A$,

86. $a = b \cos C + c \cos B$

87. $a:b:c = \sin A : \sin B : \sin C$,

88. $\sin \frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{bc}} \longrightarrow 2s = a+b+c$

89. $\cos \frac{A}{2} = \sqrt{\frac{s(s-a)}{bc}} \longrightarrow 2s = a+b+c$

90. $\sin A = \sqrt{\frac{s(s-a)(s-b)(s-c)}{bc}} \longrightarrow 2s = a+b+c$

91. $\tan \frac{1}{2}(B-C) = \frac{(b-c) \cot \frac{1}{2}A}{b+c}$

92. Area of the triangle = $\frac{1}{2} ab \sin C = \sqrt{s(s-a)(s-b)(s-c)} \longrightarrow 2s = a+b+c$

(3) Distance by sound

The speed of sound in the air at temperature of 15°C is about 340 metres per second, so the distance travelled by sound (D metres or D nautical miles) in t second is;

$$D = 340 \times t \text{ metres} \quad \text{or} \quad D' = \frac{340}{1852} \times t = \frac{t}{5.44} \text{ nautical miles}$$

Ref. The speed of sound in the air at temperature t°c is $= (331 + 0.5\,t)\,^{m}/sec.$ It is therefore possible for us to know the distance by measuring the elapsed time T between transmission of a signal and return of its echo as follows:

$$d = \frac{T}{5.44 \times 2} = \frac{T}{10.88} \text{ nautical miles}$$

Note: The traveling speed of sound is effected by atmospheric conditions (temperature, density, humidity etc.) and the reflecting material. The formula $d = \frac{T}{10.88}$ (nautical miles) is merely a reference.

When you sail in fog in a narrow channel, the speed of return of the fog signal echo using the above mentioned formula will give you some useful information, but it is not recommended to fix position by sound.

(4) Distance by binoculars

Some binoculars have divisions on their lenses. When the height of a landmark is one metre and the distance between the landmark and the observer is 1,000 metres, the visual height of this landmark in the binoculars is equal to the length of one division on the lens (Fig. 122)

When one looks at a landmark whose height is ℓ metres n divisions, the distance D from the landmark to the observer is

$$D = 1000 \times \frac{\ell}{n} \text{ metres}$$

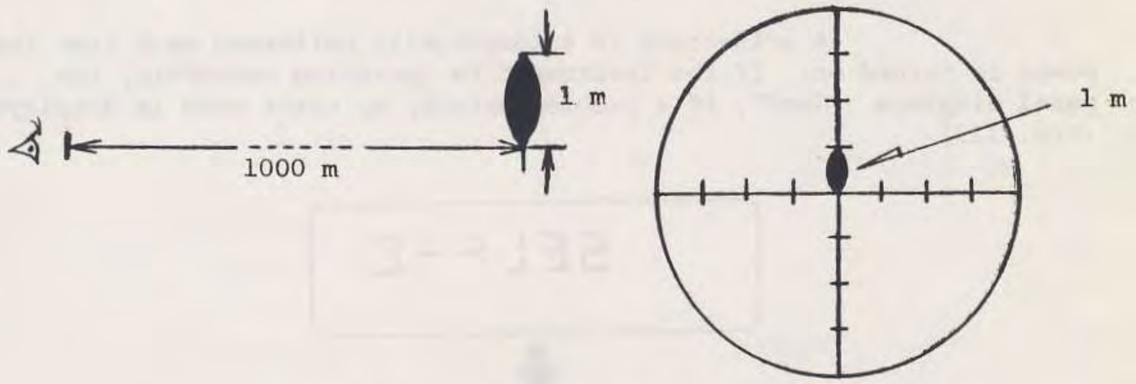


Fig. 122 Distance by binoculars.

(5) Distance by radar

We can accurately measure the distance, and fix our position by radar. The distance of isolated landmarks, such as an island elevated high above the sea level, can be easily measured, however, the distance of a mountain top, or a gently sloping cape are difficult to measure by radar. Therefore, your judgment, inference and reading of echo on a radar scope will be very important. A thorought understanding of the characteristics and limitations of a radar are most advantageous.

(6) Distance by Theodolite distance meter according to Nippon KOGAKU K.K. (NIKON)

With electronic devices, we can measure distance, according to the elapsed time between transmission of light and the return of its reflection.

Distance measurement

1. Upright Digital Display

Even if the telescope is inverted, the liquid crystal readout always displays digits upright for quick, error free reading.

2. Automatic Self-diagnosis

A self-check is automatically performed each time the power is turned on. If the instrument is operating correctly, the panel displays "Good", if a problem exists, an error code is displayed (Fig. 123)

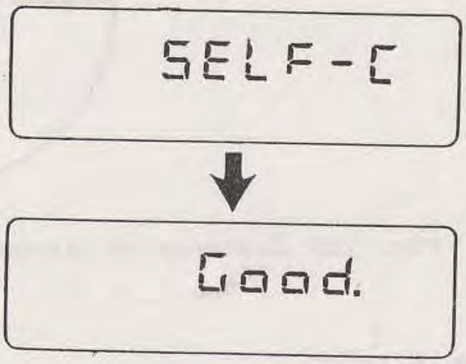


Fig. 123 Reading of distance meter.

3. Automatic Environmental Compensation

This function eliminates the need for a conversion chart. Before measuring, simply key-in the values for the barometric pressure (mm Hg or inches Hg) and temperature (^{o}C or ^{o}F). The instrument then automatically compensates for these environmental factors before displaying the distance.

Temperature range :	$-25^{o}C \sim 55^{o}C$ ($-13^{o}F \sim 131^{o}F$)
Pressure range:	400 mm Hg ~ 999 mm Hg (15.7 in Hg ~ 39.3 in Hg)

4. Automatic Curvature and Refraction

When the appropriate internal switch is set, the instrument automatically compensates for the earth's curvature and differences in air density.

5. Permanent Prism Constant Storage

Even when the main switch is turned off, the present prism constant is automatically memorized until a new prism constant is entered. The prism offset correction range is ± 999 mm.

6. Audible and Digital Target Acquisition

When the light reflected by a prism is received, a buzzer sounds and the numerical value of the returned signal's strength is displayed on the LCD board.

As a result, fine adjustment during target sighting is extremely easy. At short distances, merely point the telescope at the target prism center. When reflected light is weak, the panel displays "Lθ" to indicate that measurement is not possible (Fig. 124)

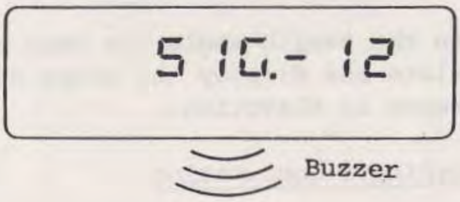


Fig. 124 Reading of distance meter.

7. Automatic Signal Level Control

The amount of reflected light is automatically attenuated after target acquisition, allowing measurement to begin immediately.

8. Distance Measurement Operation Display

At the start of distance measurement, the display's liquid crystal lights up and travels from right to left to indicate that the distance measurement function is operating (Fig. 125)

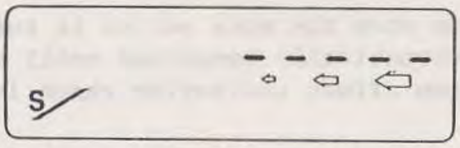


Fig. 125 Reading of distance meter.

9. Automatic Erroneous-Data Detection

When the spread of the fine readings exceeds a predetermined operating range because of haze, stray light, or any other reason, the microprocessor automatically detects and deletes the erroneous data to ensure accuracy.

10. Automatic Slope Reduction

When the zenith angle has been entered, the keyboard can be used to calculate and display the slope distance, horizontal distance, and difference in elevation.

11. Coordinate Computation

After distance measurement, the panel displays the target point's coordinates (relative to the coordinates of the instrument station).

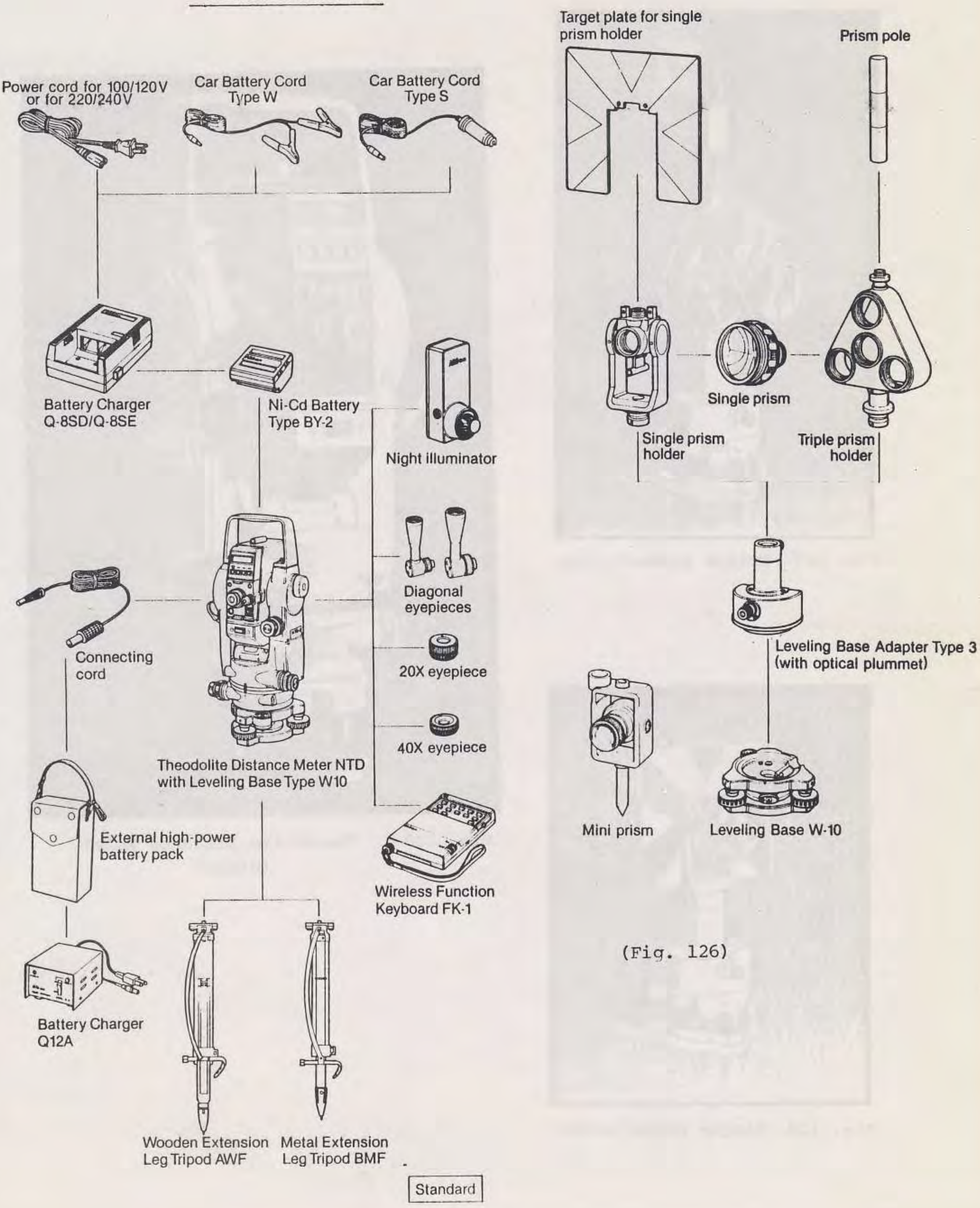
The X.Y.Z. coordinates are calculated from the previously entered horizontal and zenith angles. Either local rectangular coordinates or grid coordinates can be used (easting, northing).

Coordinates settings can range from

- 9999.999 m to + 9999.999 m or from

- 9999.99 ft. to + 9999.99 ft.

SYSTEM DIAGRAMS



(Fig. 126)

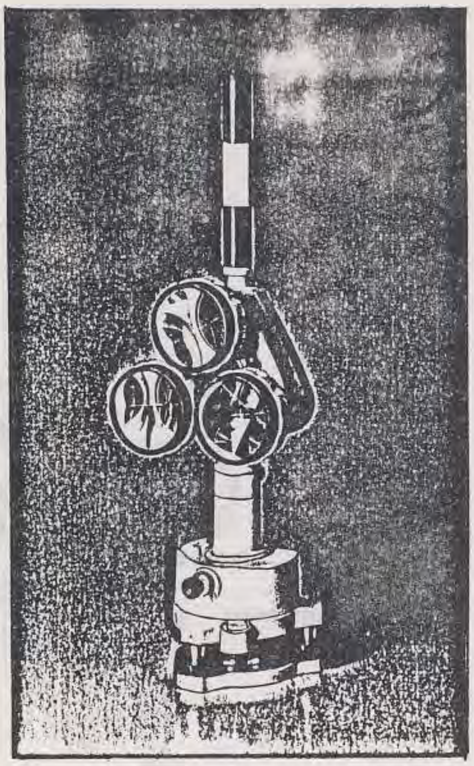


Fig. 127 Triple prism holder

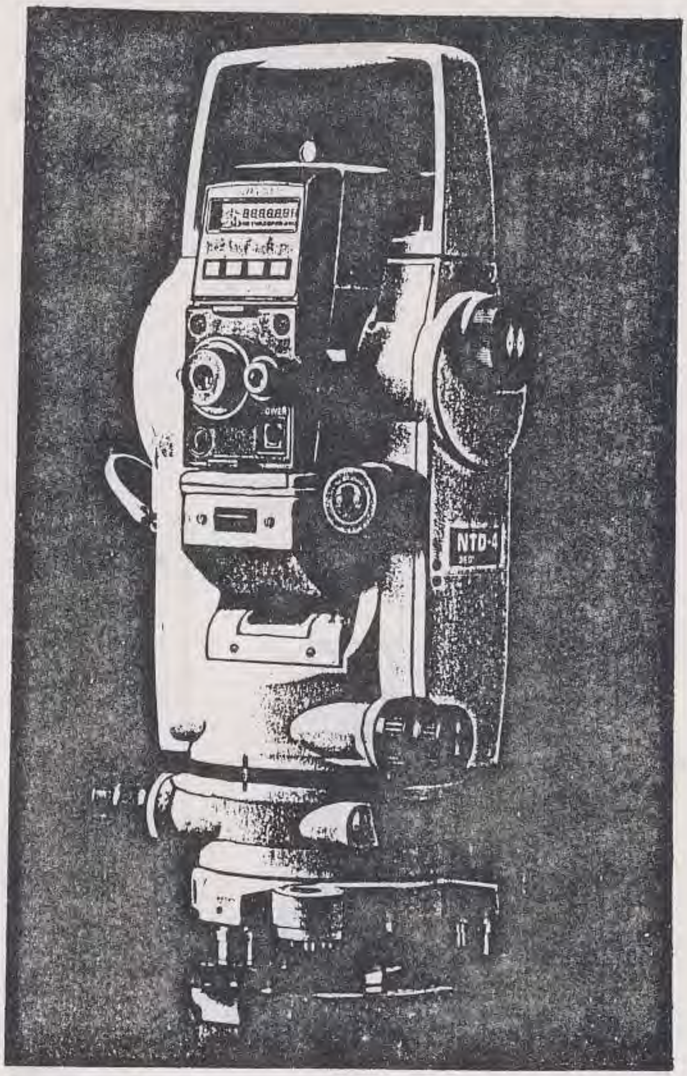
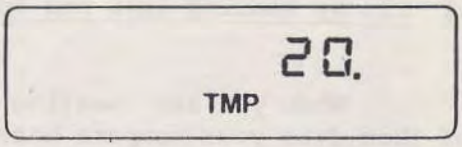


Fig. 128-1 Theodolite Distance Meter (Nikon)

Fig. 128 Single prism holder



20° input (temperature)

Fig. 131

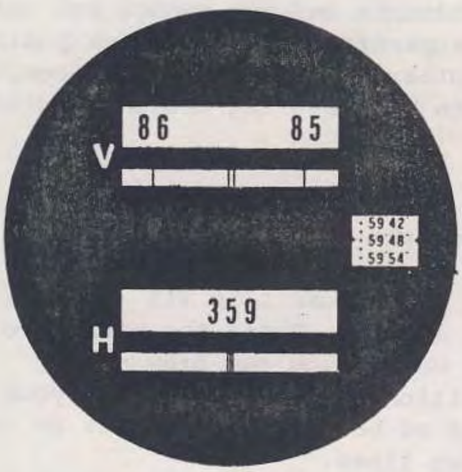


Fig. 129 Horizontal angle
= 359° - 59′ - 48″

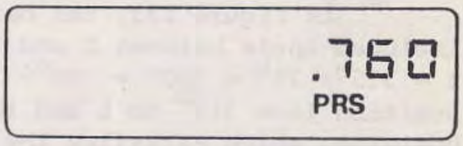
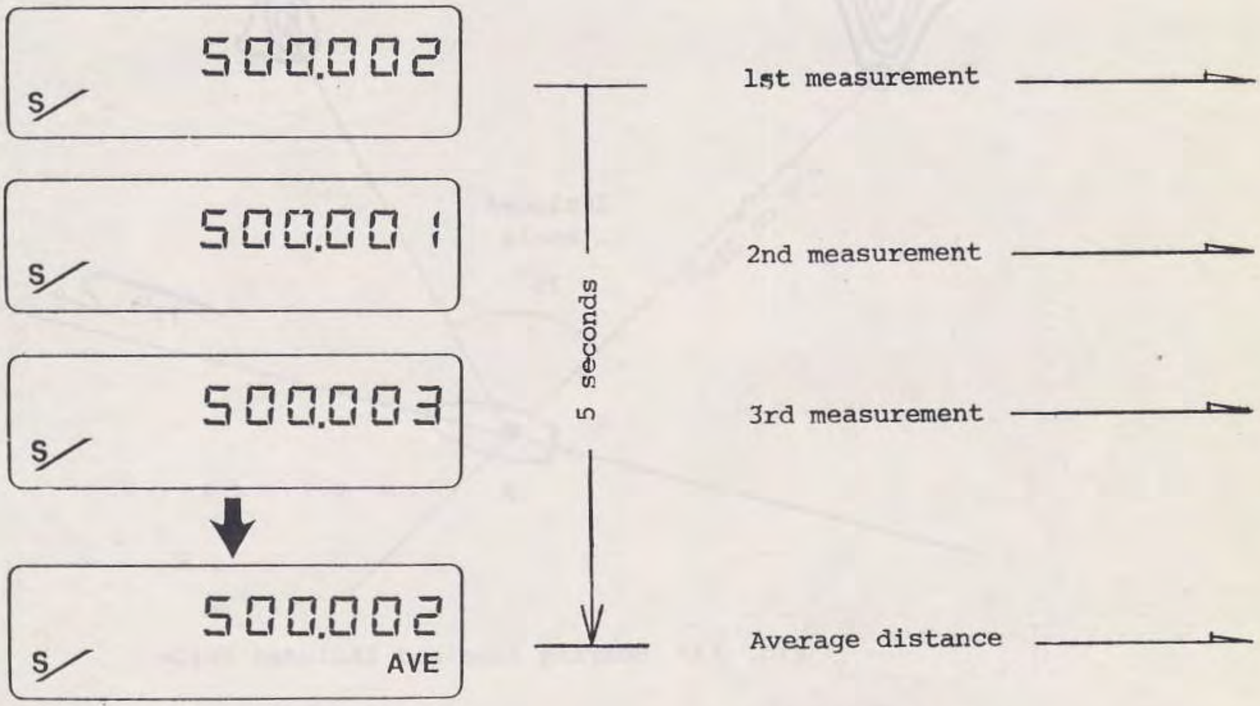


Fig. 132 760 mm Hg input
(atmospheric pressure)



500.002 metres

Fig. 130 Readings of distance meter.

* Please see page 269 as well

[2] Fix by Bearing Line and Included Angle

When you are coasting near two landmarks and you cannot see one of them from your compass because of the superstructures (derrick posts, masts, funnel etc.), yet you can see the other landmark from your compass, it is possible for you to fix your position by a bearing and the included angle of two landmarks.

First, you observe the bearing of the visible landmark, secondly you observe the included angle of two landmarks (Figs. 133 and 134)

In Figure 133, the bearing of the lighthouse L is 315° and the included angle between L and the island I is 75°. Then, the bearing of $I = 315^{\circ} + 75^{\circ} = 390^{\circ} = 390^{\circ} - 360^{\circ} = 30^{\circ}$. Hence, you can draw one position line 315° to L and the other position line 30° to I. So your position, which satisfies the requirements of being on both lines at the same time, is the intersection P of the two lines.

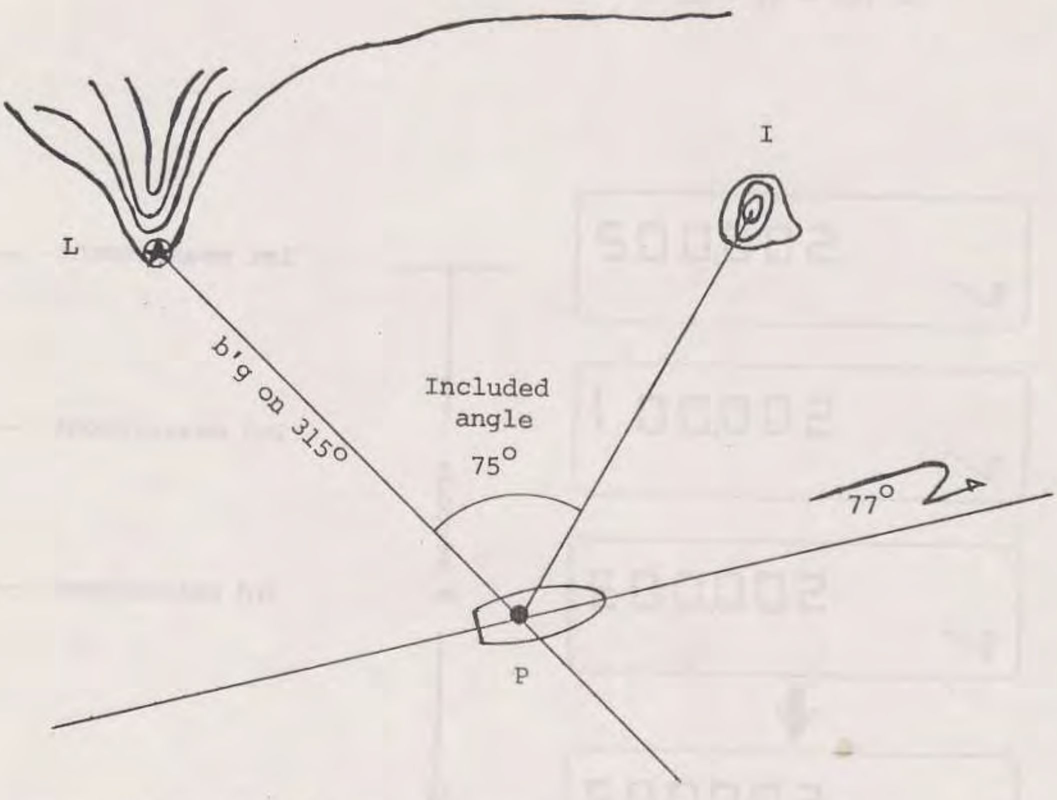


Fig. 133 Bearing line and included angle.

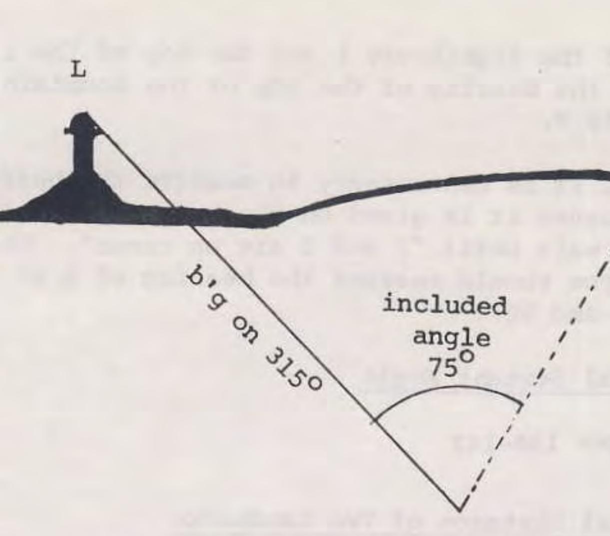


Fig. 134 Bearing line and included angle.

You cannot observe I from your compass, but you can measure the included angle from a place near the compass by a sextant.

[3] Fix by Transit and Angle (or bearing)

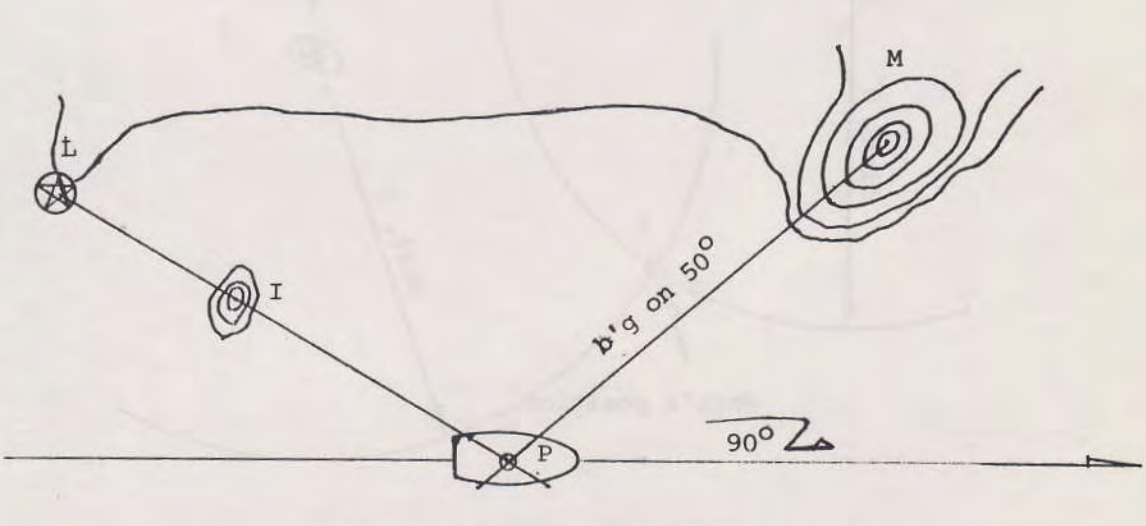


Fig. 135 Transit and angle.

In Figure 135 the lighthouse L and the top of the island I are in range of 300° and the bearing of the top of the mountain M is 50°, the ship's position is P.

In this case, it is unnecessary to measure the bearing I on range by compass, because it is given on the marine chart (Fig. 135). During coasting, you wait until "I and L are on range". When L and I are on range, you should measure the bearing of M at that instant (refer to Figures 95 and 96).

[4] Fix by Horizontal Sextant Angle

Refer to pages 138-142

[5] Fix by Horizontal Distance of Two Landmarks

Refer to Fig. 94, on page 130

If we know the distance to two or more landmarks, the intersection of equidistant circles with the landmarks as their centers is our position.

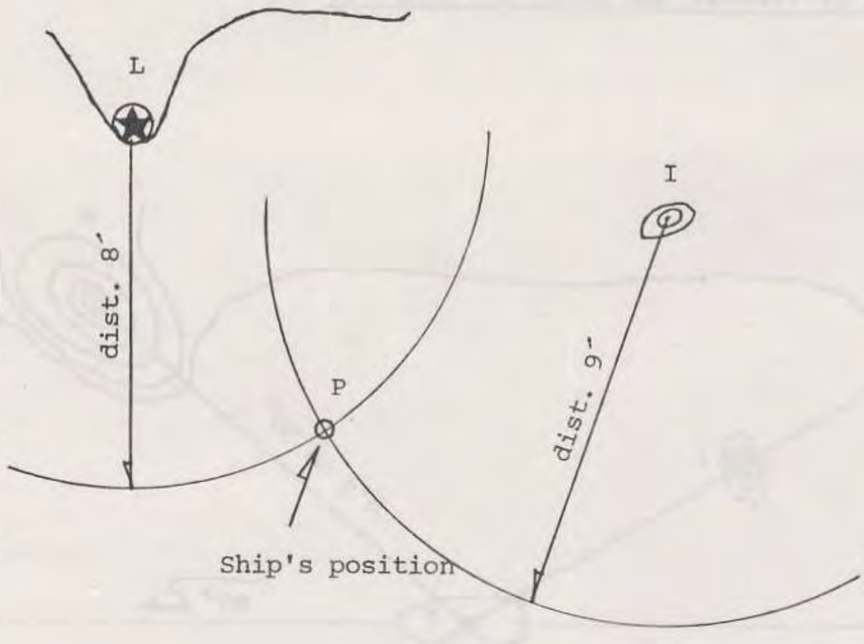
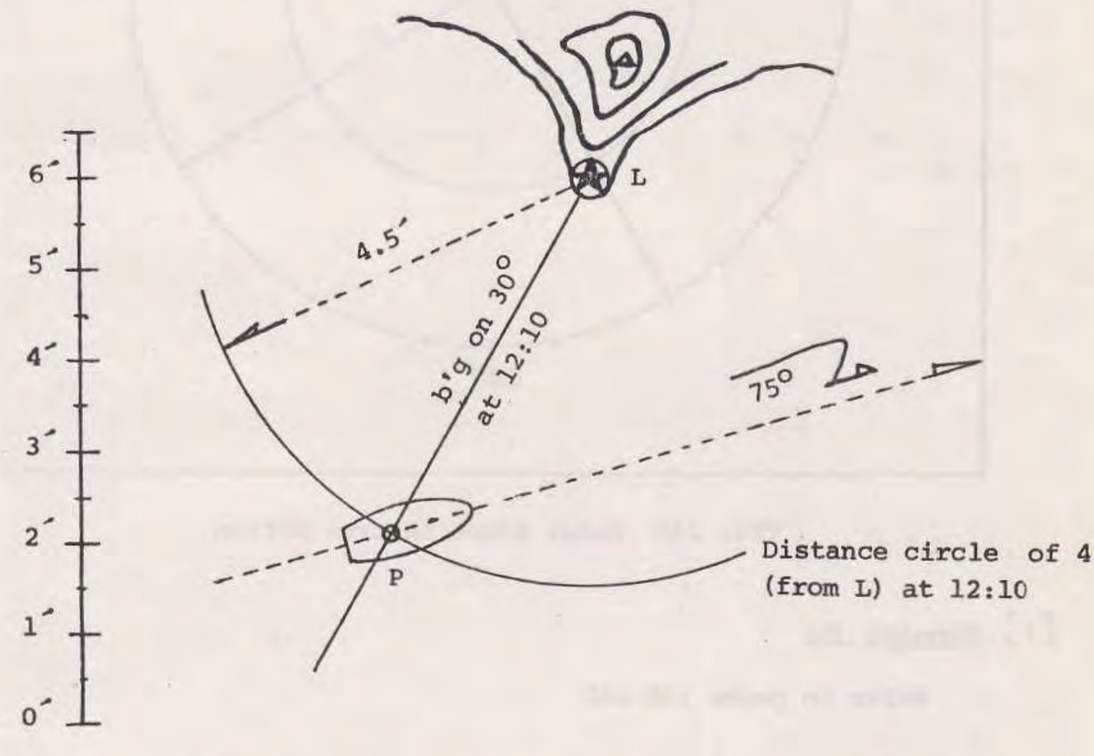


Fig. 136 Circles of position.

In Figure 136 distance from the lighthouse L is 8 nautical miles and distance from the island I is 9 nautical miles. If you draw a circle with radius 8' and the lighthouse as its center, and another circle with radius 9' and the island I as its center, then the intersection of the two circles is your position. How to obtain the distance is explained on page 150.

[6] Fix by a Bearing and a Distance Circle from a landmark

When you can measure the bearing and the distance of a landmark simultaneously, you can fix your position (refer to Figures 95 and 137)



The intersection P of the distance circle of 4.5′ at 12:10 and the bearing line 30° to L at 12:10 is your position at 12:10

Fig. 137 Bearing and distance circle

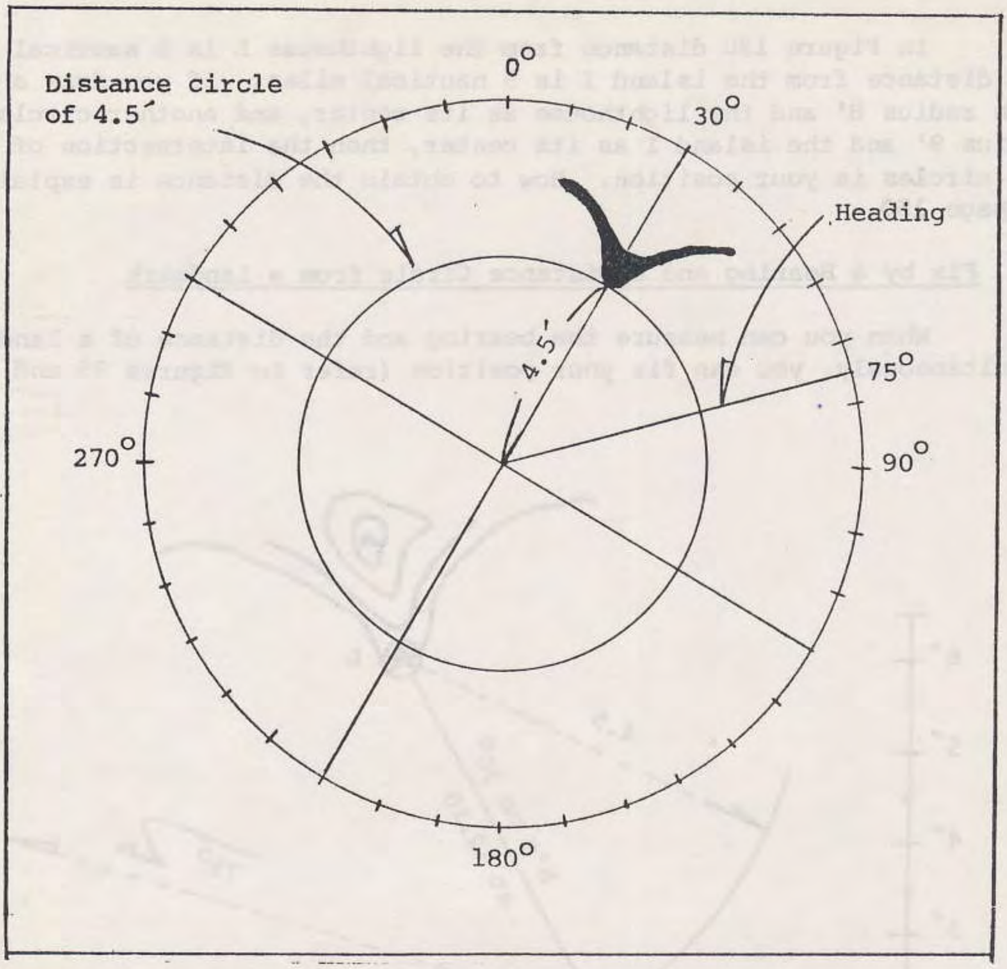


Fig. 138 Radar scope in true motion

[7] Running Fix

Refer to pages 138-142

[8] Fix by Doubling the Angle on the Bow

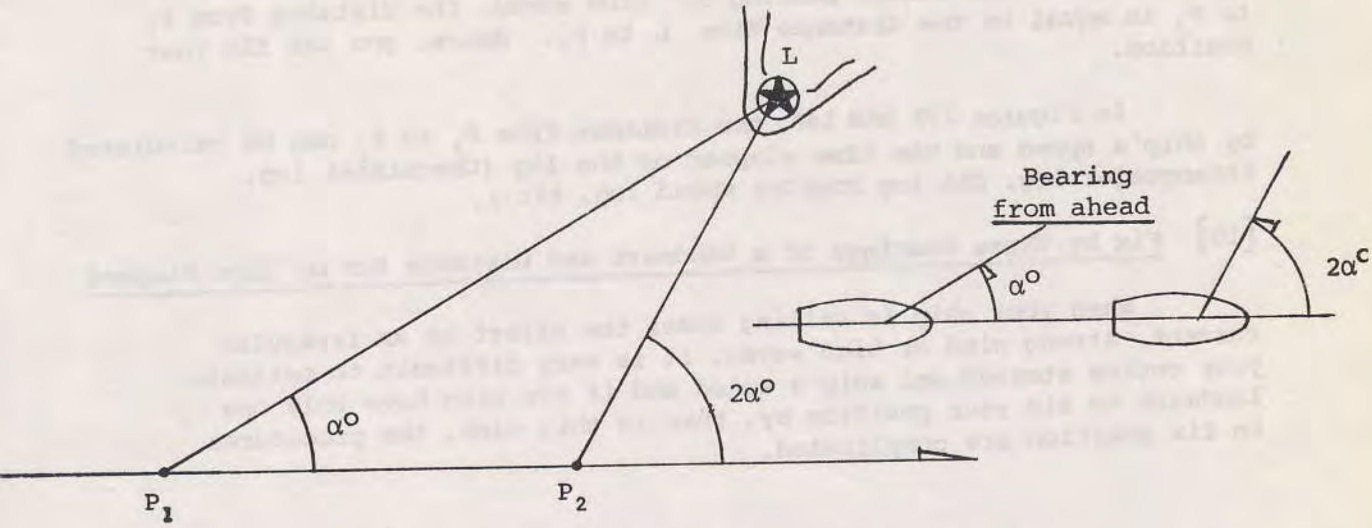


Fig. 139 Doubling the angle on the bow.

In Figure 139, the ship is proceeding from P_1 to P_2 and you can see only the lighthouse L. If you measure the first bearing α° from ahead at P_1, and the second bearing 2α° from ahead at P_2, the distance run from P_1 to P_2 is equal to the distance from L (the lighthouse) to P_2 (your position) because Δ L P_1 P_2 is the equilateral triangle on condition that your ship's course has been steered steadily and is unaffected by current.

You can observe the angle α° and 2α° by lubber's point (or line) of compass bowl.

[9] Fix by Four-point Method

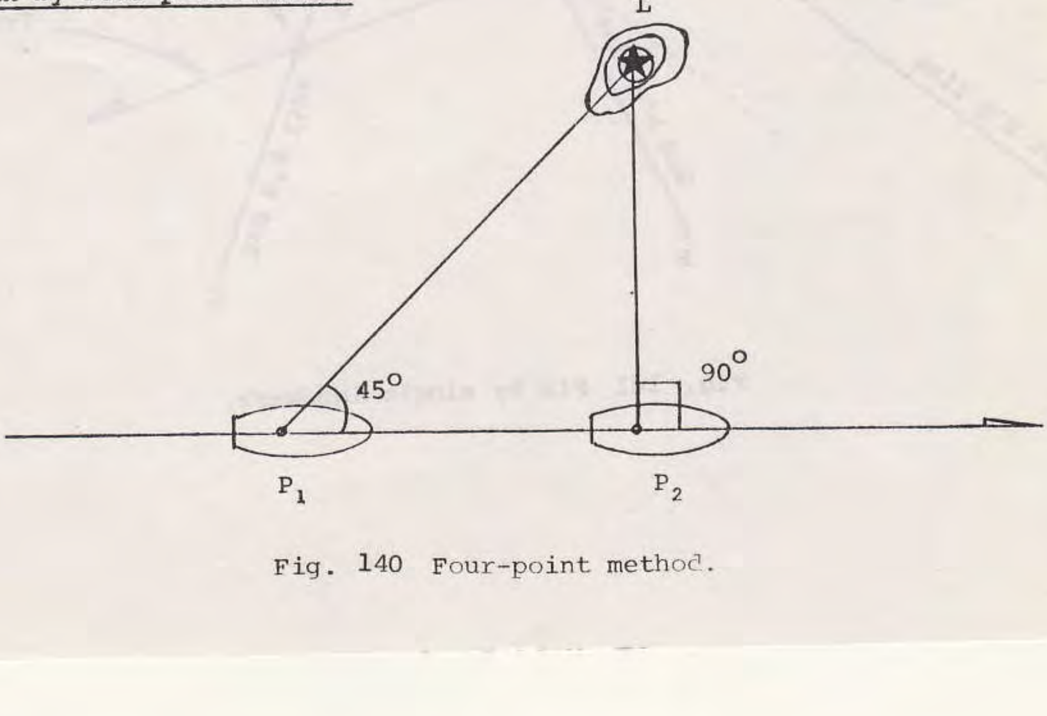


Fig. 140 Four-point method.

In Figure 140, you measure the first bearing 45° from ahead at P_1 and measure the second bearing 90° from ahead, the distance from P_1 to P_2 is equal to the distance from L to P_2. Hence, you can fix your position.

In Figures 139 and 140, the distance from P_1 to P_2 can be calculated by ship's speed and the time elapsed or the log (Chermikeef log, Strangmeier log, SAL log Doppler speed log, etc.).

[10] Fix by Three Bearings of a Landmark and Distance Run or Time Elapsed

When your ship is sailing under the effect of an irregular current, strong wind or high waves, it is very difficult to estimate your course steered and ship's speed and if you also have only one landmark to fix your position by, then in this case, the procedures to fix position are complicated.

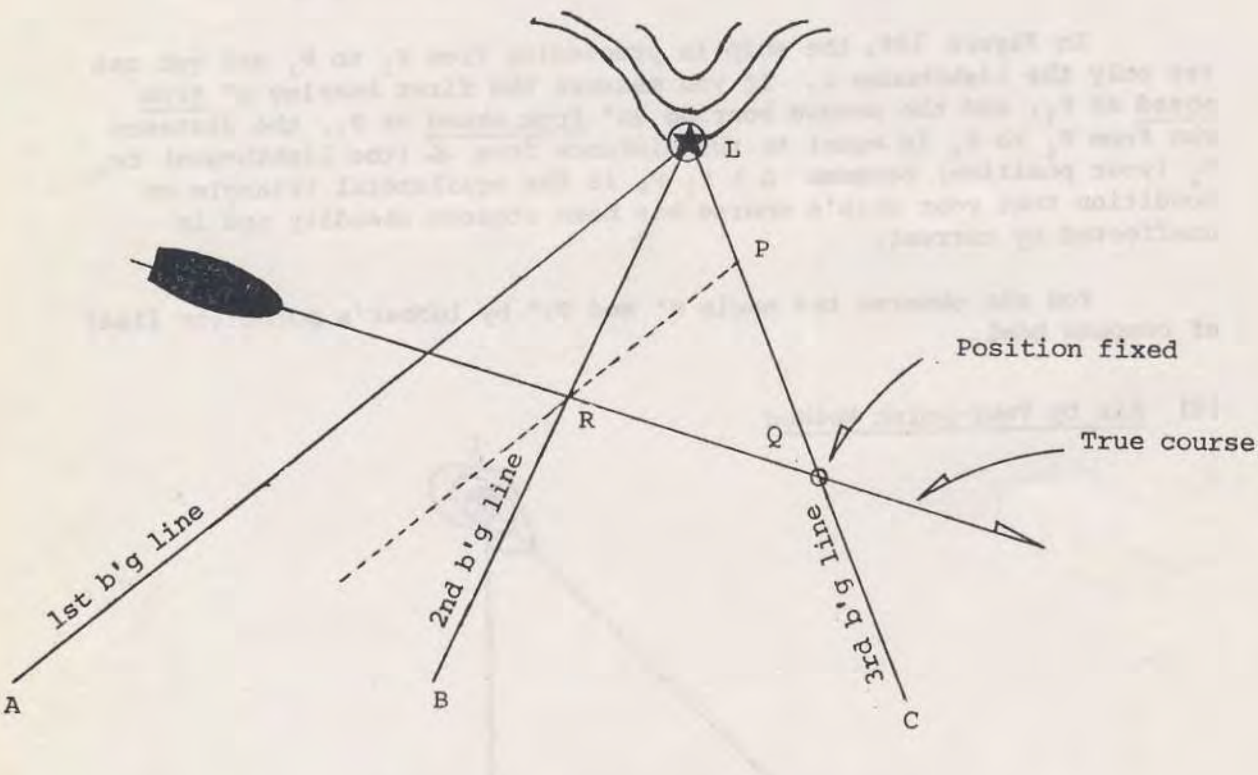


Fig. 141 Fix by single landmark.

(1) In Figure 141, AL is the first bearing line, BL is the second bearing line and CL is the third bearing line to L (the lighthouse).

$\overline{LP}$ is equal to the distance run from the 1st bearing line to the second bearing line, and $\overline{PQ}$ is equal to the distance run from the 2nd bearing line to the third bearing line.

Draw a straight line which is parallel with $\overline{AL}$ through P. The point R is the intersection of this straight line ($\overline{PR}$) and $\overline{BL}$ (the second bearing line). Connect R with Q by a straight line ($\overline{RQ}$). Then $\overline{RQ}$ is the ship's true course and point Q is your position when you observed the third bearing to the lighthouse.

(2) Provided that the distance run is directly proportional to the time elapsed, you can obtain a parallel line to the true course by using the time elapsed from the first bearing to the second bearing and from the second bearing to the third bearing without knowing the distance run.

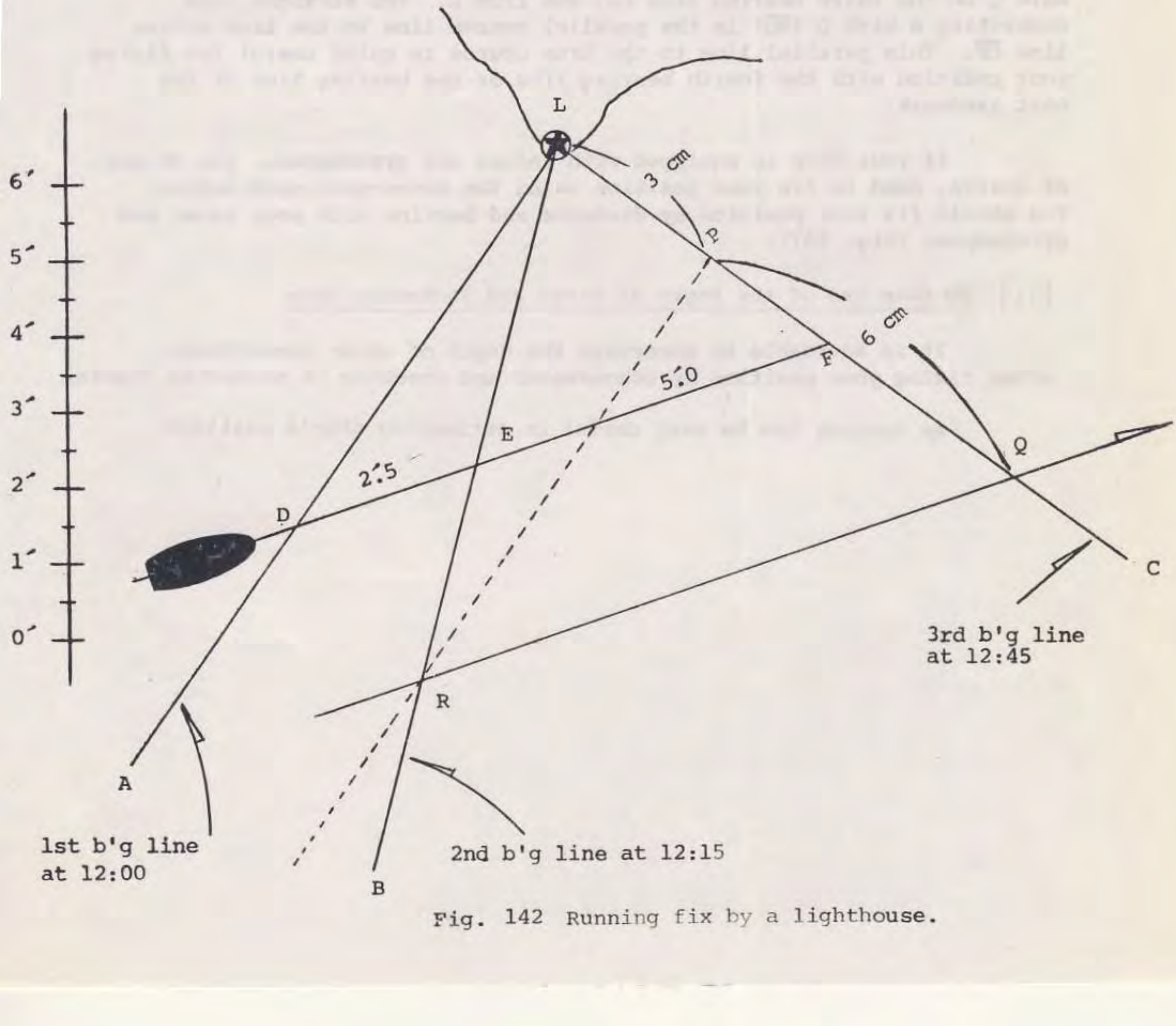


Fig. 142 Running fix by a lighthouse.

In Figure 142, your ship is proceeding from a real point A to C through B at a speed of 10 nautical miles per hour, and $\overline{AB}$ is its true course. But suppose that you do not know your ship's speed and its true course e.g.

At 12:00 you observed the first bearing of the lighthouse (L) as $\overline{AL}$, the second bearing as $\overline{BL}$ at 12:15 and the third bearing as $\overline{CL}$ at 12:45.

The elapsed time between the first and the second bearing is 15 minutes, the elapsed time between the second bearing and the third bearing is 30 minutes.

The ratio of 15 minutes to 30 minutes is the ratio 15:30 = 1:2, put the mark P on the third bearing line $\overline{CL}$, 3cm from L, and draw a straight line parallel to the first bearing line $\overline{AL}$ through P. This parallel line intersects with the second bearing line $\overline{BL}$ at point R. Next, put the mark Q on the third bearing line $\overline{CL}$, 6cm from L. The straight line connecting R with Q ($\overline{RQ}$) is the parallel course line to the true course line $\overline{DF}$. This parallel line to the true course is quite useful for fixing your position with the fourth bearing line or the bearing line of the next landmark.

If your ship is equipped with radars and gyrocompass, you do not, of course, need to fix your position using the above-mentioned method. You should fix your position by distance and bearing with your radar and gyrocompass (Fig. 137).

[11] To Make Use of the Depth of Water and Isobathic Data

It is advisable to ascertain the depth of water immediately after fixing your position by echosounder and checking it on marine charts.

An isobath can be very useful in estimating ship's position.

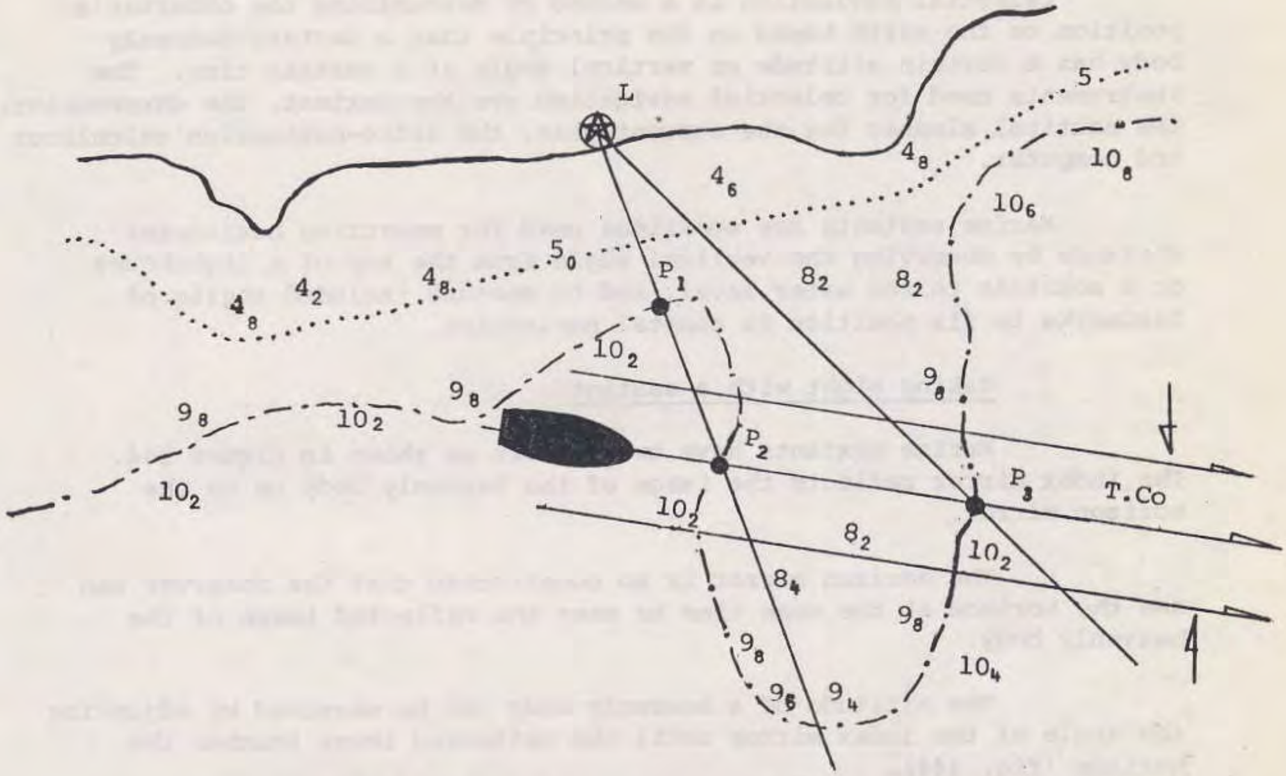


Fig. 143 Fix by a lighthouse and a contourline.

In Figure 143, when you observed the first bearing P.L or P_2L to the lighthouse L, the depth of water was 10 metres. In this case, the probable position was P_1 or P_2. Then you observed the second bearing P_3L to the lighthouse L and the depth of water was 10 metres. In this case, your position should be P_3, so your position when you observed the first bearing P_1L (or P_2L) should be P_2 because of the course line, the bearings and the isobath of 10 metres.

Marine Sextants

Celestial navigation is a method of determining the observer's position on the earth based on the principle that a certain heavenly body has a certain altitude or vertical angle at a certain time. The instruments used for celestial navigation are the sextant, the chronometer, the nautical almanac for the current year, the astro-navigation calculator and computer.

Marine sextants are sometimes used for measuring horizontal distance by observing the vertical angle from the top of a lighthouse or a mountain to the water level, and to measure included angles of landmarks to fix position in coastal navigation.

Taking sight with a sextant.

Marine sextants have two mirrors as shown in Figure 144. The index mirror reflects the image of the heavenly body on to the horizon mirror.

The horizon mirror is so constructed that the observer can see the horizon at the same time he sees the reflected image of the heavenly body.

The altitude of a heavenly body can be measured by adjusting the angle of the index mirror until the reflected image touches the horizon (Fig. 144).

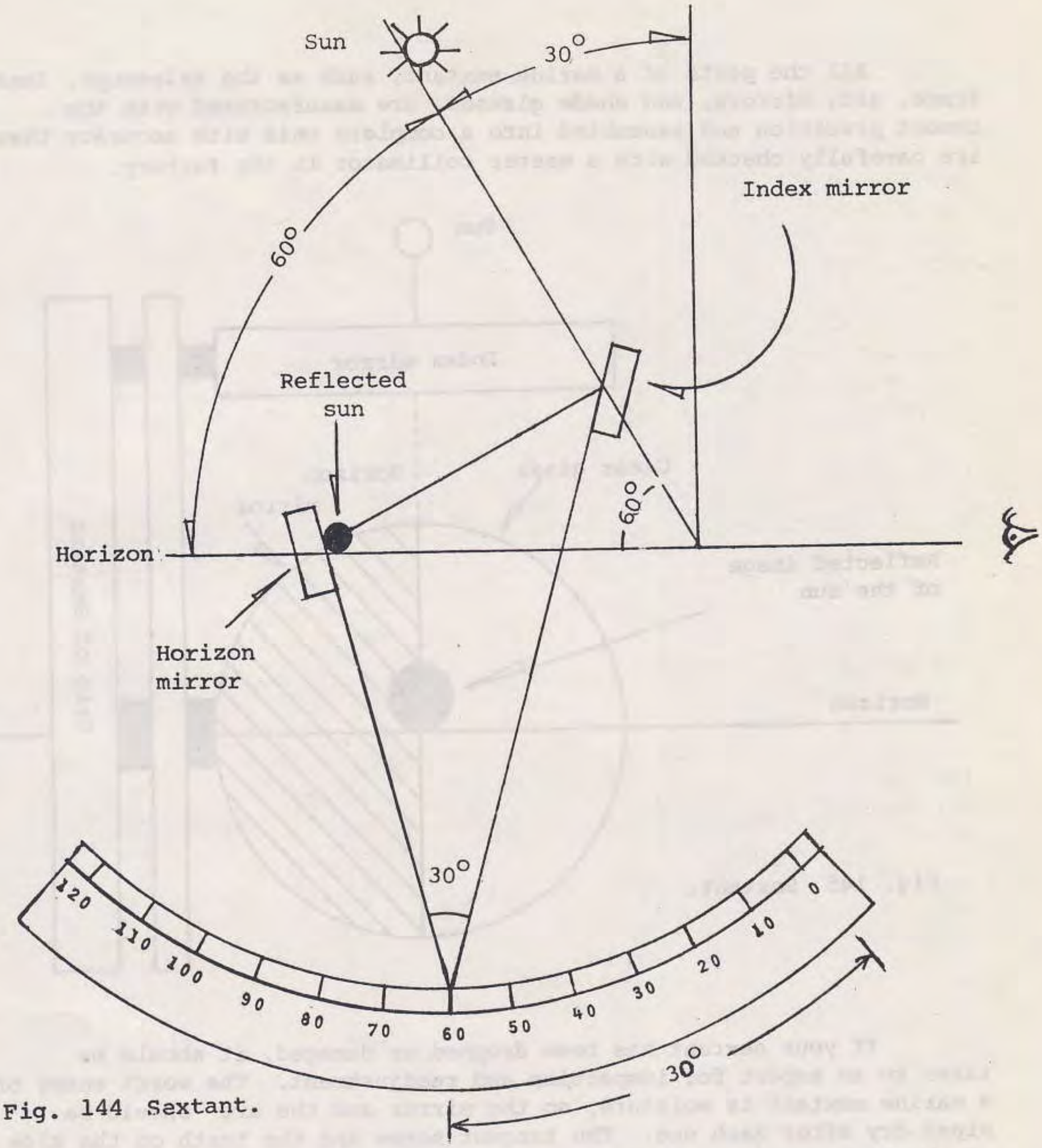


Fig. 144 Sextant.

With a marine sextant, the altitude is read in degrees, minutes and 1/10 minutes. One minute of the sextant reading represents one nautical mile. Marine sextants have been used for a long time because they are very useful for fixing position at sea in both coastal and celestial navigation.

All the parts of a marine sextant, such as the telescope, lens, frame, arc, mirrors, and shade glasses are manufactured with the utmost precision and assembled into a complete unit with accuracy they are carefully checked with a master collimator in the factory.

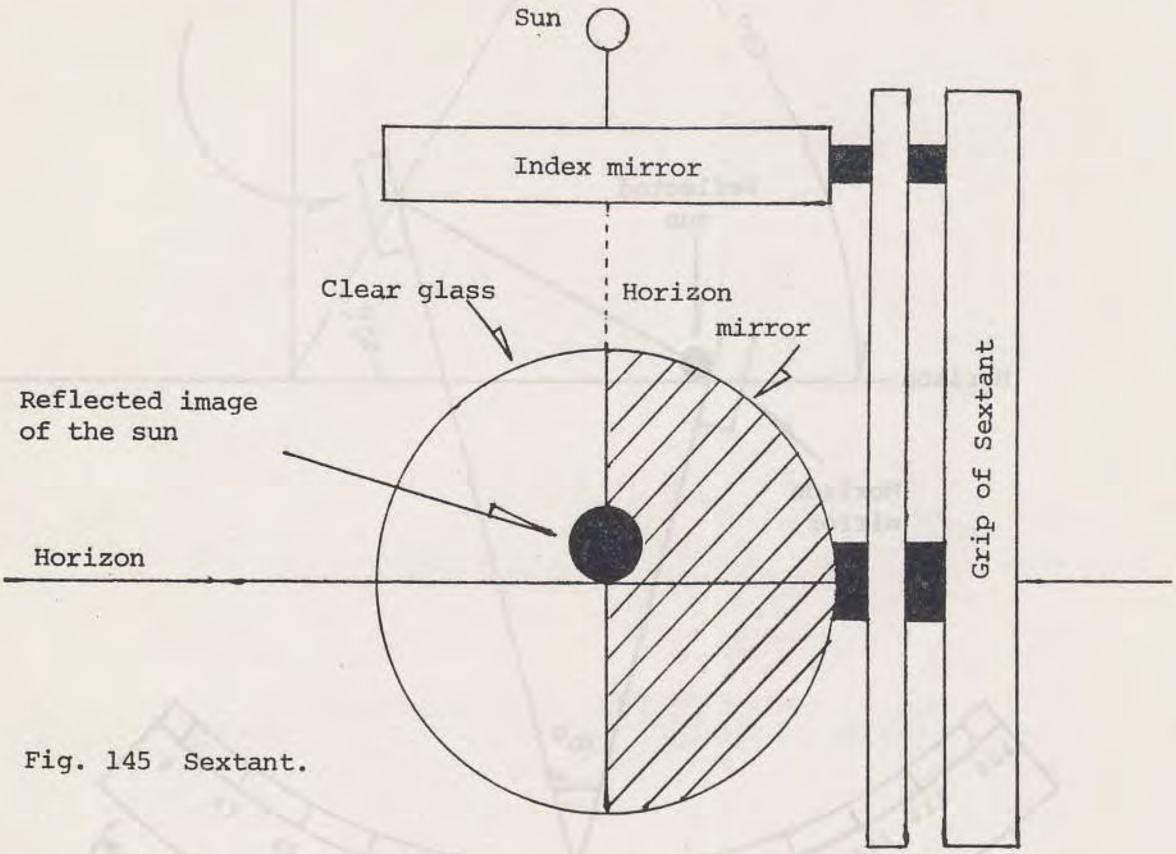


Fig. 145 Sextant.

If your sextant has been dropped or damaged, it should be taken to an expert for inspection and readjustment. The worst enemy of a marine sextant is moisture, so the mirror and the arc should be wiped dry after each use. The tangent screw and the teeth on the side of the limb should be kept clean and lightly oiled.

Study the relationship of mirrors, a heavenly body and the angle on the arc in Figures 144-147.

The construction of a marine sextant

In Figures 146 and 147:

1. Frame
2. Arc
3. Index arm
4. Grip (or Handle)
5. Telescope
6. Release
7. Micrometer drum
8. Vernier
9. Light bulb
10. Horizon shade glasses
11. Horizon mirror
12. Shade glasses of index mirror
13. Index mirror
14. Limb
15. Switch for light bulb
16. Leg-frame
17. Slide-and-lock rising piece

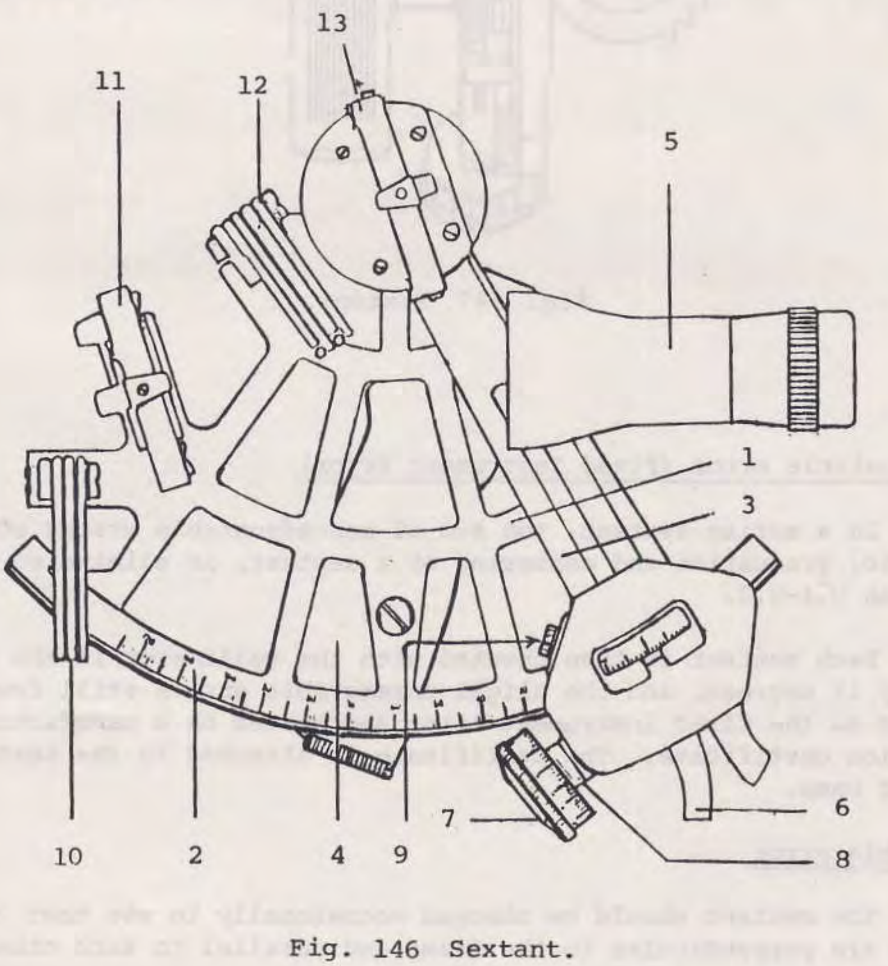


Fig. 146 Sextant.

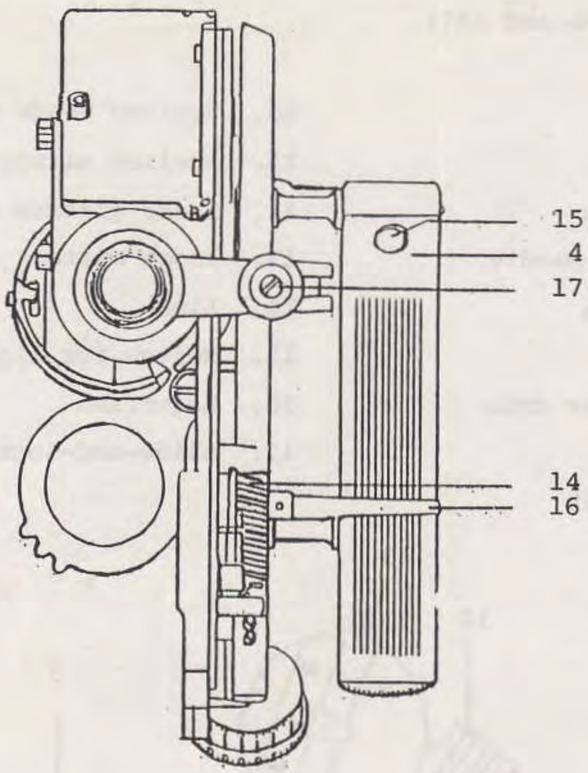


Fig. 147 Sextant.

Non-adjustable error (Fixed Instrument Error)

In a marine sextant, the sum of non-adjustable errors such as prismatic, graduation and centering of a sextant, is eliminated to less than $0'.1$-$0'.3$.

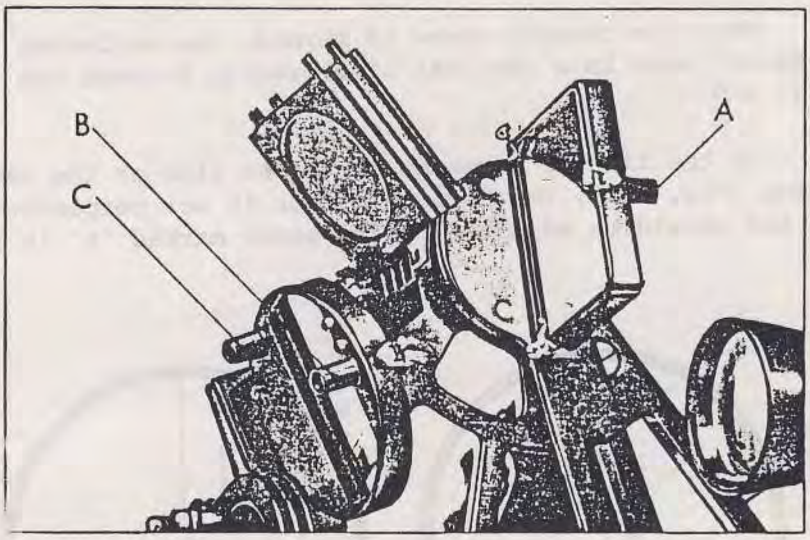
Each sextant is then checked with the collimator in the factory at every 15 degrees, and the slight unremovable errors still found are regarded as the fixed instrument error and stated on a manufacturer's inspection certificate. The certificate is attached to the sextant's carrying case.

Adjustable error

The sextant should be checked occasionally to see that the mirrors are perpendicular to the frame and parallel to each other. If misalignment is found, adjustment is necessary.

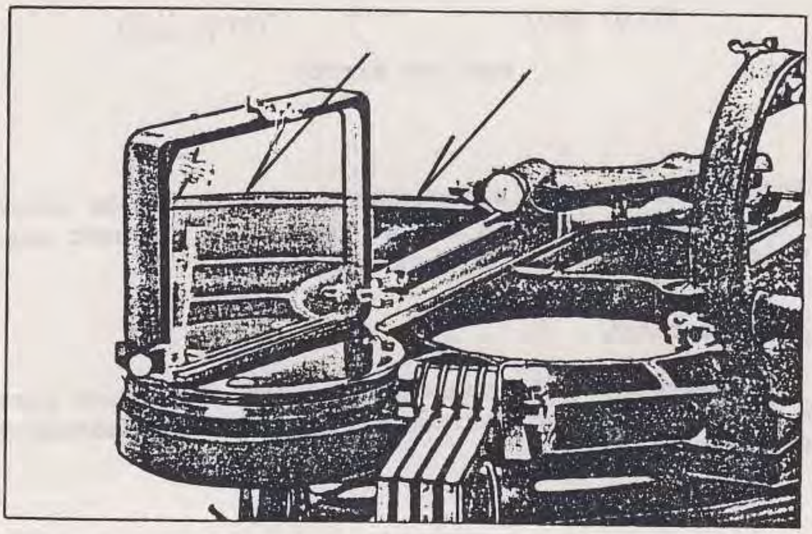
A. Perpendicularity of the index mirror

If the two lines of sight appear to be joined in a straight line, the mirror is perpendicular. If the line is not straight, it requires an adjustment of the screw marked "A" in Figure 148.



by courtesy of "TAMAYA Co., LTD"

(Fig. 148)



by courtesy of"TAMAYA Co., LTD."

(Fig. 149)

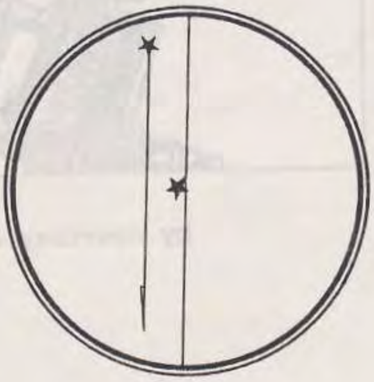
B. Side error

An error resulting from the horizon mirror not being perpendicular is called the side error. To make the adjustment, the sextant is sighted directly at a star at night with the index set at 0°.

When the tangent screw is turned, the reflected image of the star should move in a vertical line exactly through the direct image (Fig. 150)

If the line of movement is to one side or the other of the direct image (Fig. 151), the horizon mirror is not perpendicular to the frame and should be adjusted by the screw marked "B" in Figure 148.

(Fig. 150)



(Fig. 151)

Horizon mirror

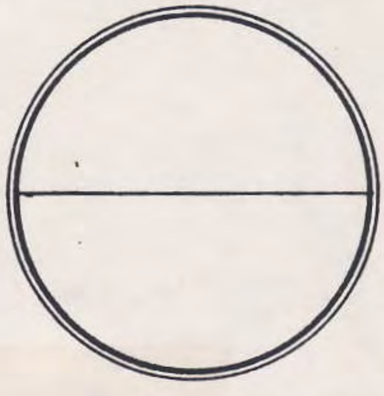
Frequent manipulation of the adjustment screws should be avoided, as it may cause excessive wear. A slight lack of adjustment has minimal effect on calculation and can be ignored.

C. Index error

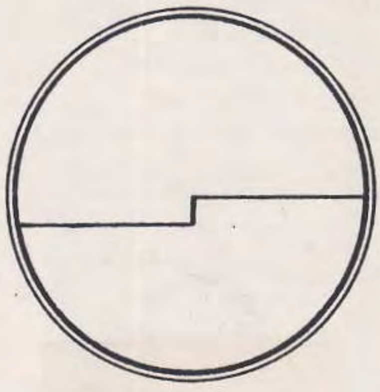
An index error is caused by a lack of perfect parallelism between the index mirror and horizon mirror when the sextant is set at 0°- 00.'0.

With sextants the horizon mirror is adjusted within the mirror frame. To eliminate or reduce excessive index error, the horizon mirror must be adjusted by the screw marked "C" in Figure 148.

If the index error is small (less than 4.0), it is advisable not to try to remove it. Instead subtract or add this error in your calculations.



Horizon in alignment



Horizon out of alignment
Index error present

Fig. 152

The reading of a sextant

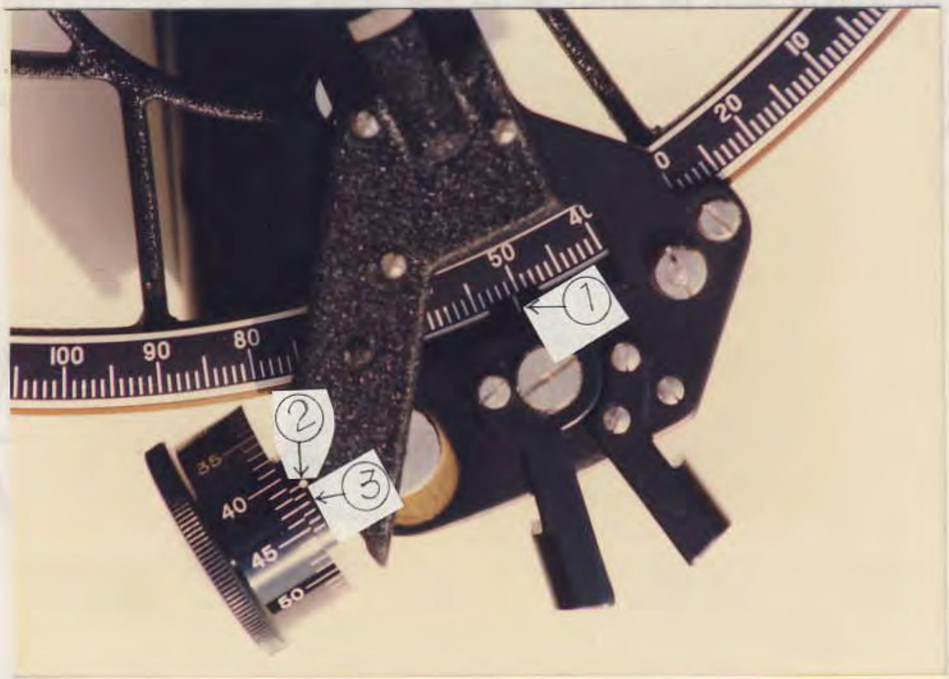


Fig. 152-1 The sextant reading is 50° 41.'4

In Fig. 152-1 above, the degree scales on the arc are read first.

The scale marked by ① (white line) shows the value of the reading as more than 50° and less than 51°. Next the minute scales marked by ②, should be read.

The minute scale shows the reading value as more than 41′ and less than 42′. Finally the tenth scales marked by ③ on the micrometer drum are checked. The scale on the micrometer drum that lies in a line with the scale on the minute scale drum shows 0.4 minutes.

In the case of Fig. 152-1, the 43 minutes scale lies in a line with the second tenth minute scale (second scale from upper most scale on the micrometer drum). So the reading of the tenth scale is 0.4 minutes, because the tenth scale has 5 intervals, and each interval shows 0.2 minutes. See Figs. 152-2 ~ 152-3.

Fig. 152-2 The sextant reading is 20° 08'.2

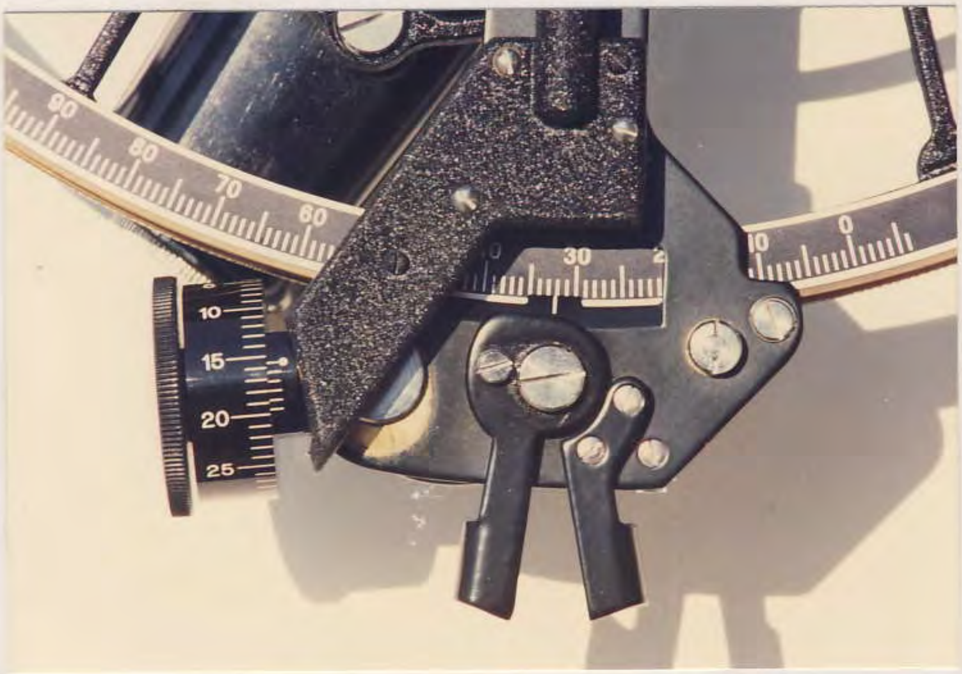


Fig. 152-3 The sextant reading is 32° 15'.6

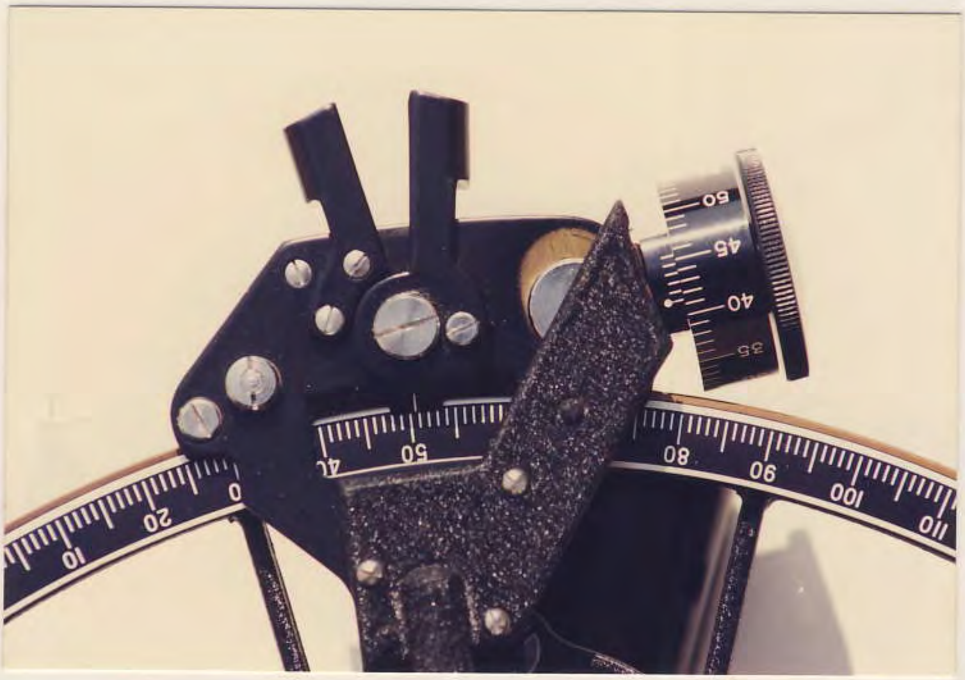


Fig. 152-4 Sextant reading = 50° 41'.4

(On arc)

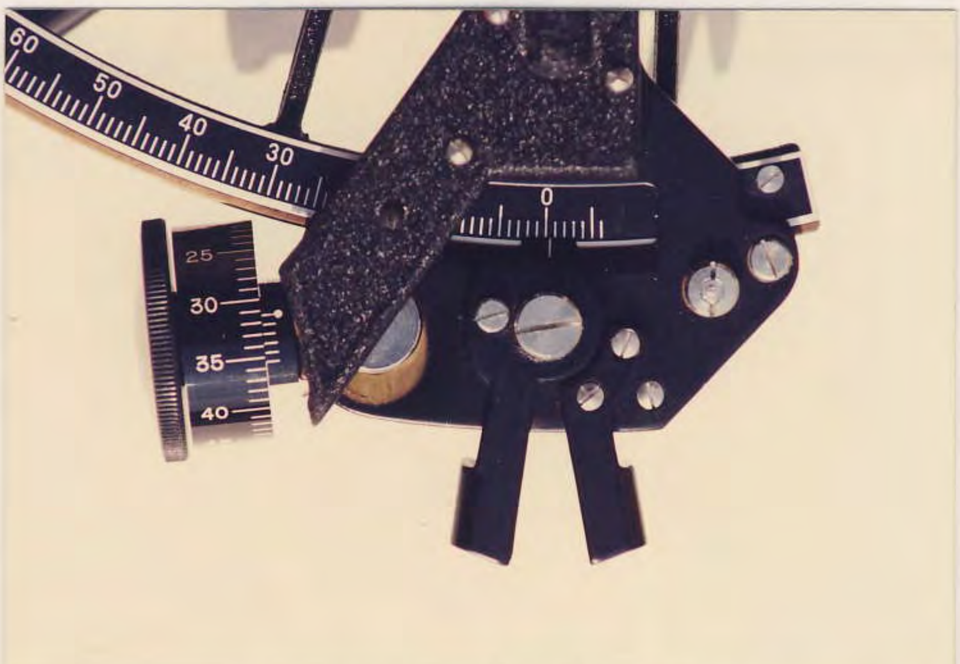


Fig. 152-5 Sextant reading = Θ 28'.6

Note: 60' - 31'.4 = 28'.6 (Off arc)

Navigational Triangle Rulers

How to use a Navigational Triangle Ruler

1. In the case of a landmark existing between your position and the visible horizon. (Scale of distance should be A, see Figs. 154-156).

Example

Included angle between the top of the lighthouse and sea level is 0°- 58' and the height of the top of lighthouse is 400 feet above sea level. What is the distance from the lighthouse to your position? (Ans: about 3.9 miles)

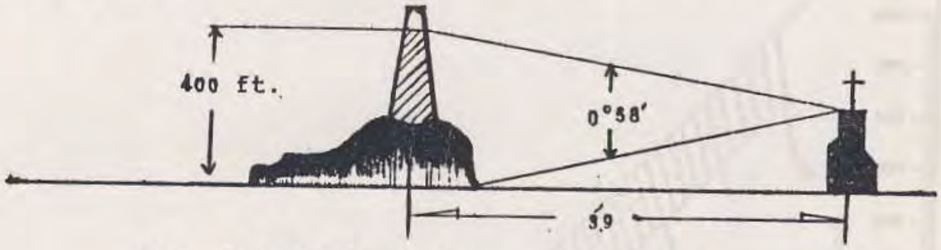


Fig. 153 Distance from a lighthouse.

Solution

The reference point of the triangle ruler should be on the longitudinal line of the nautical chart, and the division of 0°- 58' of scale of altitude should be on the same longitudinal line.

Move the triangle ruler parallel so that the division of 400 feet of scale of height of landmark is just on the latitudinal line. Then you can read the division 3.9 miles on the intersection of the latitudinal line and the scale of distance (A). (Fig. 156, 157 and 158).

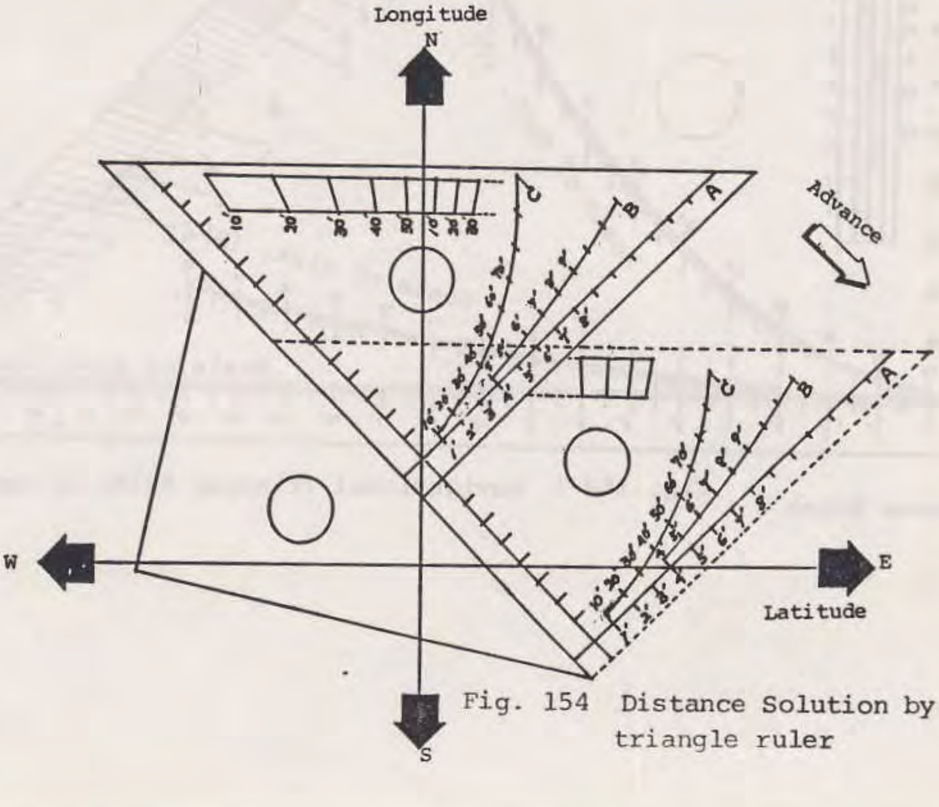


Fig. 154 Distance Solution by triangle ruler

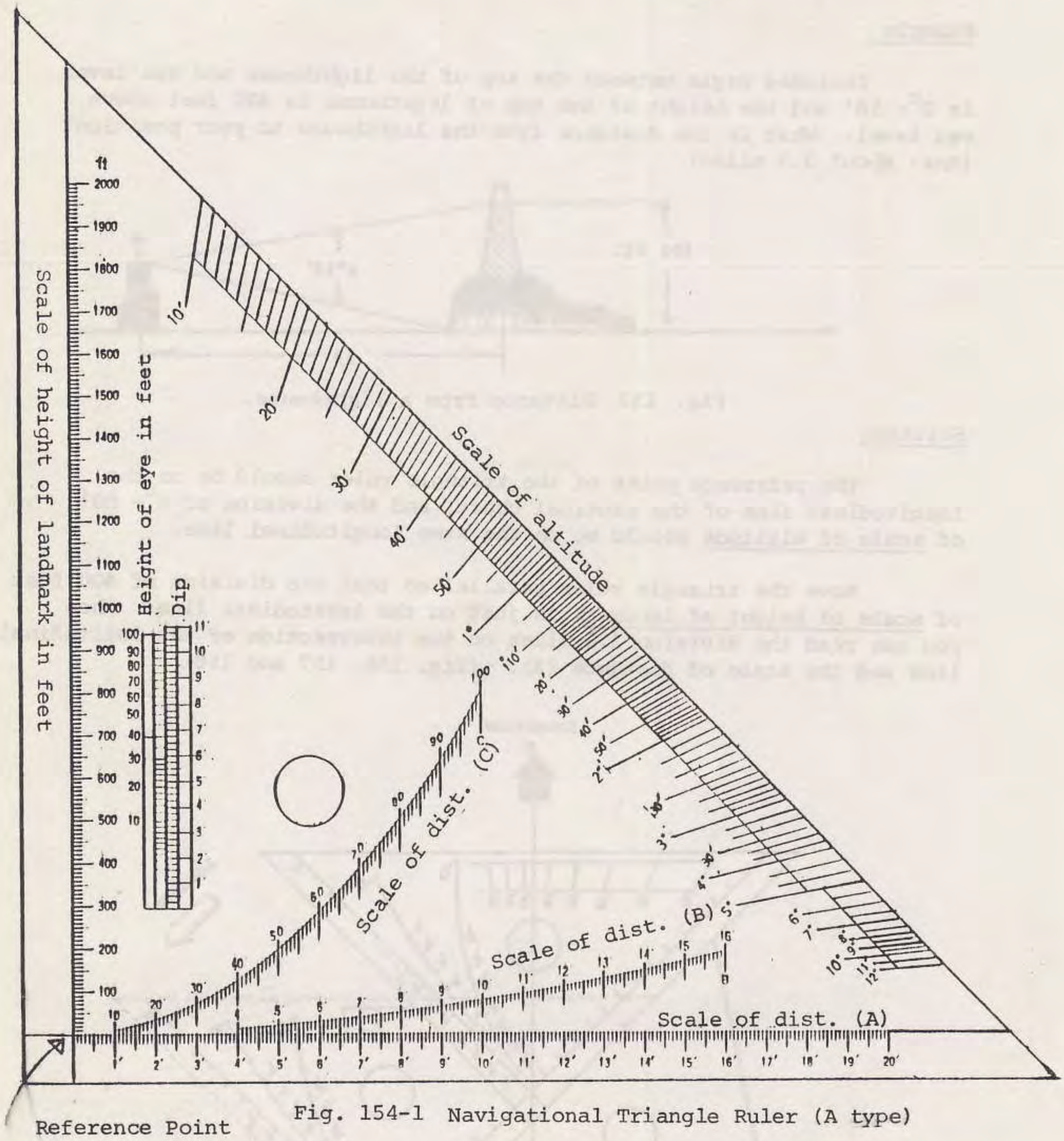


Fig. 154-1 Navigational Triangle Ruler (A type)

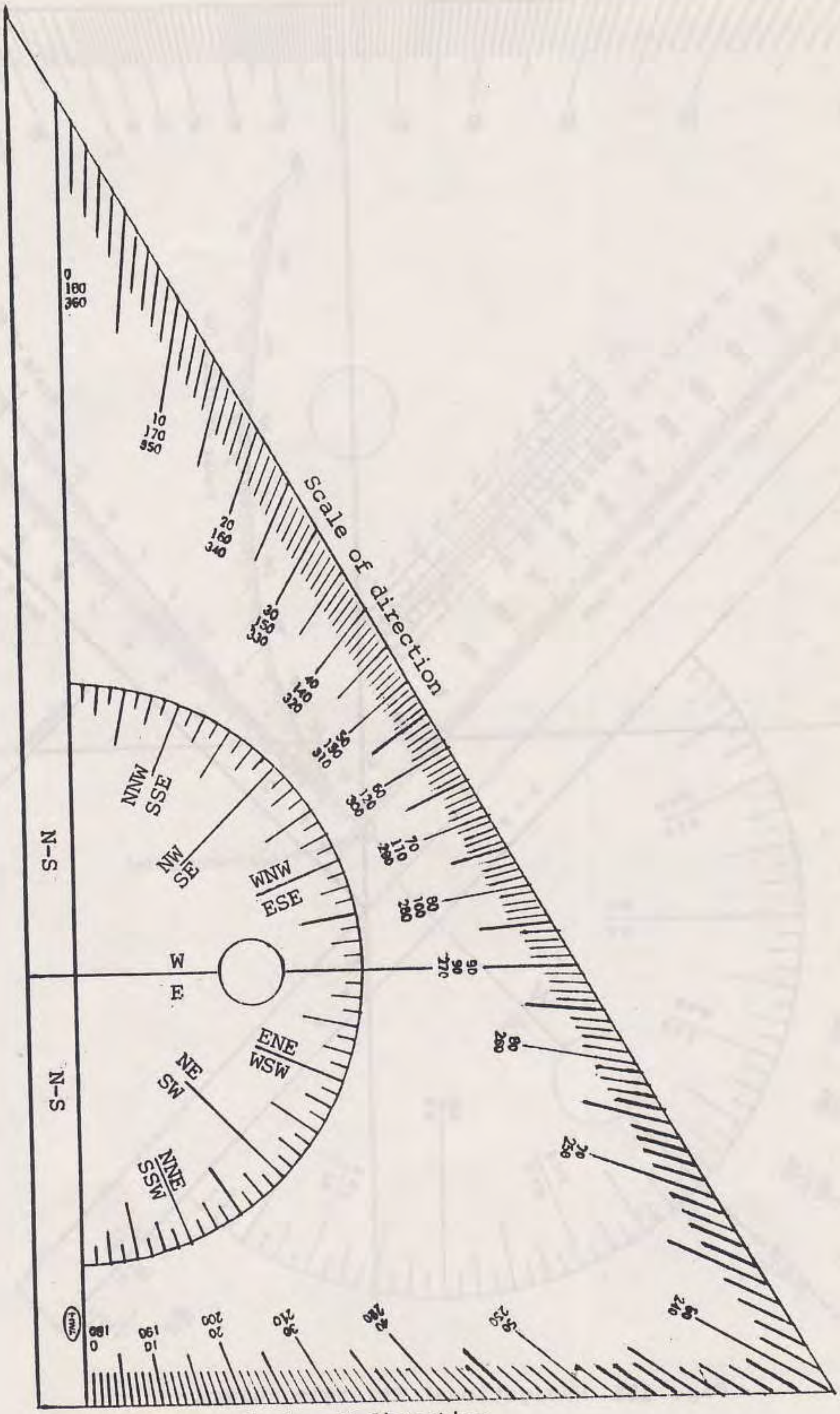


Fig. 154-2 Navigational Triangle Ruler (B type)

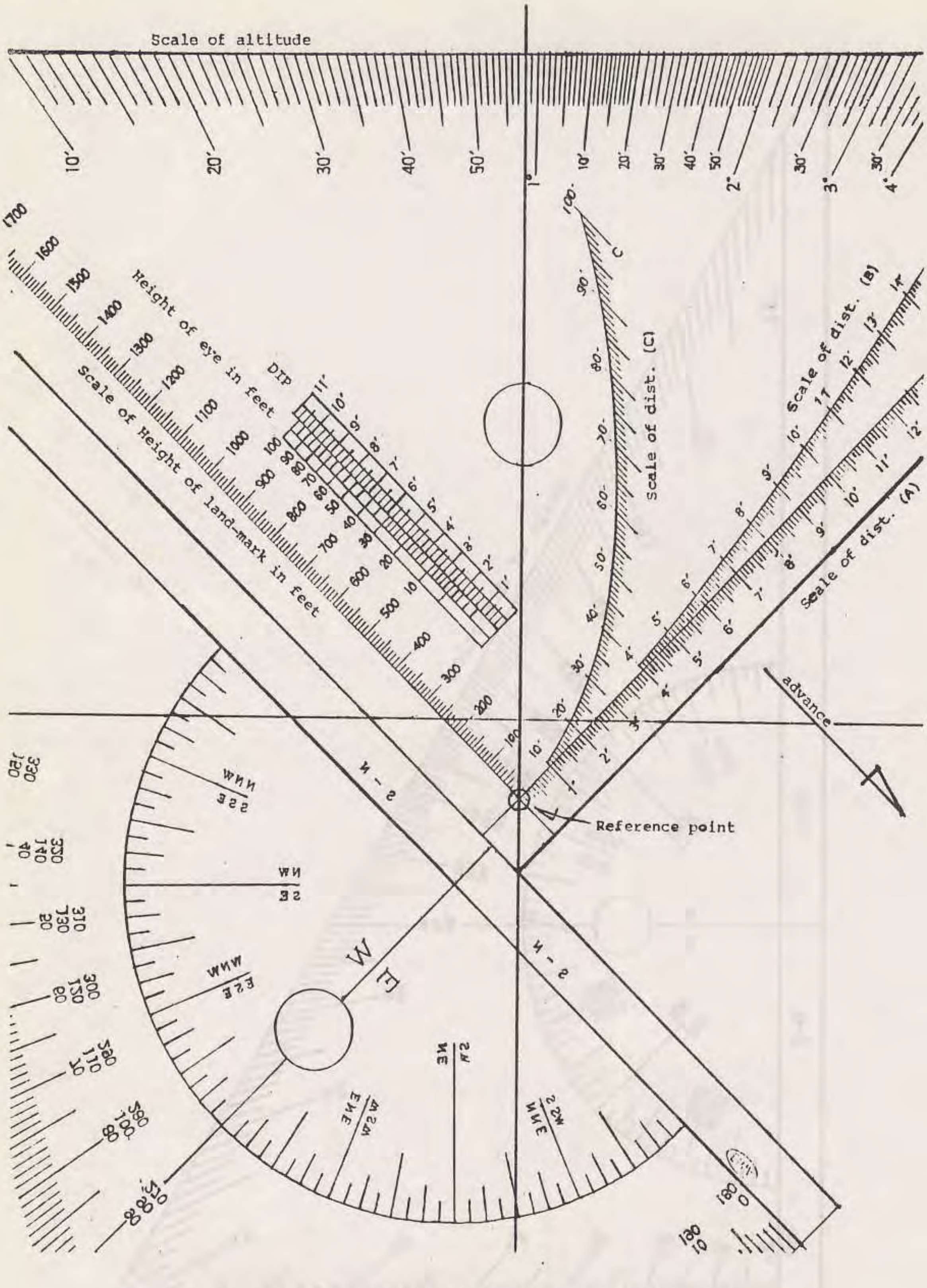


Fig. 155 Navigational Triangle Rulers

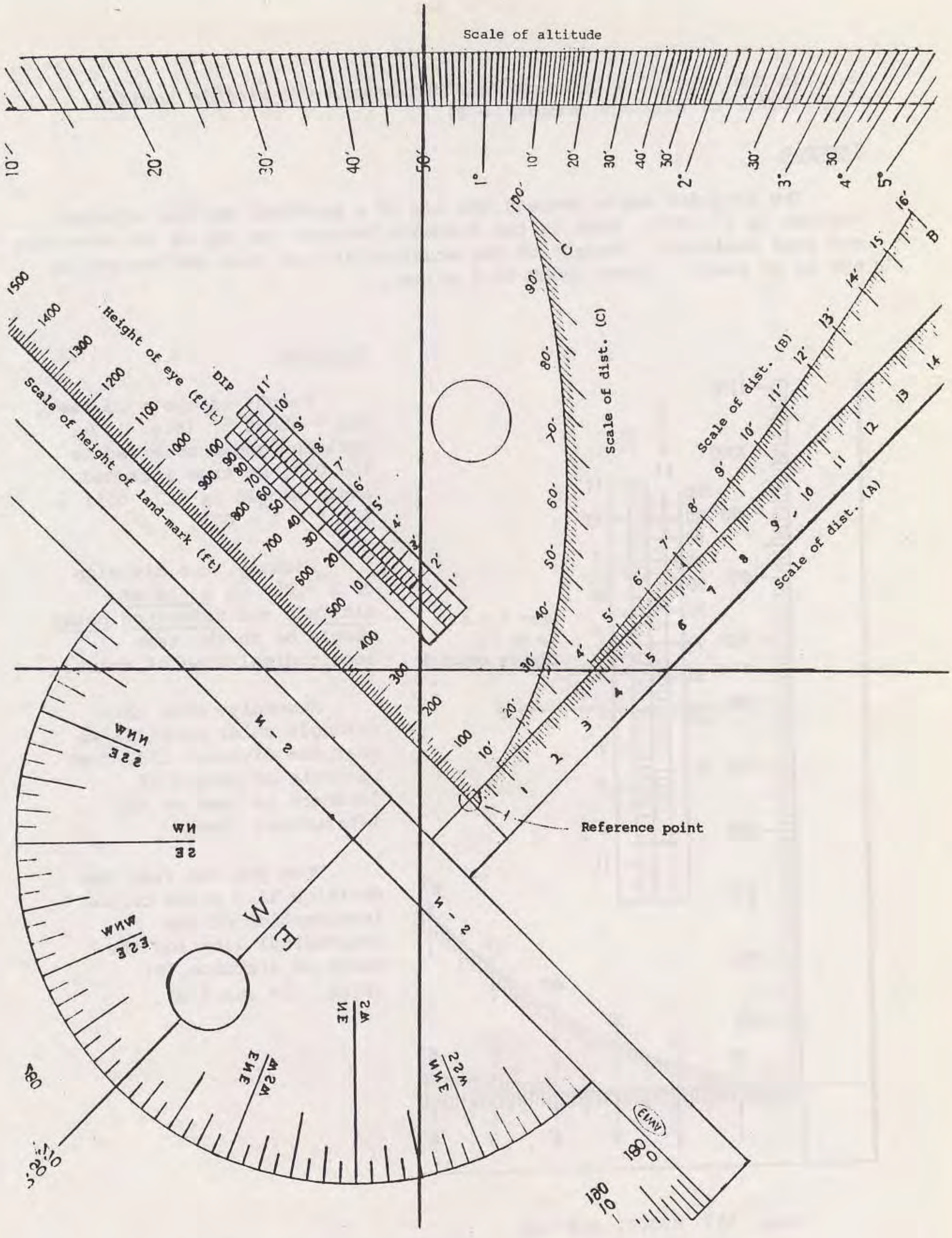


Fig. 156 Navigational Triangle Rulers

2. In the case of a landmark existing beyond the visible horizon. (Scale of distance should be B).

Example

The included angle between the top of a mountain and the visible horizon is $1^\circ - 05'$. What is the distance between the top of the mountain and your position? (Height of the mountain is 1300 feet and height of eye is 22 feet). (Ans: about 11.3 miles).

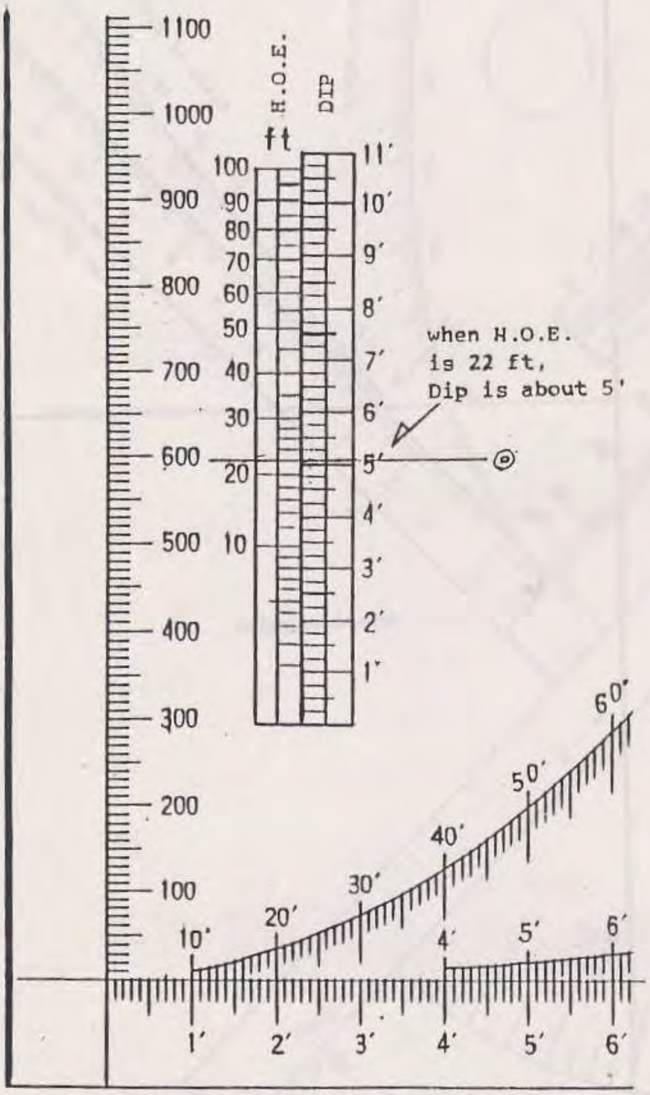


Fig. 157 H.O.P. and Dip.

Solution

Height of eye = 22 feet, Dip = about 5' (Fig. 157). Included angle observed is $1^\circ - 05'$, so true included angle should be $(1^\circ - 05') - (05') = 1^\circ - 00'$.

Firstly, the division of $1^\circ - 00'$ on scale of altitude and reference point should be on the same longitudinal line of chart.

Secondly, move this triangle ruler parallel so that the division 1300 feet of scale of height of landmark is just on the latitudinal line.

Then you can read the division 11.3 miles on the intersection of the latitudinal line and the scale of distance (B). (Figs. 157 and 158).

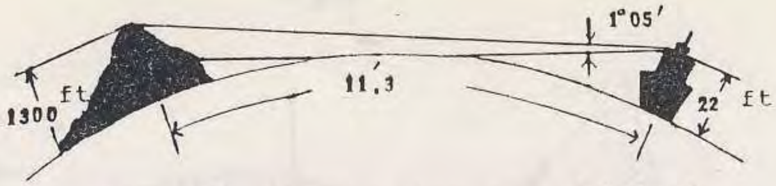


Fig. 158 Distance from top of island.

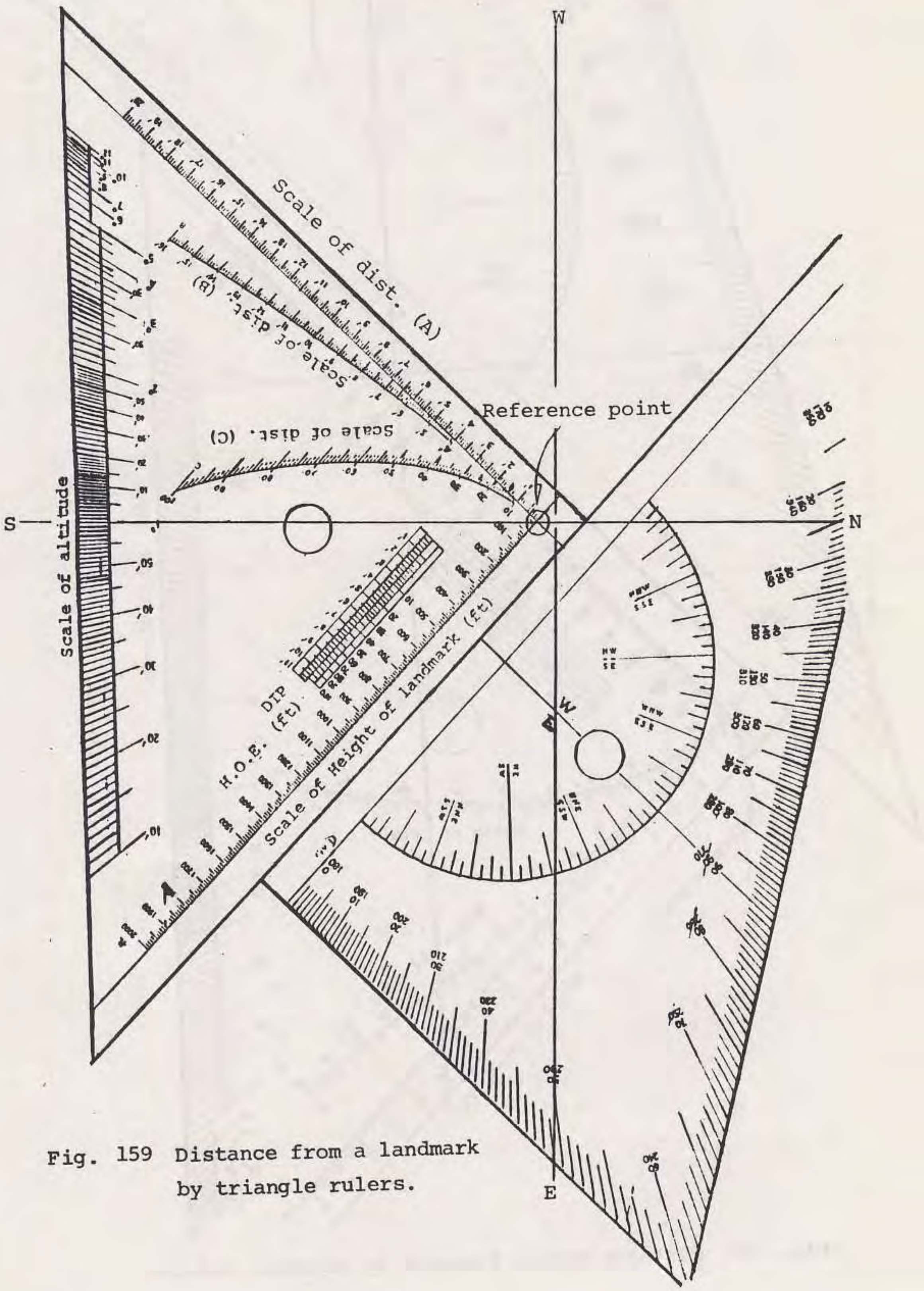


Fig. 159 Distance from a landmark by triangle rulers.

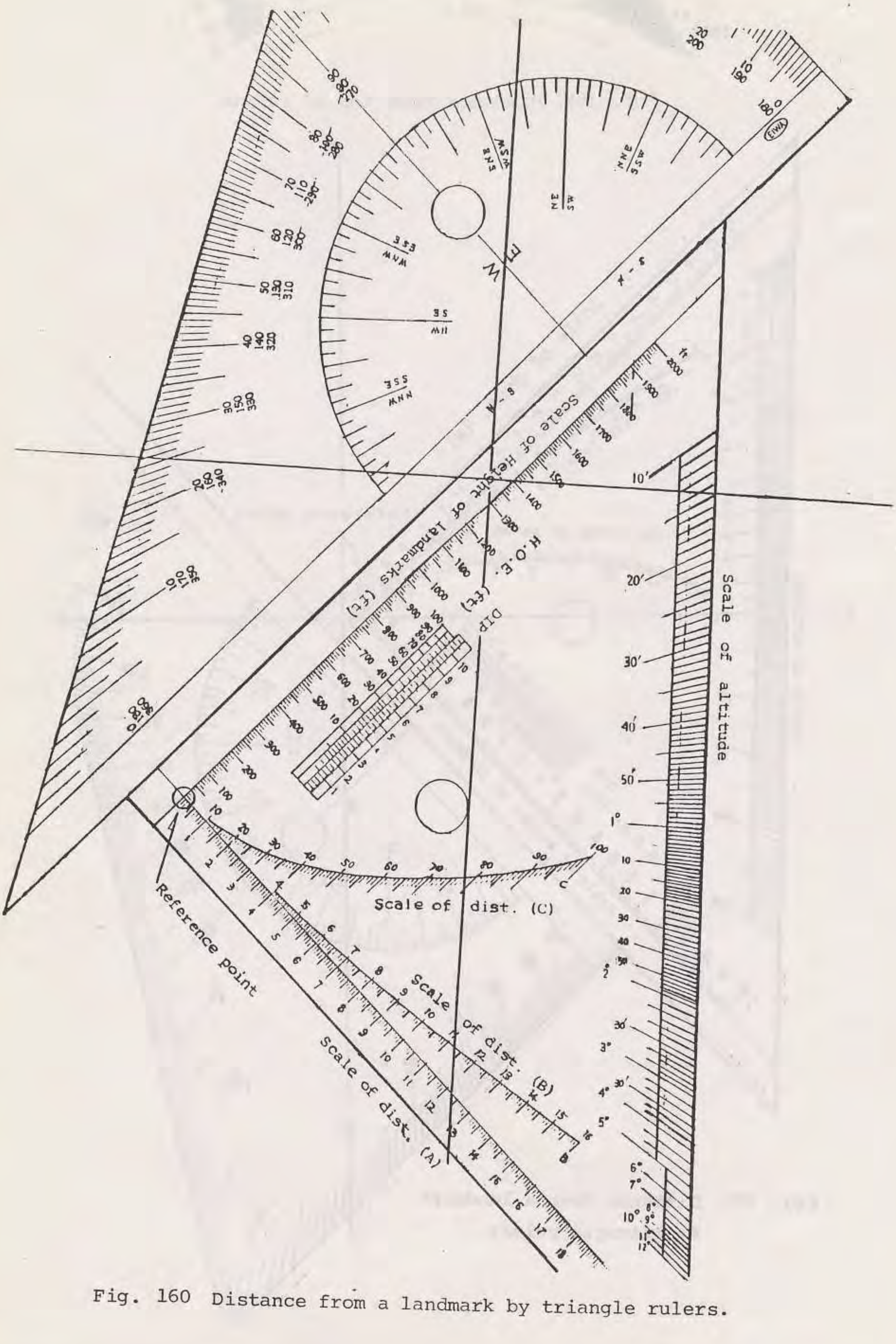


Fig. 160 Distance from a landmark by triangle rulers.

Fixing a Position in the Vicinity of the Shore

In the areas of off-shore fishery activities, such as artificial fish shelters or reefs, stationary trapnets, fishery conservation zones, prohibited fishing areas, etc., it is important that their positions be fixed both on charts and at sea. Firstly the areas or the zones should be drawn by points and lines on a chart. Secondly the task of fixing positions should be done by observation on board. Once charted and the positions fixed all the information should be published. Therefore, typical methods to fix position with depth of water by small observation boats are introduced here.

1) A transit line and an included angle

A transit line is a line of position towards a chosen landmark and, if there is a second landmark, it is possible to observe an included angle between the transit line to the chosen landmark and a transit line to the second landmark from the ship's position. A straight line from the ship's positon to either landmark is also a line of position. A ship should be somewhere along these lines of position. If two simultaneous and non-parallel lines of position are available, the only position that satisfies the requirements of being on both lines at the same time is the intersection of the two lines (see Fig. 161).

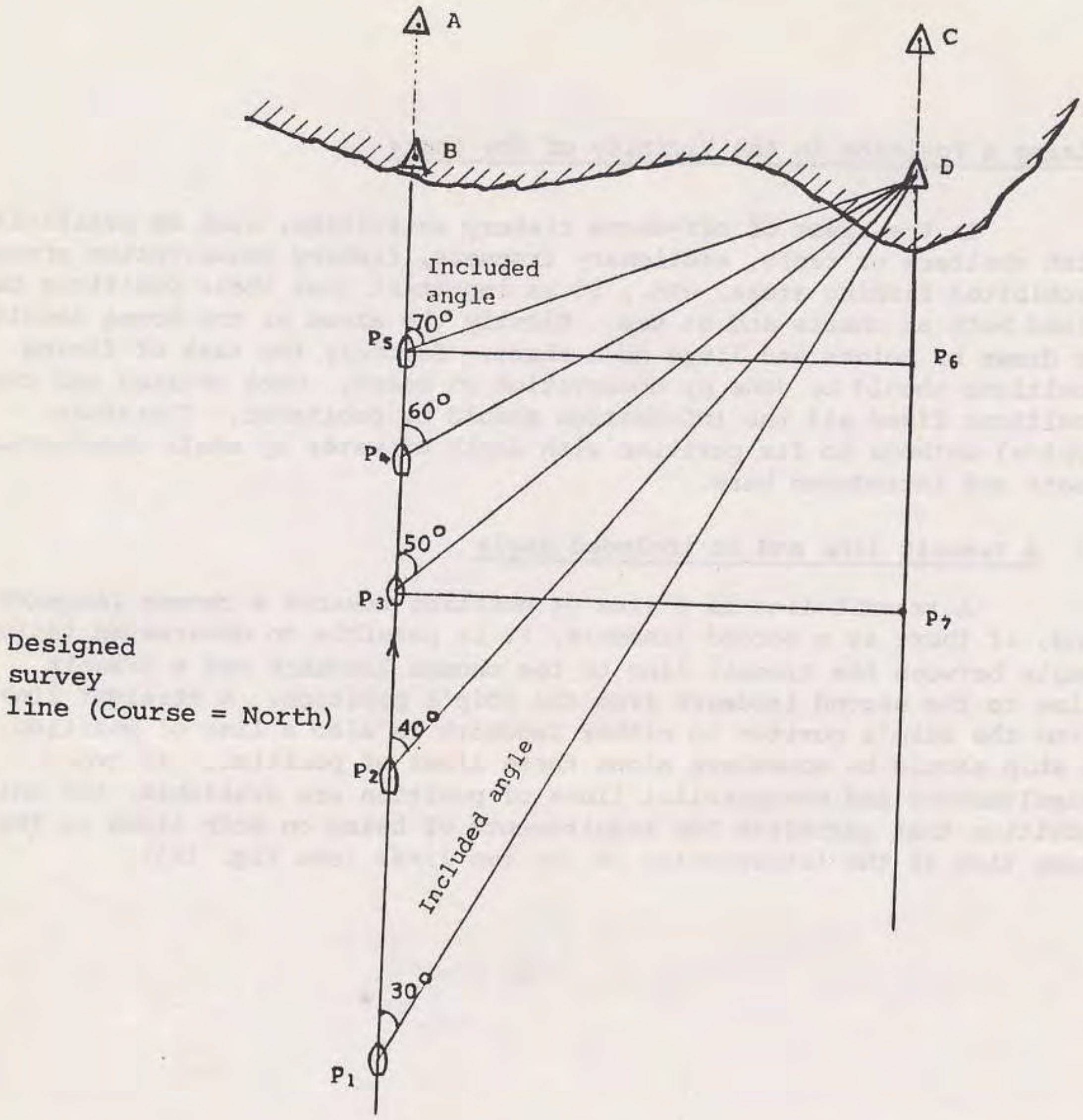


Fig. 161 Fix by a transit line and included angles.

The area within the points P_3, P_5, P_6 and P_7 represents the survey area:

1. Position (P_1): Ship is on the transit line $\overline{ABP_1}$ and the included angle is 30° between $\overline{ABP_1}$ and $\overline{DP_1}$

2. Position (P_2): Ship is on the transit line $\overline{ABP_2}$ and the included angle is 40° between $\overline{ABP_2}$ and $\overline{DP_2}$

3. Position (P_3): Ship is on the transit line $\overline{ABP_3}$ and the included angle is 50° between $\overline{ABP_3}$ and $\overline{DP_3}$

4. Position (P_4): Ship is on the line $\overline{ABP_4}$ and the included angle is 60° between $\overline{ABP_4}$ and $\overline{DP_4}$

5. Position (P_5): Ship is on the transit line $\overline{ABP_5}$ and the included angle is 70° between $\overline{ABP_5}$ and $\overline{DP_5}$

6. The included angles are horizontally measured by a sextant, both poles A and B should be fixed points on the shore

7. A close communication link by way of portable radio-telephones should be arranged between shore staff and a crew member on board.

2) A course (bearing of landmark) and an included angle

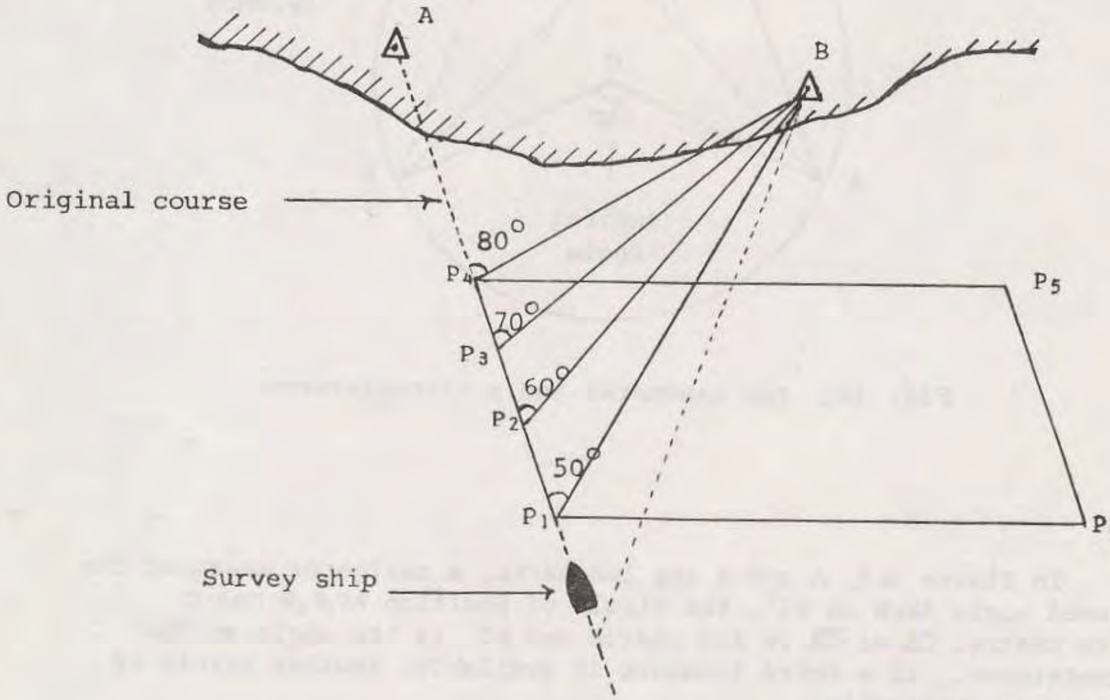


Fig. 162 Fix by a course line and included angles.

In Figure 162, a survey ship's course is 340° to landmark A (a pole) and the included angles were observed between AP_1 and BP_1 as 50°, AP_2 and BP_2 as 60°, AP_3 and BP_3 as 70°, AP_4 and BP_4 as 80° when the survey ship's position was P_1, P_2, P_3 and P_4 respectively. In such a case, a navigator must observe the bearing of A continuously by the ship's compass, if the ship deviates from the designed or original course, the deviation should be amended by steering.

3) Three points (landmarks) and two angles at the circumference

If the angle at the circumference is obtainable, it is possible to draw a circle of position passing through two points (see Fig. 163).

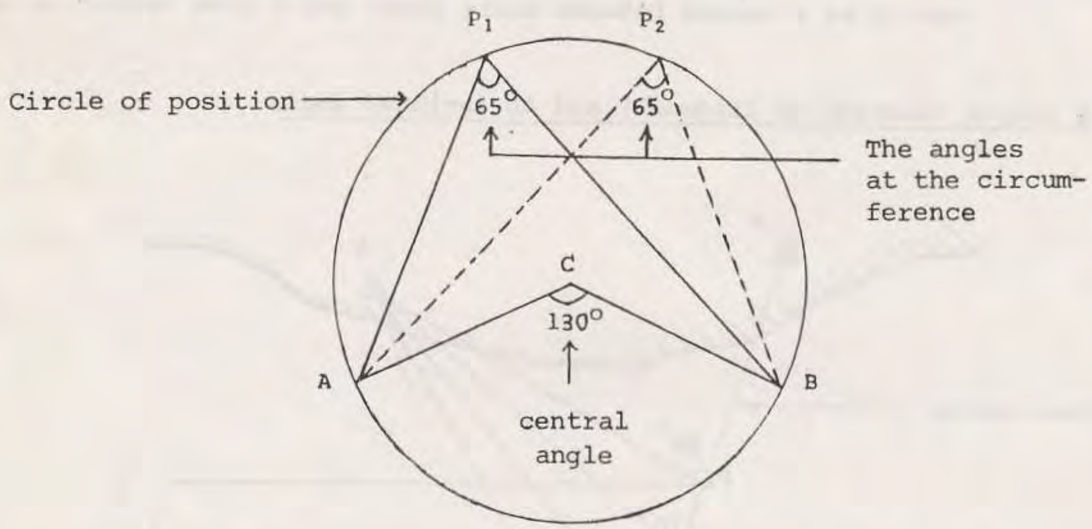


Fig. 163 Two landmarks and a circumference.

In Figure 163, A and B are landmarks, a navigator measured the included angle AP_1B as 65°, the circle of position AP_1P_2B has C as its centre, $\overline{CA}$ or $\overline{CB}$ is its radius and 65° is its angle at the circumference. If a third landmark is available, another circle of position is obtainable.

When a navigator measures simultaneously two included angles by using three landmarks, the intersection of the two circles of position is the fixed position (see Figs. 164 a and b)

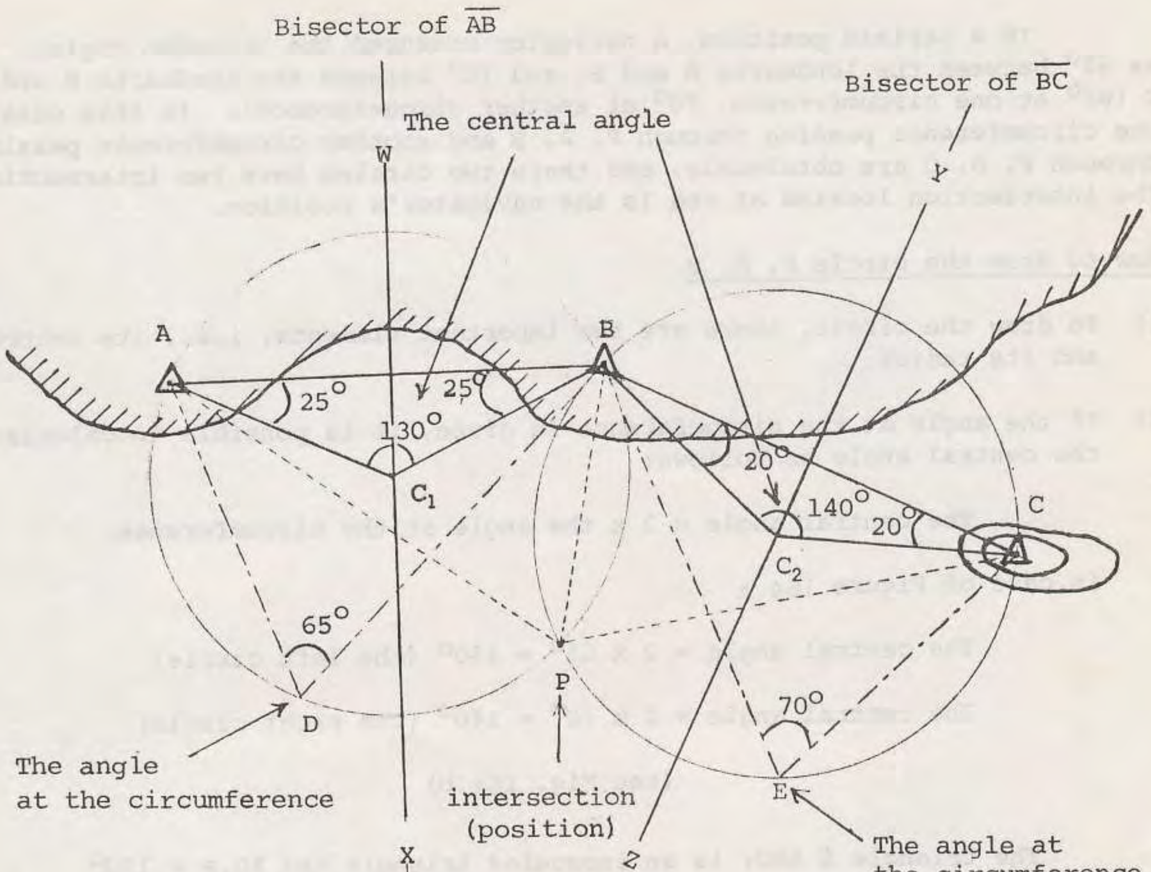
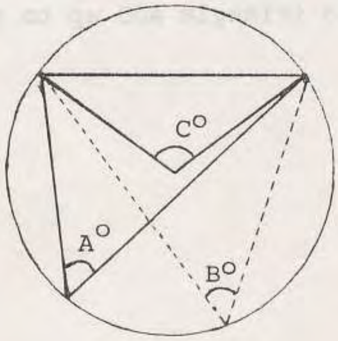


Fig. 164a Fix by three landmarks.



Note: $A^{\circ} = B^{\circ} = \frac{C^{\circ}}{2}$ or

$C^{\circ} = 2 \times A^{\circ} = 2 \times B^{\circ}$

C° = the central angle

A° and B° = the angles at the circumference

Fig. 164b A central angle and angles at the circumference.

In a certain position, a navigator measured the included angles as 65° between the landmarks A and B, and 70° between the landmarks B and C (65° at one circumference, 70° at another circumference). In this case, one circumference passing through P, A, B and another circumference passing through P, B, C are obtainable, and these two circles have two intersection. The intersection located at sea is the navigator's position.

How to draw the circle P, A, B

1) To draw the circle, there are two important elements, i.e., its centre and its radius.

2) If the angle at the circumference is given, it is possible to calculate the central angle as follows:

   The central angle = 2 x the angle at the circumference

   In case of Figure 164 a

   The central angle = $2 \times 65^{\circ} = 130^{\circ}$ (the left circle)

   The central angle = $2 \times 70^{\circ} = 140^{\circ}$ (the right circle)

   (see Fig. 164 b)

The triangle ΔABC_1 is an isosceles triangle and $AC_1B = 130^{\circ}$ (the central angle), so the angle $\angle BAC_1 = \angle ABC_1 = 25^{\circ}$. The straight line $\overline{AC_1}$ should be drawn and the intersection of AC_1 and the straight line $\overline{WX}$ (perpendicular to $\overline{AB}$ and passing through the midpoint of $\overline{AB}$) is the centre of the circle and AC_1 is the radius of the circle required. Then it is possible to draw the circle, its angle at the circumference is 65° (the left circle in Figure 164 a). The right circle in Figure 164 a can also be drawn following the same method and procedure.

Note: The three interior angles of a triangle add up to two right angles ($= 90^{\circ} \times 2 = 180^{\circ}$)

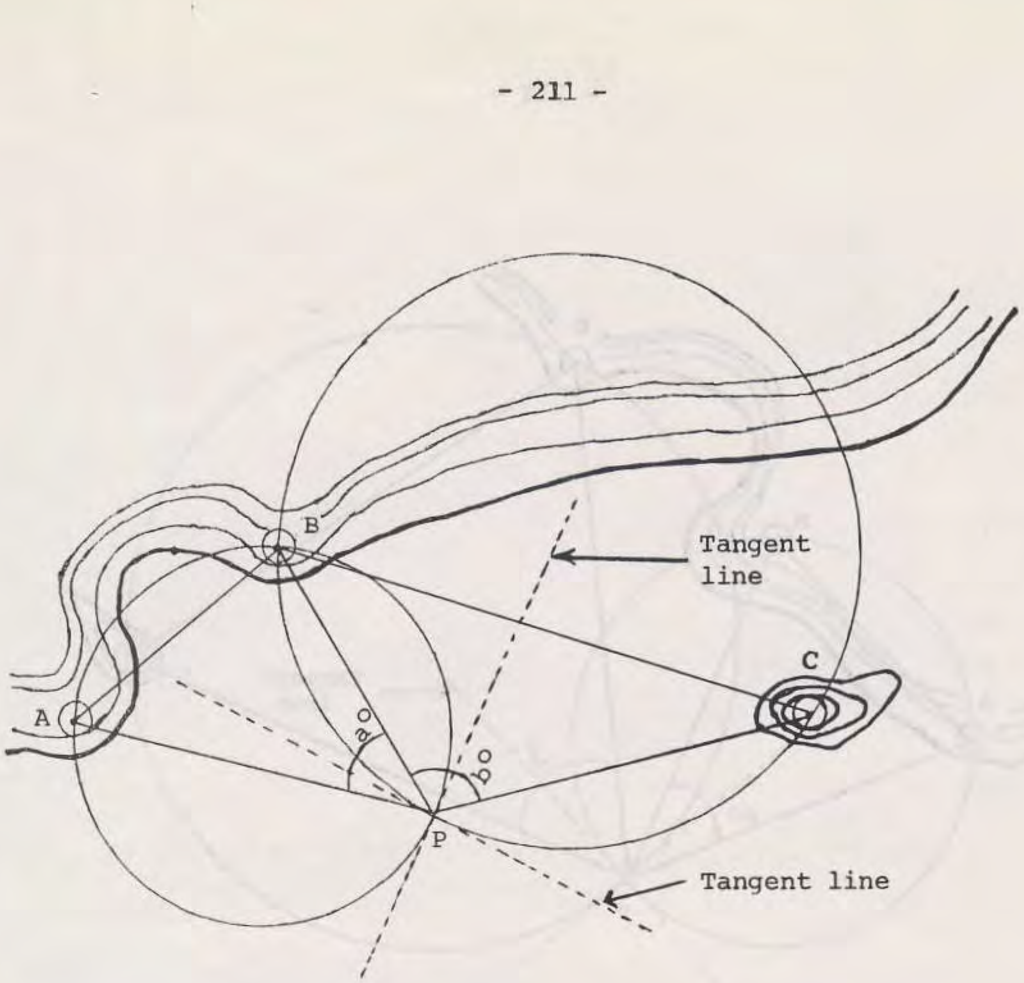


Fig. 164c Condition of included angles.

In Figure 164c, the included angles a and b are measured, we can therefore fix our position by using a three-arm protractor or three radial lines drawn on a piece of transparent paper. In such a case, to maintain a high degree of accuracy when determining the position both a and b should be greater than 30 degrees. When the tangent line at P on the circle PBA meets the tangent line at P on the circle PBC at right angles, the determined position at the intersection of the two circles of position is extremely accurate, but if the included angles are very small in value, e.g. only 10 or 20 degrees, the accuracy of the determined position is less reliable. Therefore, the selection of the three landmarks should be done carefully.

In the case of Figure 164d, the abovementioned explanation can also be applied.

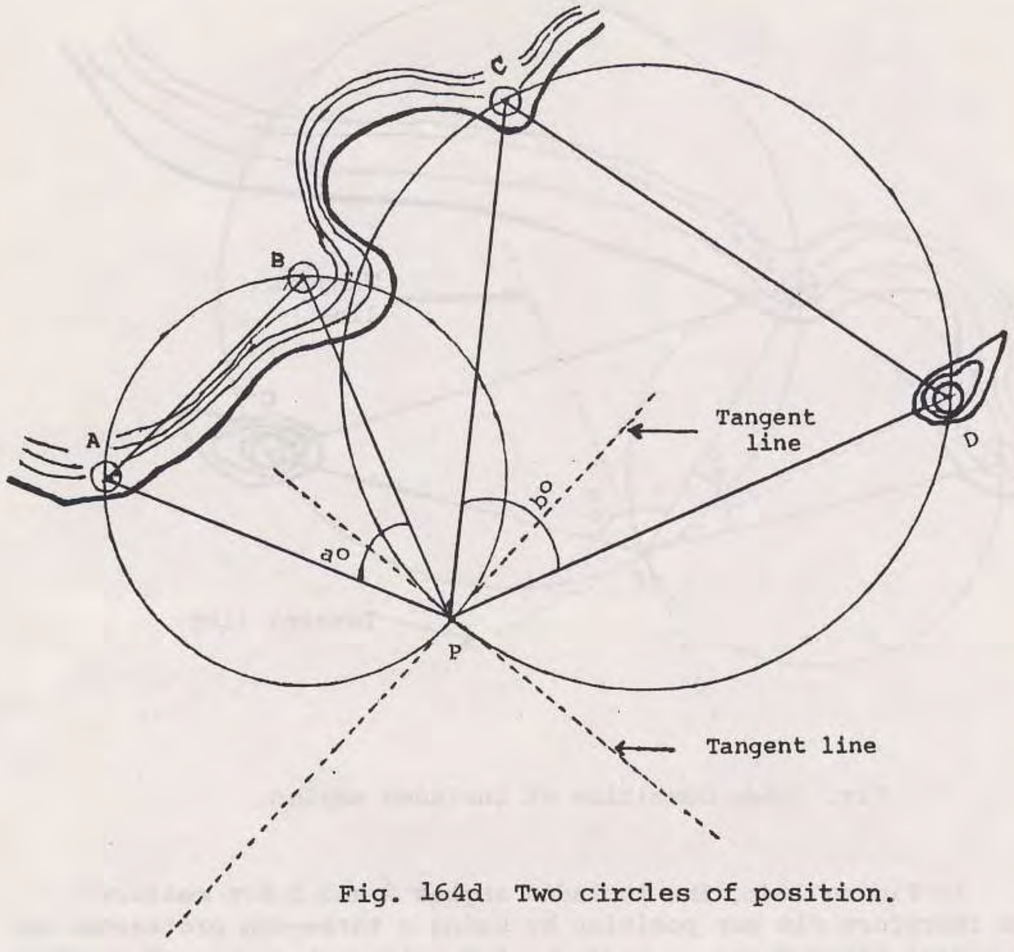


Fig. 164d Two circles of position.

In Figure 164 d, A, B, C and D are the landmarks. The included angle between A and B is a (or a at the circumference PAB) and b between C and D (or b at the circumference PCD).

By the same method and procedure as in Figure 164 c, two circles of position are available. The position at the intersections of the two circles of position is the ship's position.

Questions

1) (Fig. 165)

From an observation boat on the transit line AB (A= Light house TUMPAT, B = Buoy), an observer measured the included angles between the transit line and lighthouse C (SABAK) as 20^{o} at 11:00, and 56^{o} at 13:00.

Required - : The position at 11:00 and 11:30 in latitude and longitude.

2) (Fig. 166)

An observation boat on the transit line AB measured the angle at the circumference as 34 degrees between B (Buoy) and C (Lighthouse SABAK) by a sextant at 14:00.

Required - : The position at 14:00.

3) (Fig. 167)

During sailing, a navigator measured the included angle between the lighthouse and A (Top of the north island) as 46 degrees and A and B (Top of the south island) as 34 degrees.

Required - : Fix position on the chart.

4) (Fig. 168)

A ship is coasting, with course set on 80 degrees. AT 10:40 navigators measured the included angle as 32 degrees between A (Lighthouse) and B (Top of the mountain) and as 28 degrees between C (Top of island) and D (Top of another island).

Required - : Position at 10:40 on the chart.

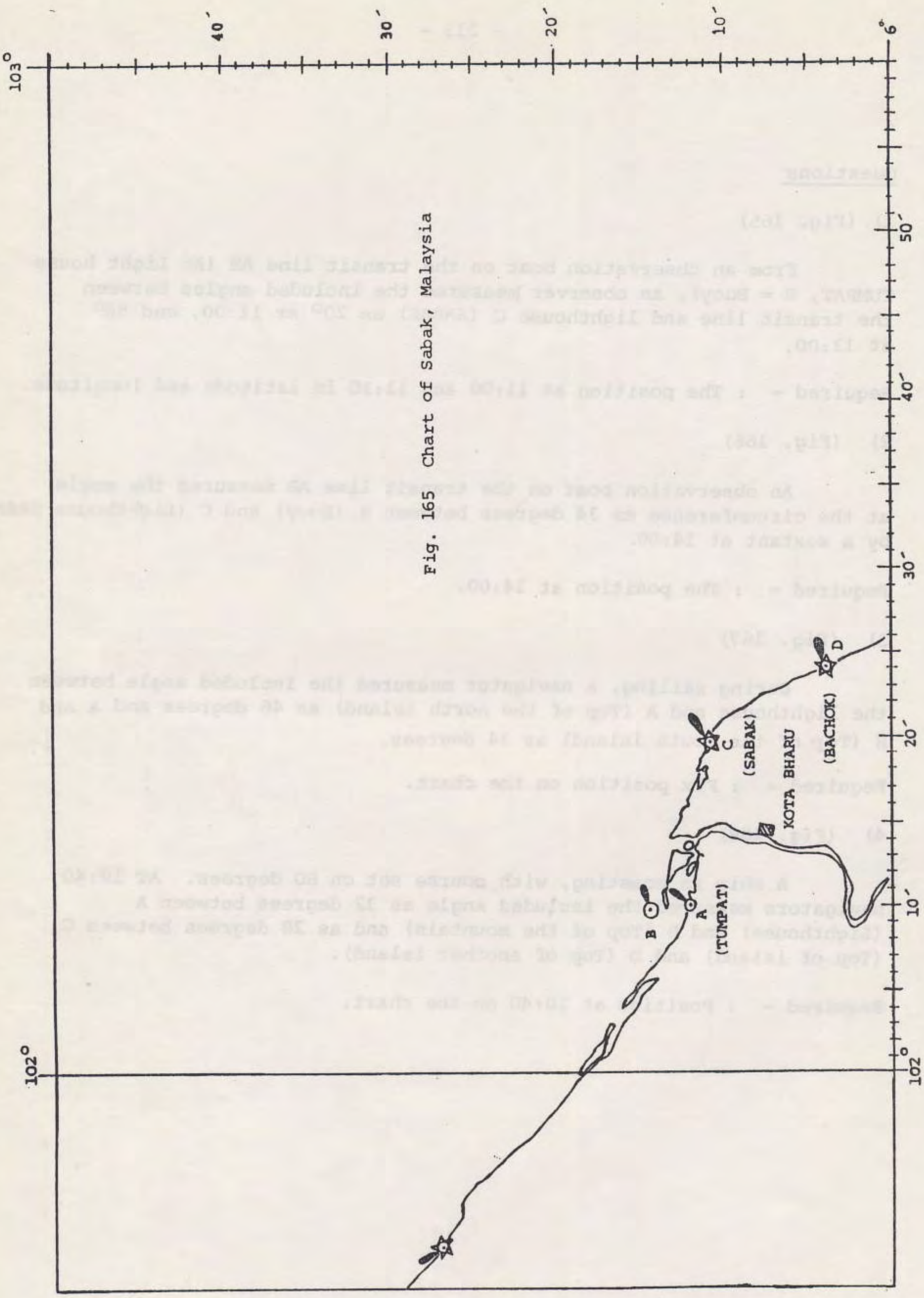


Fig. 165 Chart of Sabak, Malaysia

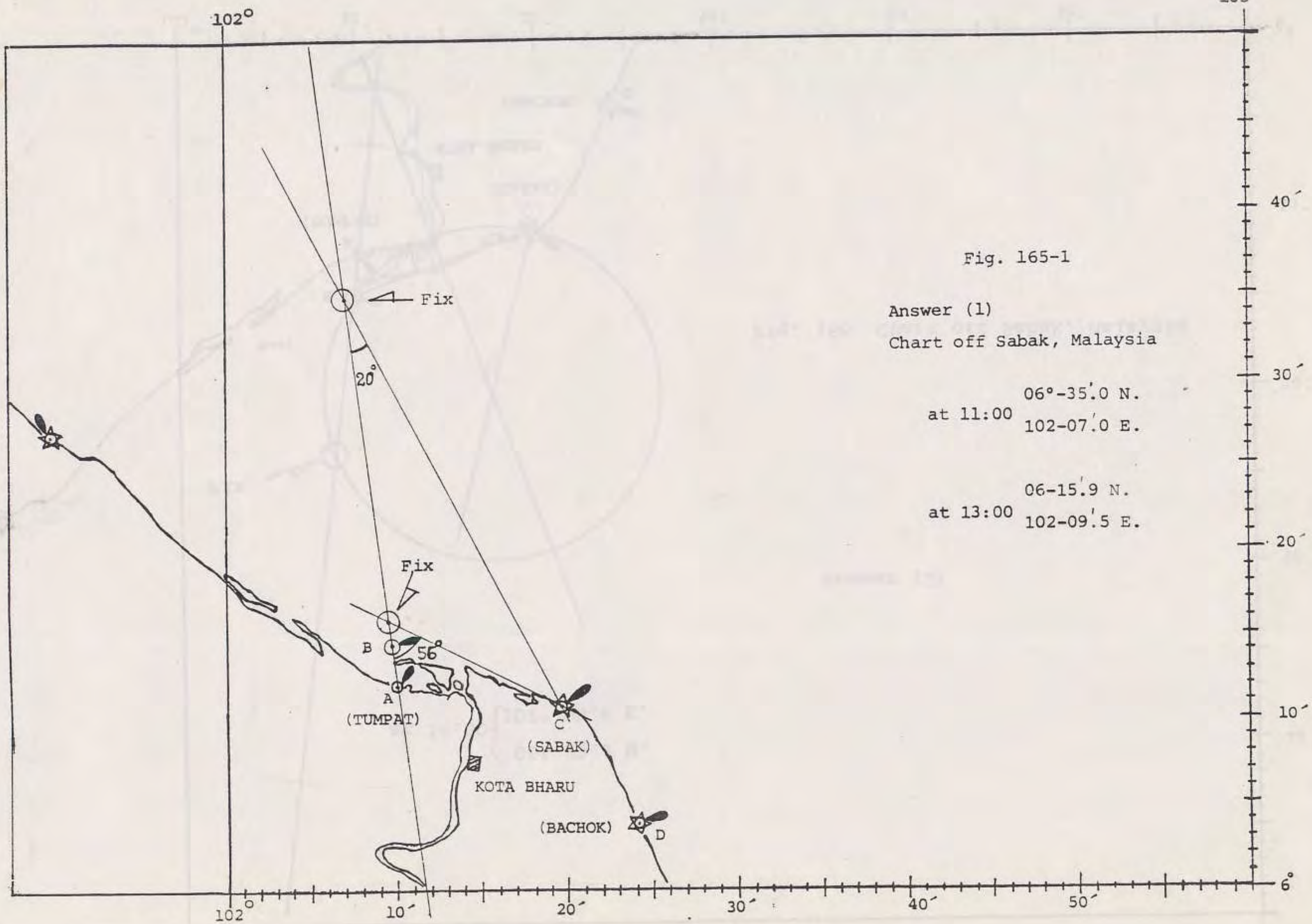


Fig. 165-1

Answer (1)
Chart off Sabak, Malaysia

at 11:00 06°-35'.0 N.
102-07'.0 E.

at 13:00 06-15'.9 N.
102-09'.5 E.

Answer (2)

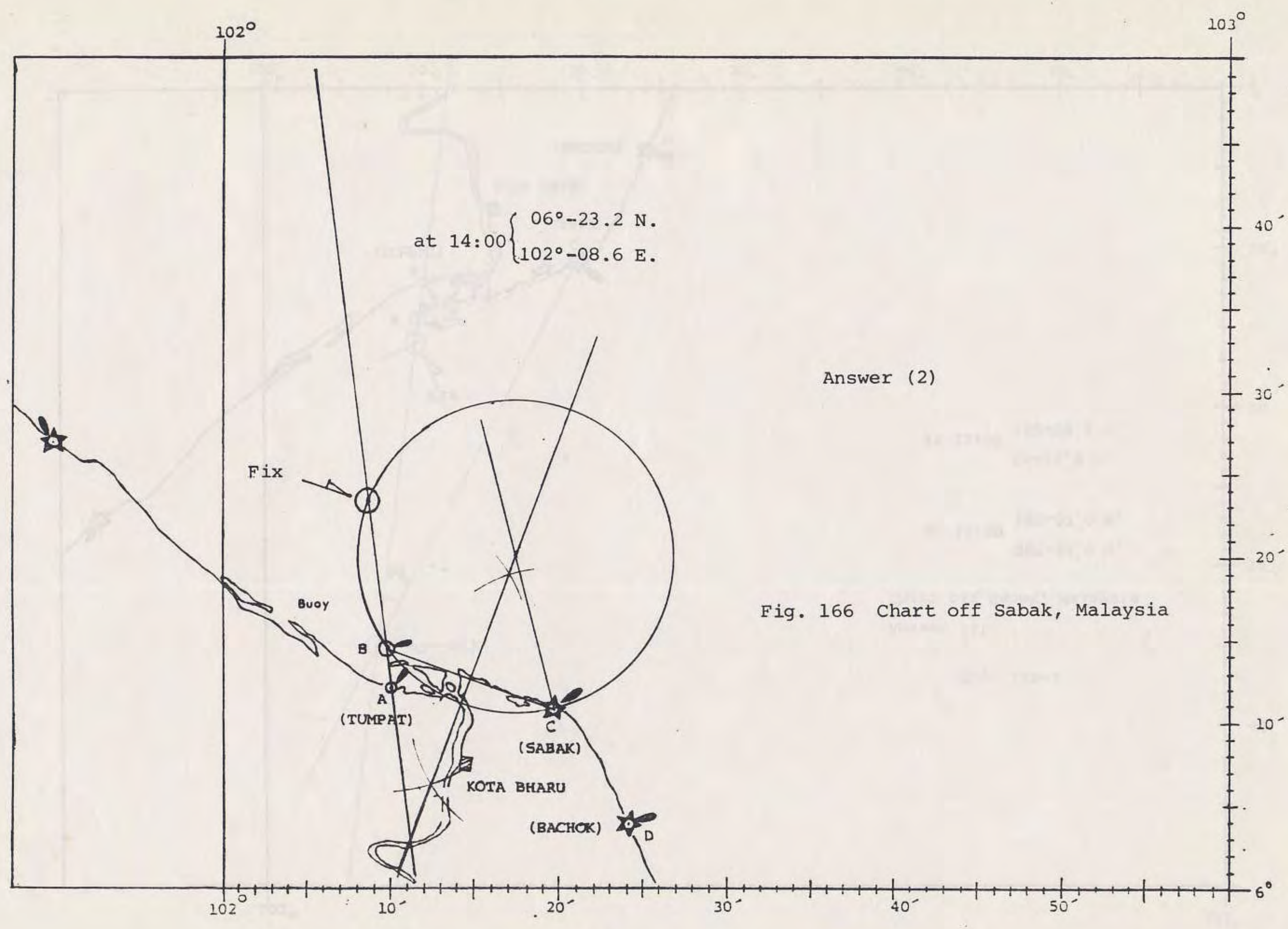


Fig. 166 Chart off Sabak, Malaysia

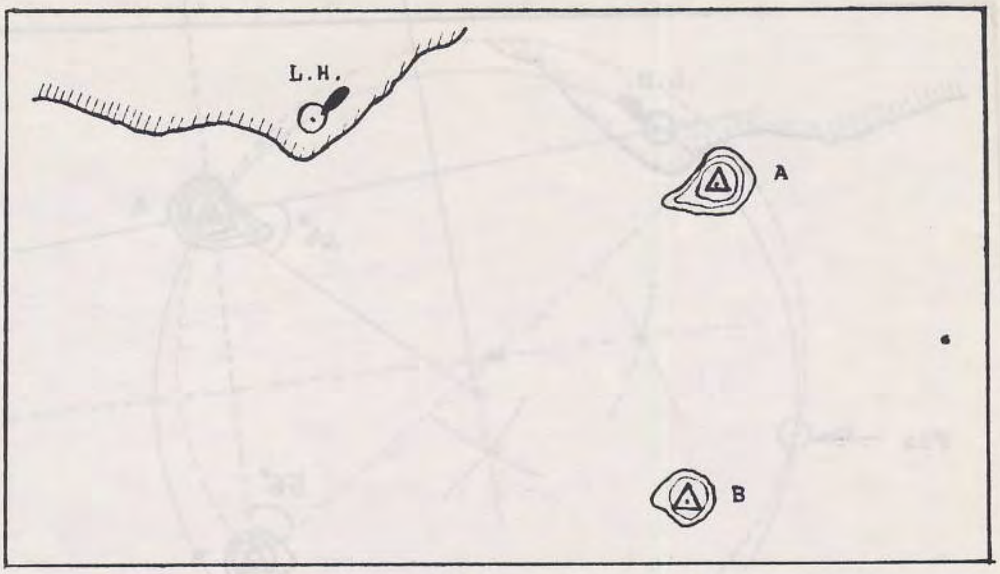


Fig. 167 Fix position on the above chart.

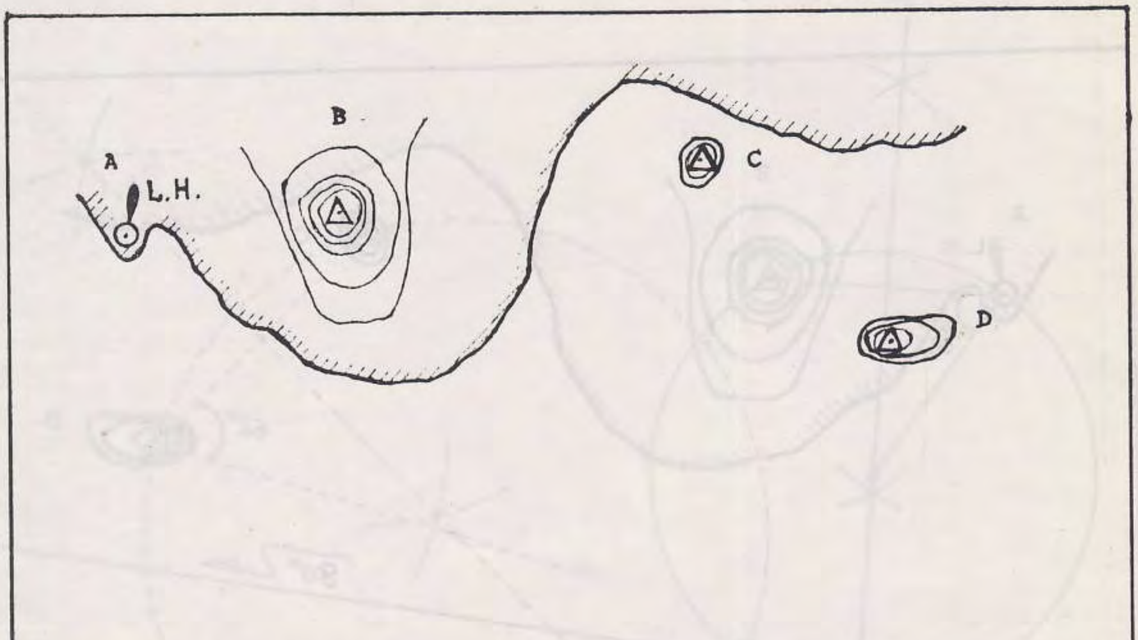


Fig. 168 Fix position on the above chart.

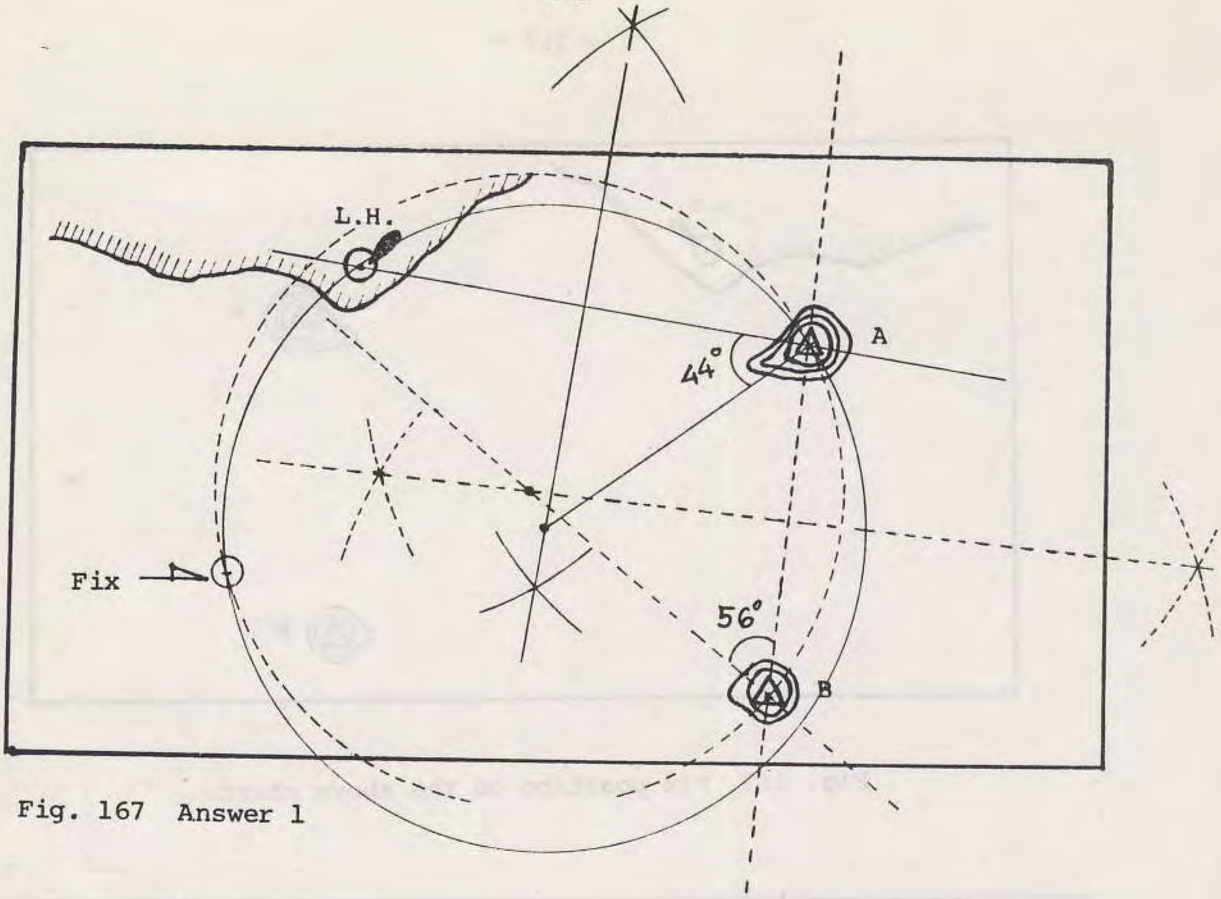


Fig. 167 Answer 1

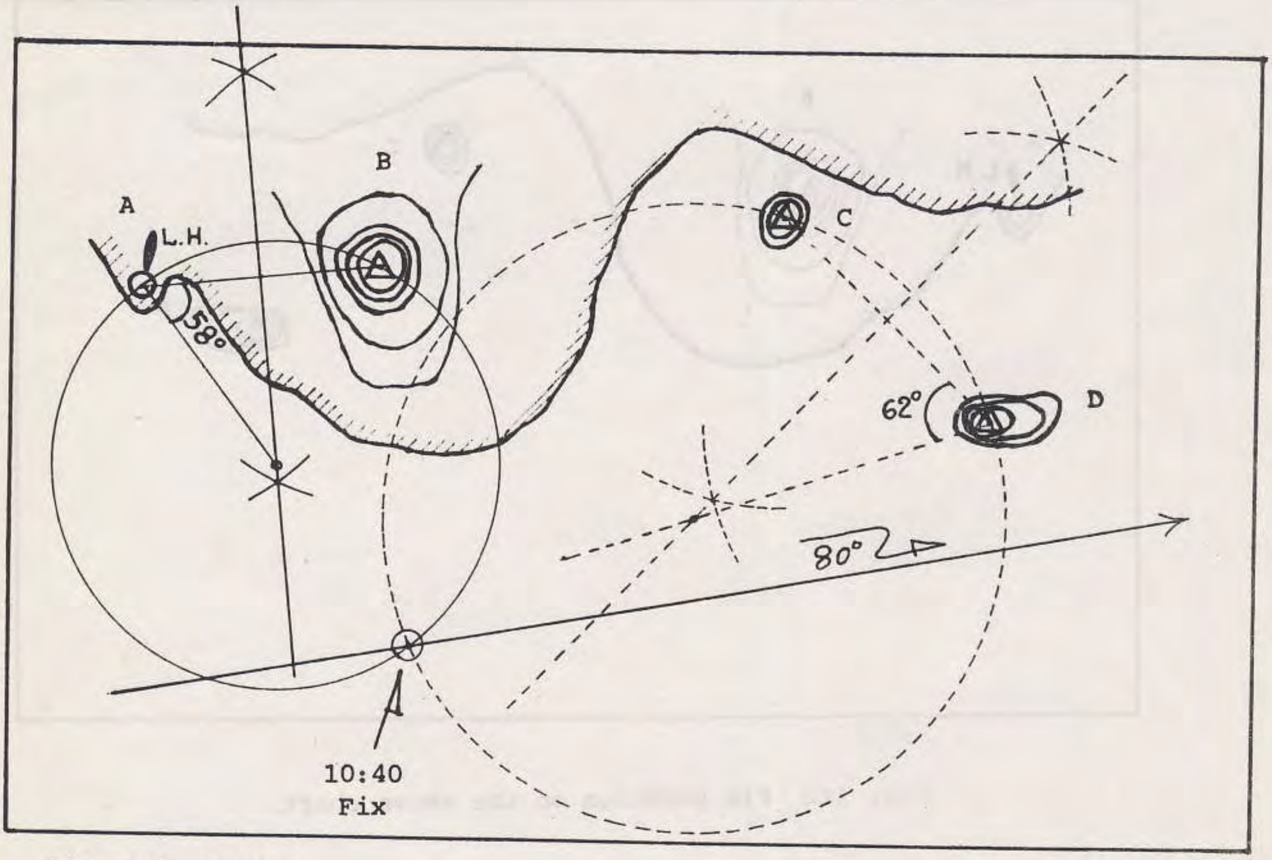


Fig. 168 Answer

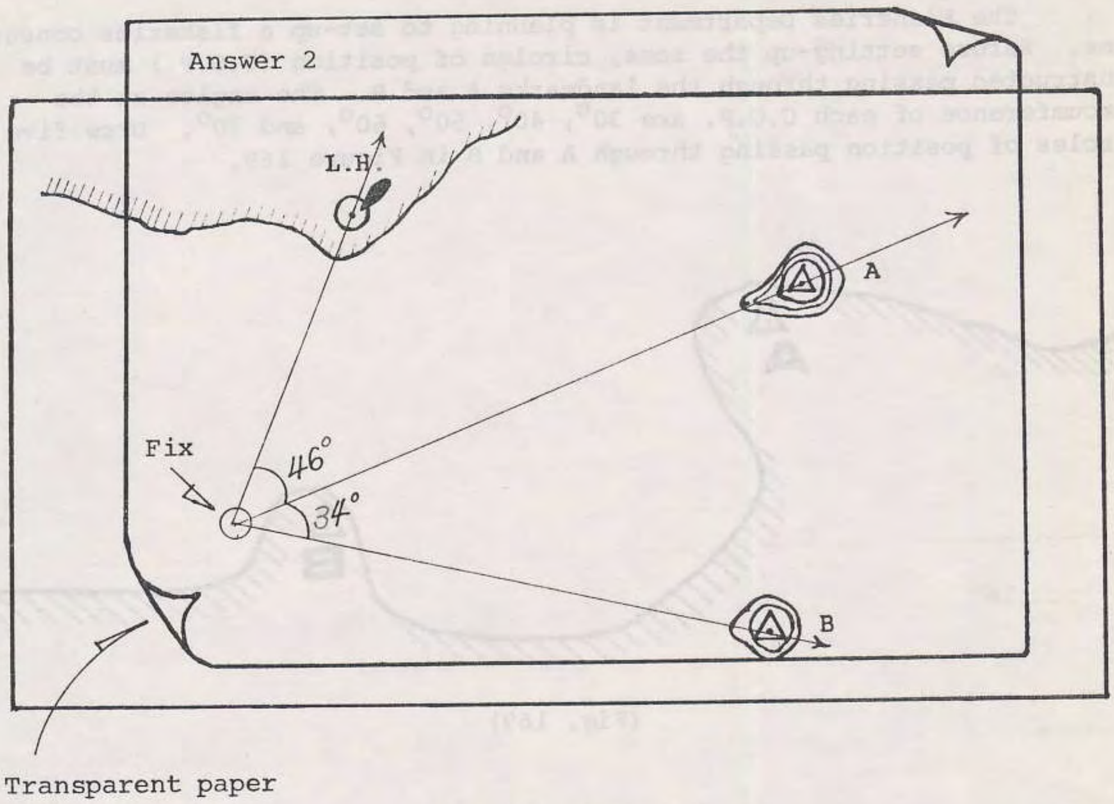


Fig. 167 Answer 2

Question

The Fisheries Department is planning to set-up a fisheries conservation zone. Before setting-up the zone, circles of position (C.O.P.) must be constructed passing through the landmarks A and B. The angles at the circumference of each C.O.P. are 30°, 40°, 50°, 60°, and 70°. Draw five circles of position passing through A and B in Figure 169.

(Fig. 169)

Note:

1. The perpendicular bisector of $\overline{AB}$ must be constructed.
2. The central angle of a circle = 2 × the angle at the circumference.
3. The three interior angles of a triangle add-up to two right angles (i.e. 180°).

Solution

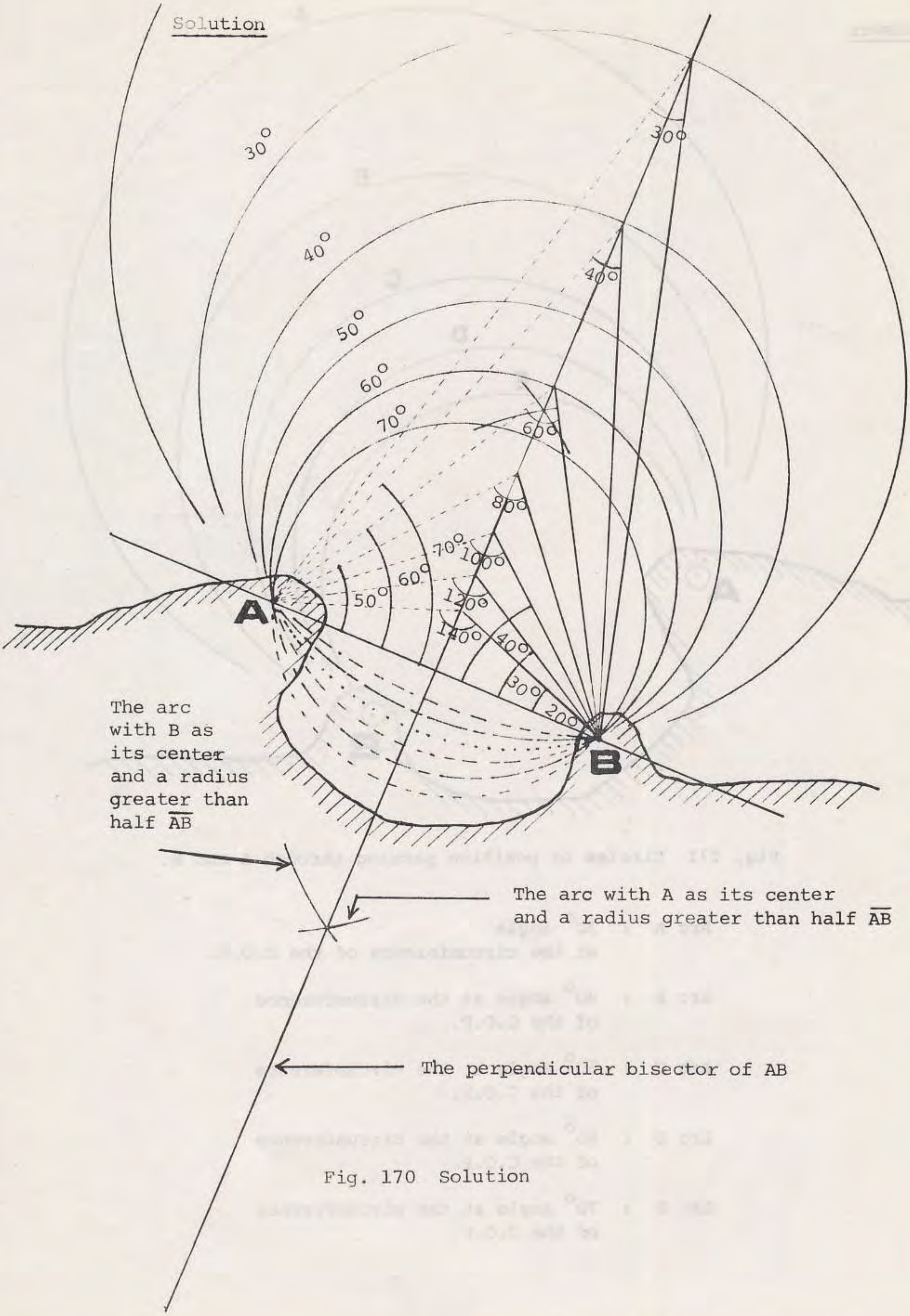


Fig. 170 Solution

Answer

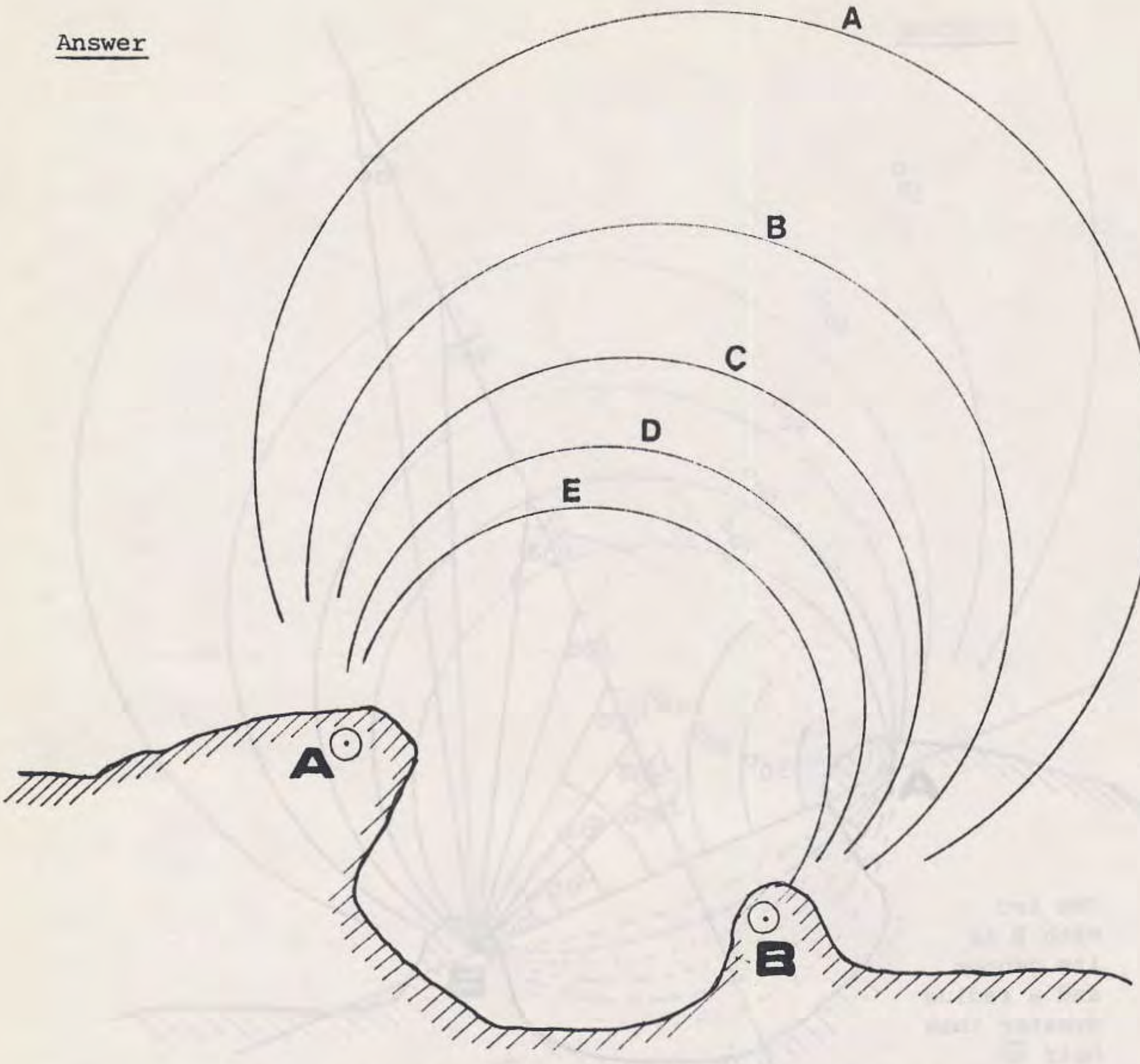


Fig. 171 Circles of position passing through A and B.

Arc A : 30° angle at the circumference of the C.O.P.

Arc B : 40° angle at the circumference of the C.O.P.

Arc C : 50° angle at the circumference of the C.O.P.

Arc D : 60° angle at the circumference of the C.O.P.

Arc E : 70° angle at the circumference of the C.O.P.

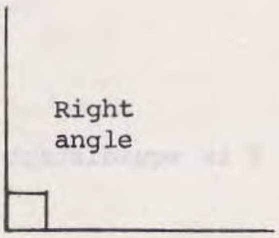
Review of the Triangle

Definitions

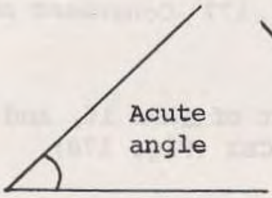
1) A right angle measures exactly 90° (Fig. 172)

2) An acute angle measures less than 90° (Fig. 173)

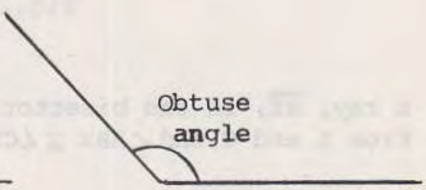
3) An obtuse angle measures more than 90° (Fig. 174)



(Fig. 172)



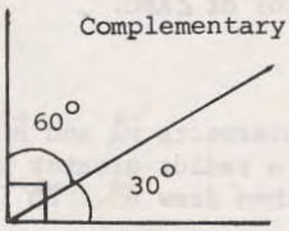
(Fig. 173)



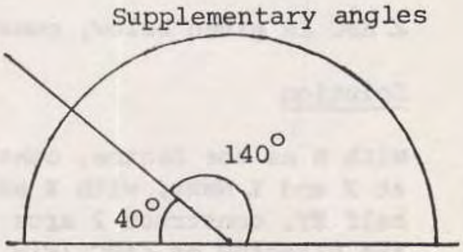
(Fig. 174)

4) Two angles are complementary if, and only if, they add-up to 90°. Each angle is a complement to the other (Fig. 175)

5) Two angles are supplementary if, and only if, they add-up to 180°. Each is a supplement to the other (Fig. 176)



(Fig. 175)



(Fig. 176)

6) Angles with an equal number of degrees are called congruent angles, i.e.: ∠A ≅ ∠B, (Fig. 177)

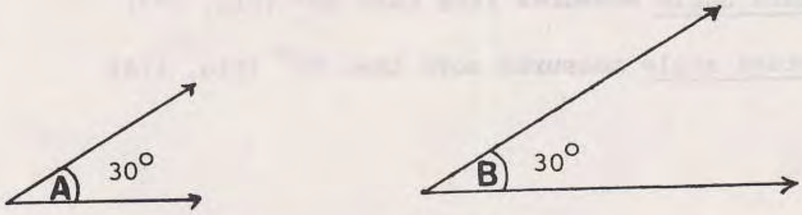


Fig. 177 Congruent angles.

7) A ray, $\overrightarrow{BX}$, is the bisector of ∠ABC if, and only if, X is equidistant from A and C, and ∠ABX ≅ ∠CBX (Fig. 178)

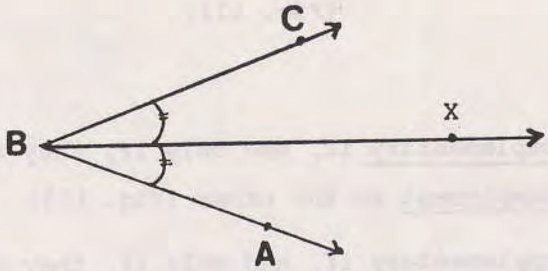


Fig. 178 Bisector.

Question

∠ABC is given below, construct $\overrightarrow{BD}$, the bisector of ∠ABC.

Solution

With B as the centre, construct an arc that intersects $\overrightarrow{BA}$ and $\overrightarrow{BC}$ at X and Y. Next, with X and Y as centres, and a radius greater than half XY, construct 2 arcs intersecting at D, then draw $\overrightarrow{BD}$. $\overrightarrow{BD}$ is the bisector of ∠ABC (Fig. 179)

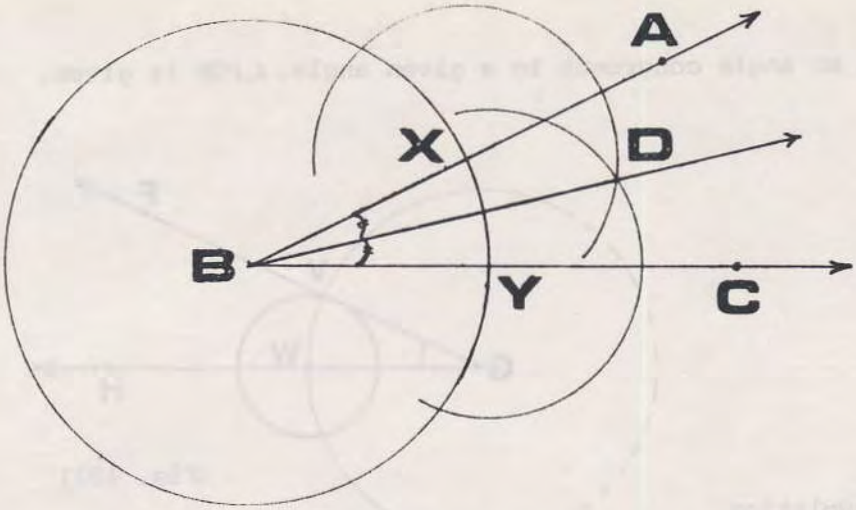


Fig. 179 Bisector.

Question

Construct the perpendicular bisector of $\overline{AB}$. $\overline{AB}$ is given.

Solution

With A and B as centres and a radius greater than half AB, construct a pair of circles, intersecting at P and Q. Next, draw $\overleftrightarrow{PQ}$. $\overleftrightarrow{PQ}$ is the perpendicular bisector of $\overline{AB}$ (Fig. 180).

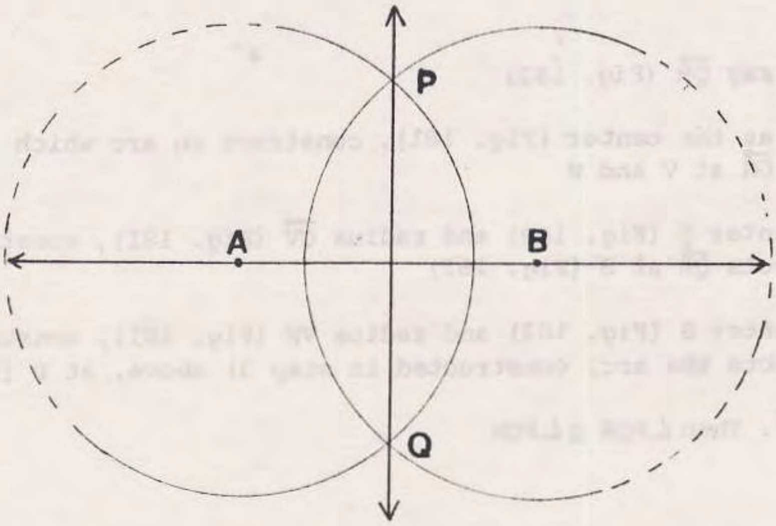
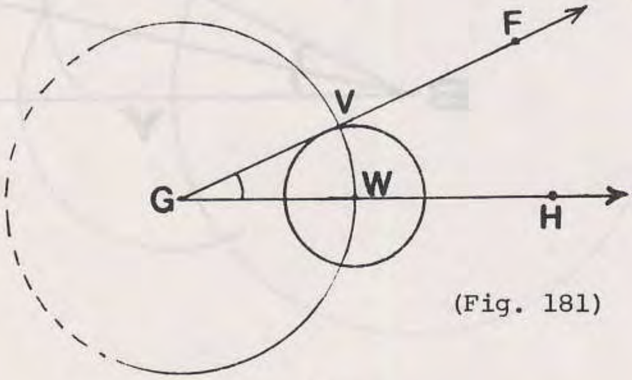


Fig. 180 Perpendicular bisector.

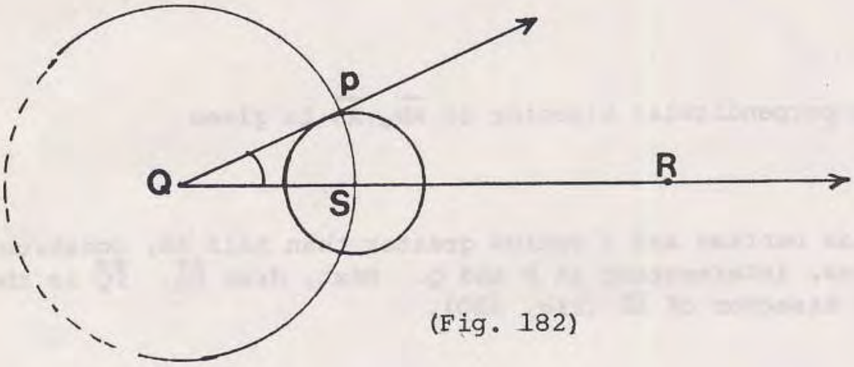
Question

Construct an angle congruent to a given angle. ∠FGH is given.



(Fig. 181)

Solution



(Fig. 182)

Solution

1) Draw a ray $\overrightarrow{QR}$ (Fig. 182)

2) With G as the center (Fig. 181), construct an arc which intersects $\overrightarrow{GF}$ and $\overrightarrow{GH}$ at V and W

3) With center Q (Fig. 182) and radius $\overrightarrow{GV}$ (Fig. 181), construct an arc which intersects $\overrightarrow{QR}$ at S (Fig. 182)

4) With center S (Fig. 182) and radius VW (Fig. 181), construct an arc which intersects the arc, constructed in step 3) above, at P (Fig. 182)

5) Draw $\overrightarrow{QP}$. Then ∠PQR ≅ ∠FGH

8) An acute triangle is a triangle with three acute angles (Fig. 183)

9) A right triangle is a triangle with a right angle (Fig. 184)

10) An obtuse triangle is a triangle with an obtuse angle (Fig. 185)

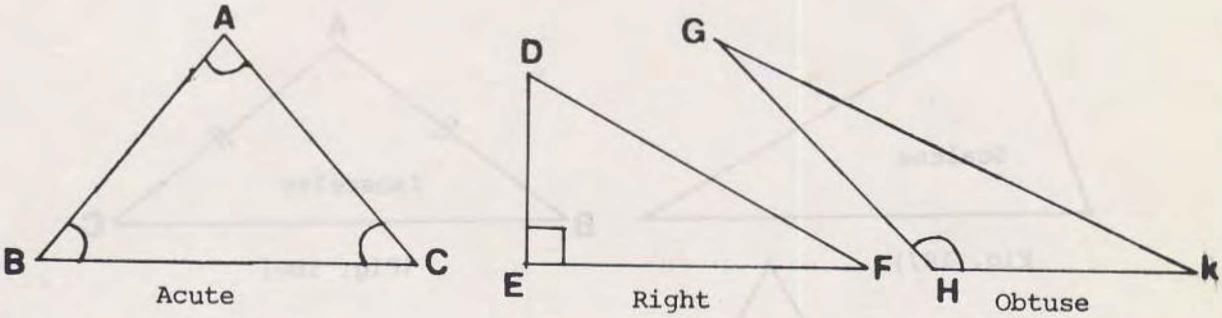


Fig. 183) (Fig. 184) (Fig. 185)

In a right **triangle** the side opposite the right angle is called the hypotenuse. The hypotenuse of ΔDEF is DF, the legs are ED and EF.

11. An equiangular triangle is a triangle with three congruent angles (Fig. 186)

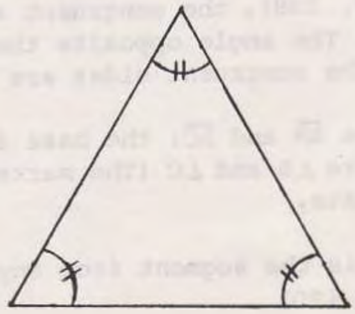


Fig. 186 Equiangular triangle.

12. A scalene triangle is a triangle with no two sides congruent (Fig. 187).

13. An isosceles triangle is one with at least two sides congruent (Fig. 188).

14. An equilateral triangle is one with three congruent sides (Fig. 189).

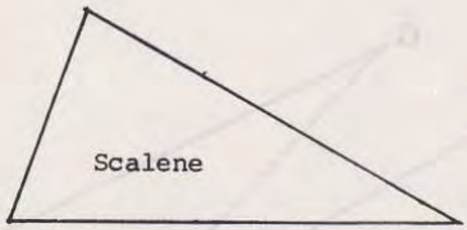
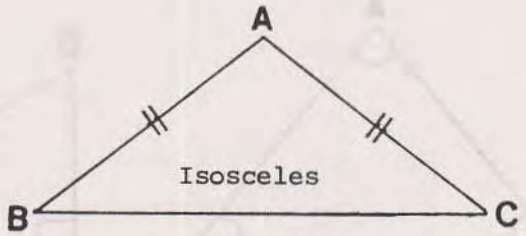
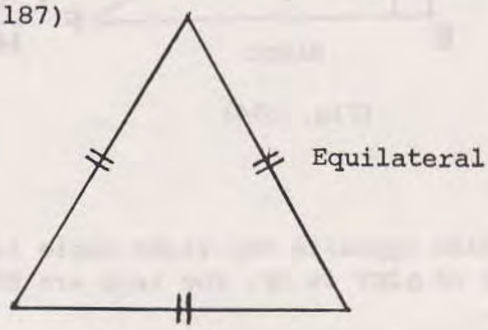


Fig. 187)



(Fig. 188)



(Fig. 189)

In an isosceles (Fig. 188), the congruent sides are called the legs; the third side is the base. The angle opposite the base is the vertex angle. The angle opposite the congruent sides are base angles.

In $\triangle ABC$, the legs are $\overline{AB}$ and $\overline{AC}$: the base is $\overline{BC}$. $\angle A$ is the vertex angle, and the base angles are $\angle B$ and $\angle C$ (The marks on the sides of $\triangle ABC$ indicate the congruent segments.

15. A median of a triangle is the segment from any vertex to the midpoint of the opposite side (Fig. 190).

16. An altitude of a triangle is the segment from any vertex perpendicular to the line that contains the opposite side (Fig. 190).

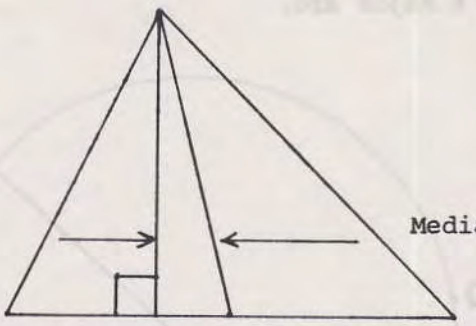


Fig. 190 Altitude and Median.

Review of Arcs of the Circle

1. A central angle of a circle is an angle whose vertex is the center of the circle. In Figure 191, $\angle\alpha$ is a central angle of circle O.

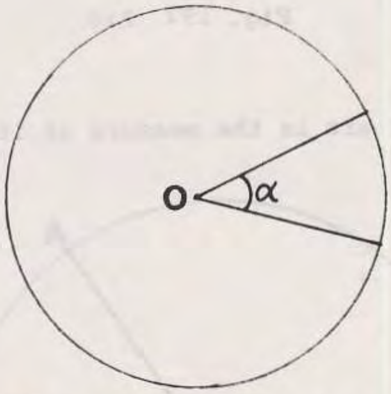


Fig. 191 Central angle.

2. A minor arc of a circle is the set of points on the circle which lie on a central angle or in the interior of the central angle (Fig. 192).

   In Figure 192, in arc AB (Written $\widehat{AB}$ $\widehat{BA}$) is a minor arc. The arc formed by A and B and the points of the circle in the exterior of APB is called a major arc.

AB will mean minor arc $\widehat{AB}$. To avoid confusion, three letters are necessary to name a major arc.

$\widehat{AQB}$ (or $\widehat{BQA}$) is a major arc.

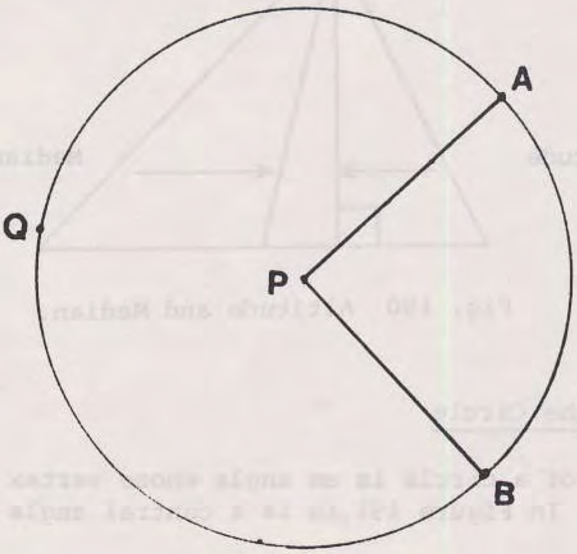


Fig. 192 Arc

3. The degree of a minor arc is the measure of its central angle (Fig. 193)

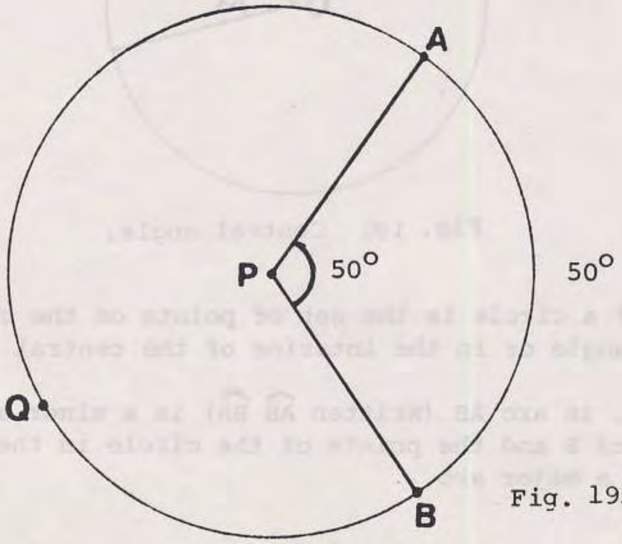


Fig. 193 Arc

In Figure 193, if $\angle P = 50^\circ$, then $\widehat{AB} = 50^\circ$.

4. An angle inscribed in an arc is an angle whose sides contain the endpoints of the arc and whose vertex is a point on the arc other than the endpoints.

In the Figure 194. $\angle C$ is an inscribed angle. $\overset{\frown}{AB}$ is called the intercepted arc.

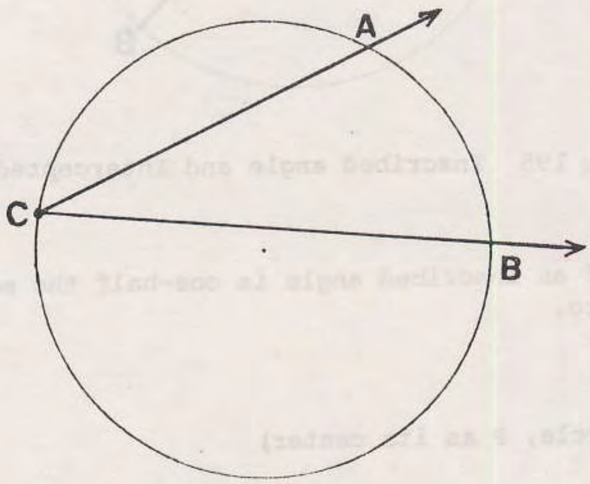


Fig. 194 Inscribed angle and Intercepted arc.

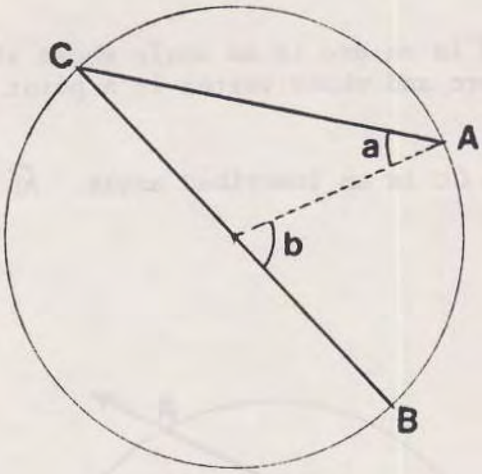


Fig. 195 Inscribed angle and Intercepted arc.

5. The measure of an inscribed angle is one-half the measure of its intercepted arc.

In Figure 195

Given ⊙P (Circle, P as its center)

Prove $\angle ACB = \frac{1}{2} \times AB$

Plan for→Three possibilities must be considered:

Proof Case 1 (Fig. 195)

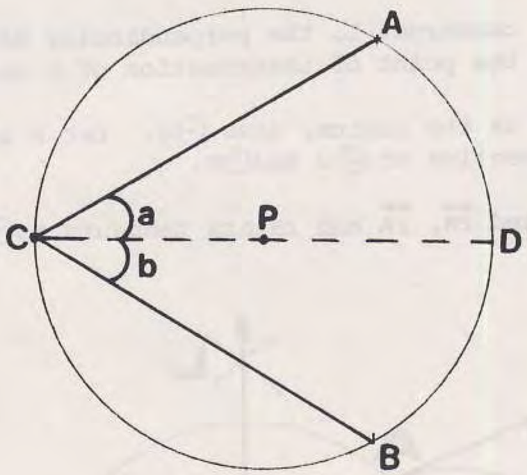
P lies on $\angle ACB$. Draw $\overline{AP}$

Show $\angle ACB \cong \angle a$

Show that $\angle ACB + a = \angle b$. But since $\angle b = \widehat{AB}$, We have $2 \times \angle ACB = \widehat{AB}$.

Case 2.

P lies in the interior of $\angle ACB$. Draw $\overrightarrow{CP}$. Use Case 1 to show $\angle a = \frac{1}{2} \times \widehat{AB}$, and $\angle b = \frac{1}{2} \times \widehat{BD}$. Show $\angle ACB = \angle a + \angle b$, so $\angle ACB = \frac{1}{2} \times \widehat{AD} + \frac{1}{2} \times \widehat{BD}$. Finally, show $\widehat{AD} + \widehat{DB} = \widehat{AB}$ and use transitivity.

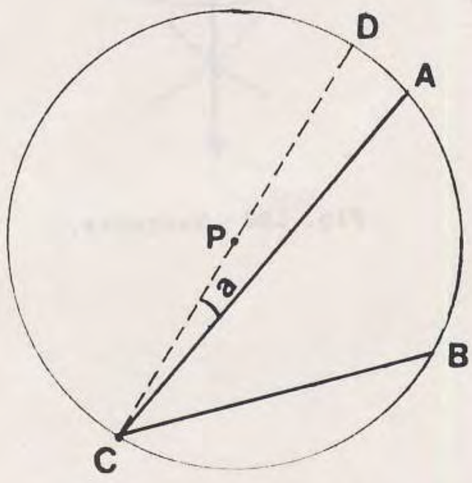


(Fig. 196)

Case 3.

P lies in the exterior of $\angle ACB$. Draw $\overline{CP}$. Show $\angle ACB = \angle DCB - a$.

Use Case 1 to show $\angle DCB = \frac{1}{2} \times \widehat{DB}$ and $a = \frac{1}{2} \times \widehat{DA}$. Finally, show $\angle ACB = \frac{1}{2} \times \widehat{DB} - \frac{1}{2} \times \widehat{DA}$, and $\widehat{AB} = \widehat{DB} - \widehat{DA}$. Then use substitution.



(Fig. 197)

6. Construct the tangents to a circle from a given point on the exterior of the circle

Given: ⊙P and point r on its exterior

Construct: Tangents to ⊙P from r.

Method: (Fig. 198)

Step 1. Draw $\overline{Pr}$, construct L, the perpendicular bisector of $\overline{Pr}$. Let O be the point of intersection of L and $\overline{Pr}$.

Step 2. Using OP as the radius, draw ⊙O. Let A and B be the points of intersection of ⊙O and ⊙P.

Step 3. Draw $\overleftrightarrow{rA}$ and $\overleftrightarrow{rB}$, $\overleftrightarrow{rA}$ and rB are tangents to ⊙P from point r.

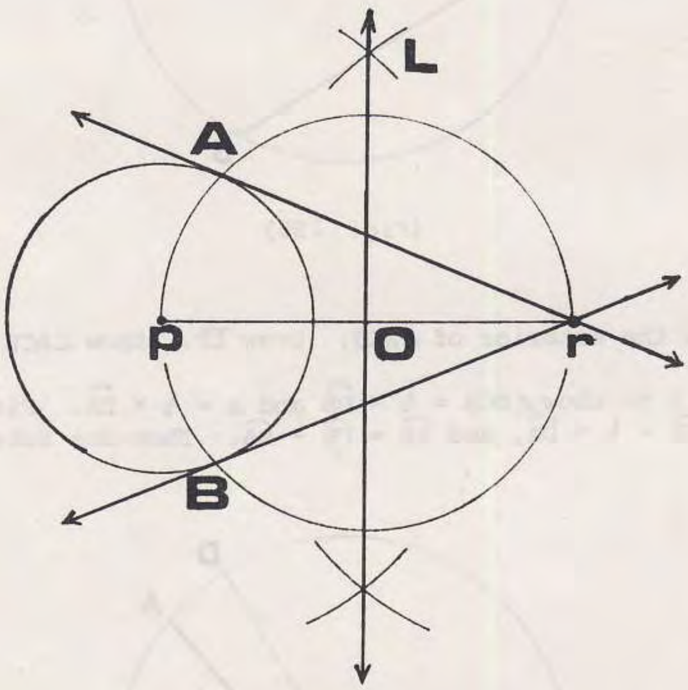
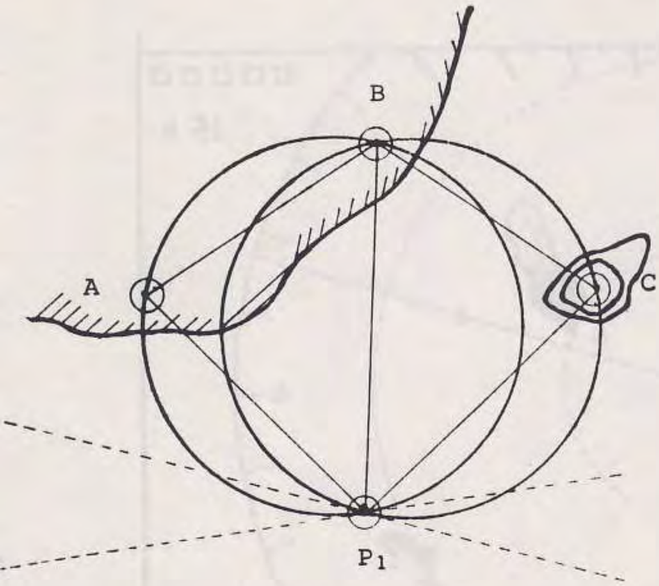
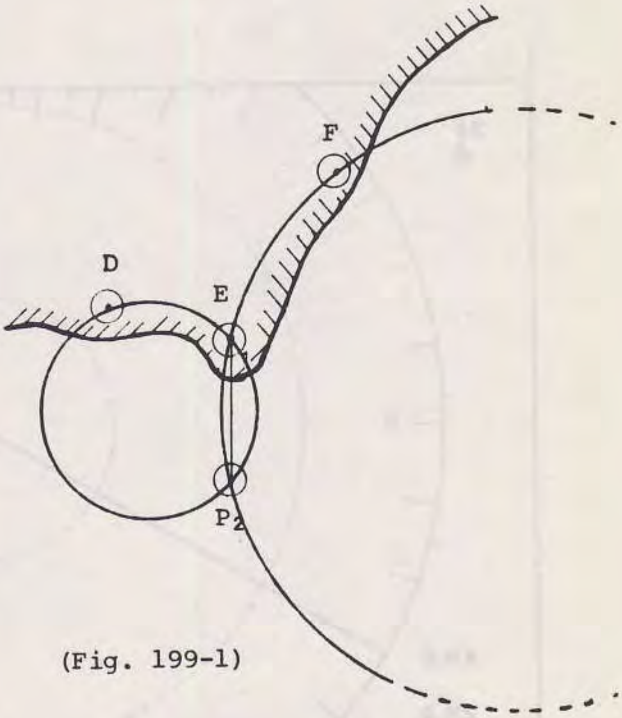


Fig. 198 Tangents.



(Fig. 199)



(Fig. 199-1)

The intervals of the two circles of position are very close together.

Position Fix by Two Circles of Position

1) In Fig. 199, the intervals of the two circles are very close together and the two tangent lines meet at angles with a very small value (about 23 degrees). This means the **determined** position P_1 is inaccurate. Such circles are not recommended to fix position.

2) In Fig. 199-1, the landmark E located between D and F is nearer the observer's position (P_2) than D and F. The accuracy of determined position (P_2) is slight. Therefore, it is not recommended to select such a landmark as E to fix position.

Radar

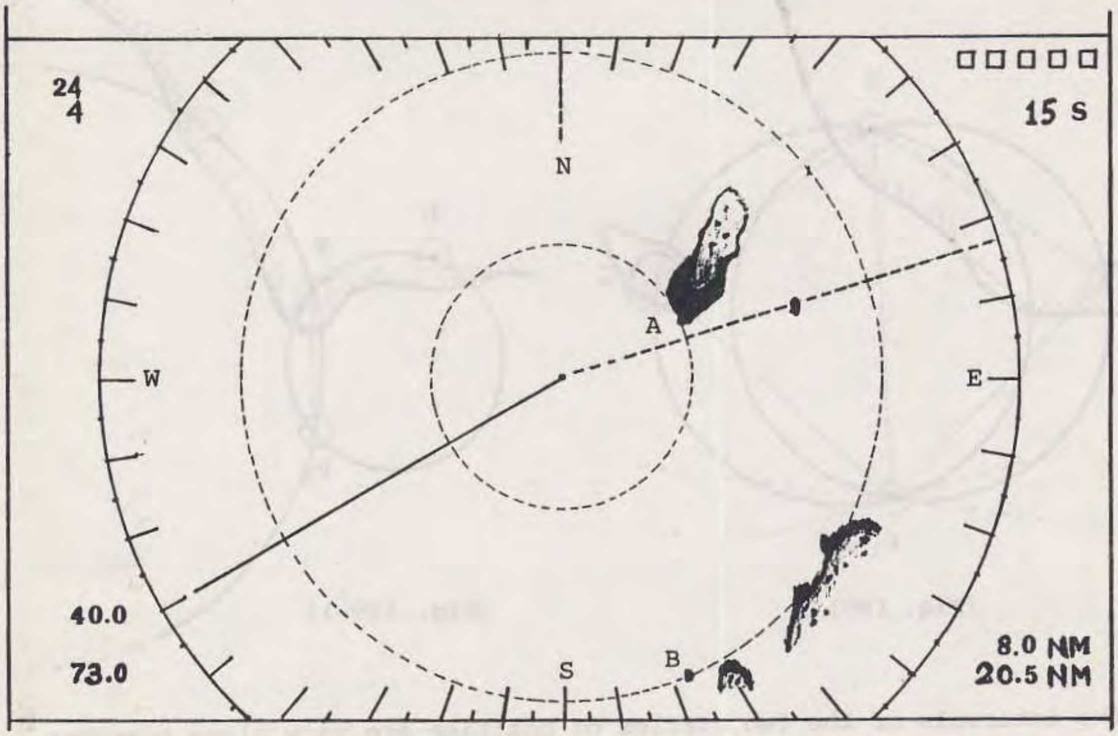


Fig. 200 Radar Display

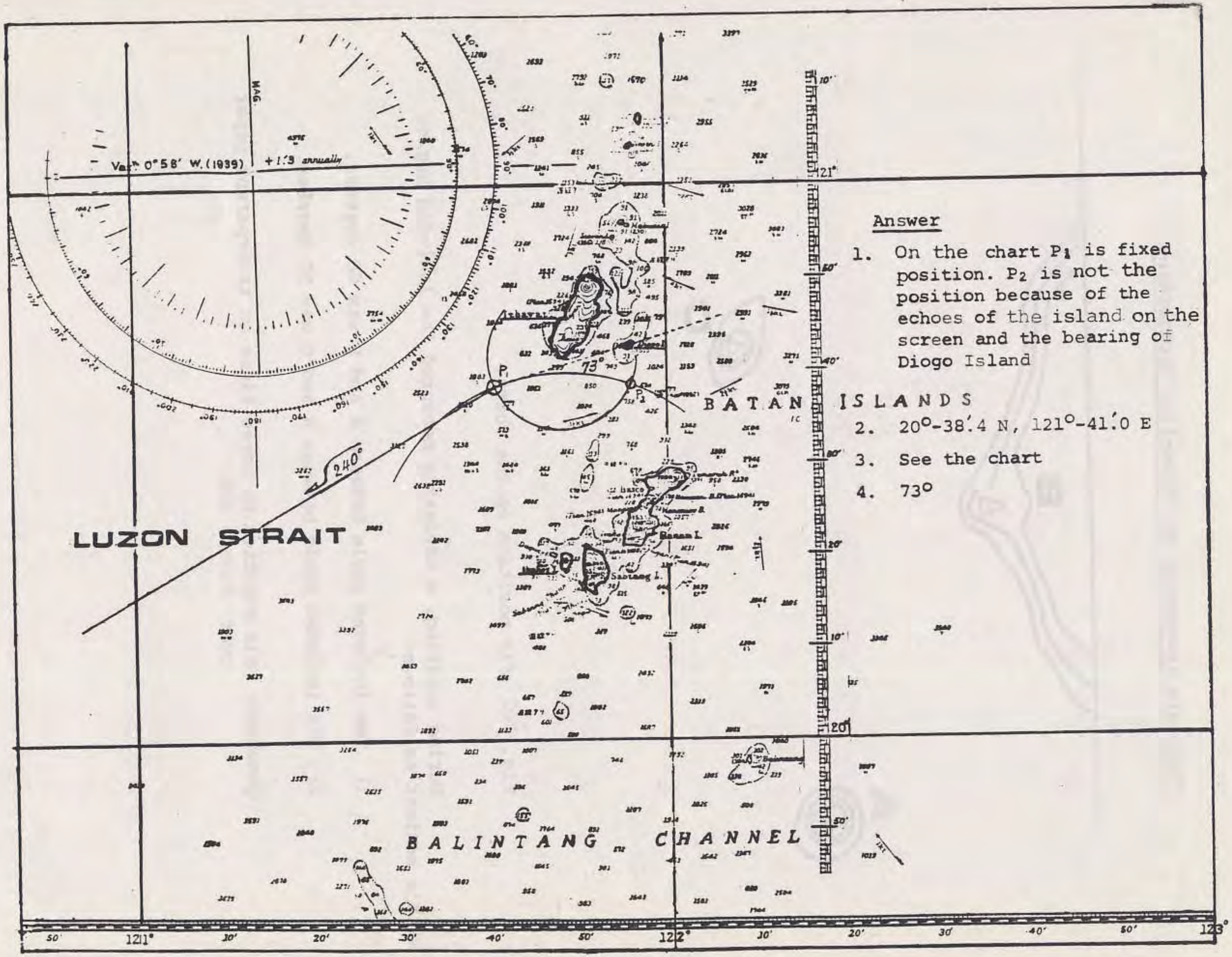
In Figure 200, A ship's radar screen is showing diget data and **echoes from islands. Check the echoes and digits on the screen carefully, then answer the questions.**

Question:

1. **Fix ship's position on the attached chart (Figure** 201).
2. **And record its longitude and latitude.**
3. **Draw the course line from the fixed position on the attached chart (Figure** 201).
4. **Record the bearing of Diogo Island.**

Ref.

On the radar screen, A is the southern end of Itbayat Island, and B is the top of the mountain on Ibahos Island.



Answer

1. On the chart P_1 is fixed position. P_2 is not the position because of the echoes of the island on the screen and the bearing of Diogo Island
2. 20°-38′.4 N, 121°-41′.0 E
3. See the chart
4. 73°

Fig. 201 Northern part of the Philippines.

Three-arm protractor or Three-line Protracting

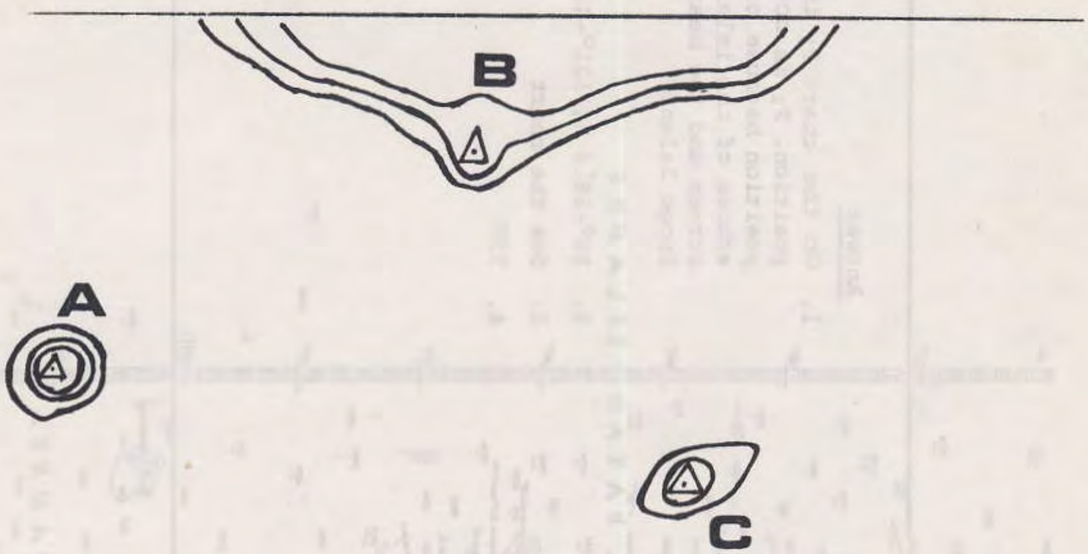


Fig. 202 Fix position on the chart.

During coasting, a navigator measured the included angles with a sextant as follows:

1) The included angle between A and B was 60 degrees.

2) The included angle between B and C was 50 degrees.

Question: Fix position by three lines on transparent paper over Figure 202.

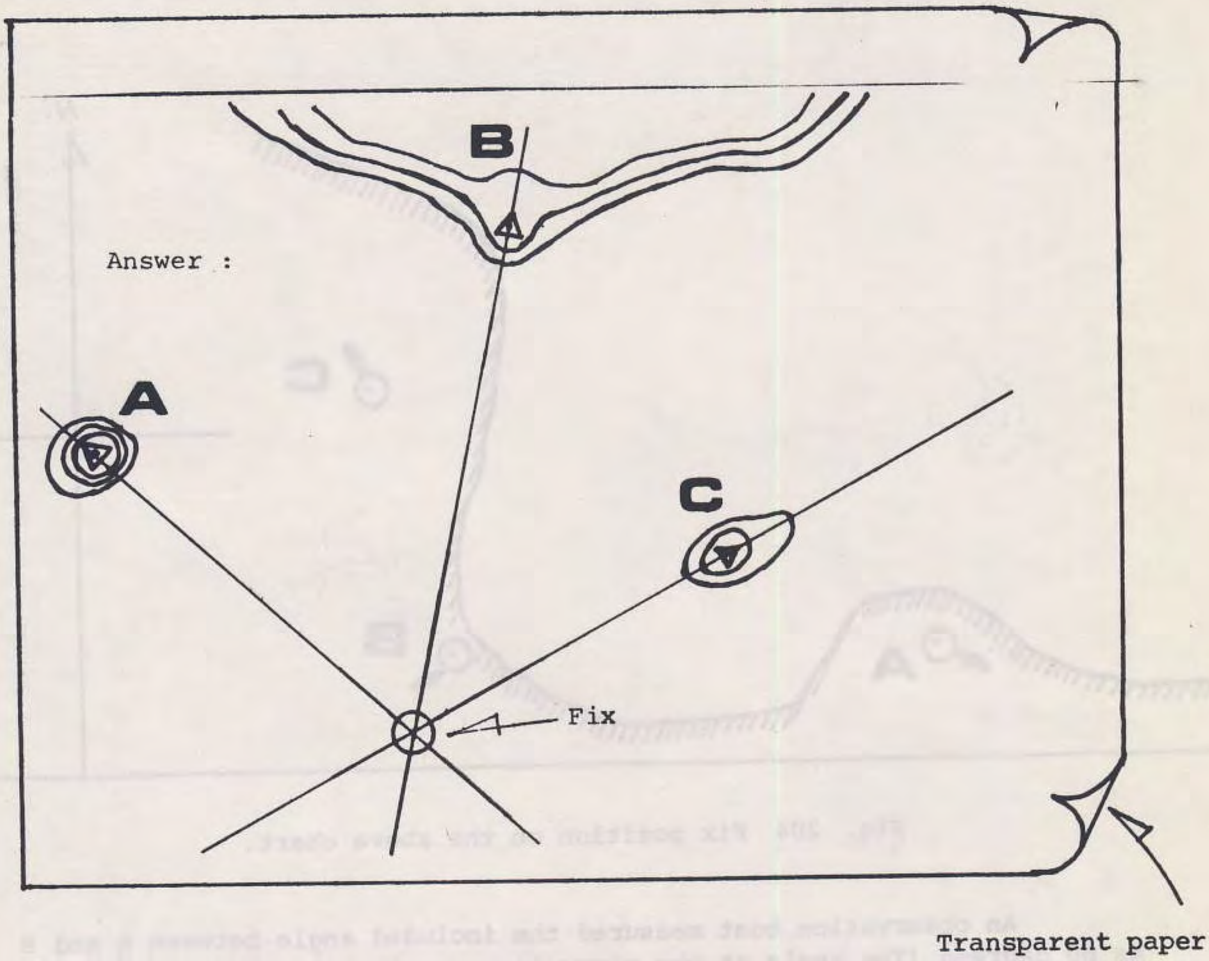


Fig. 203 Fix using Transparent paper

< PAB 60°

< PBC 50°

Fix Using the Angle at the Circumference and a Bearing Line

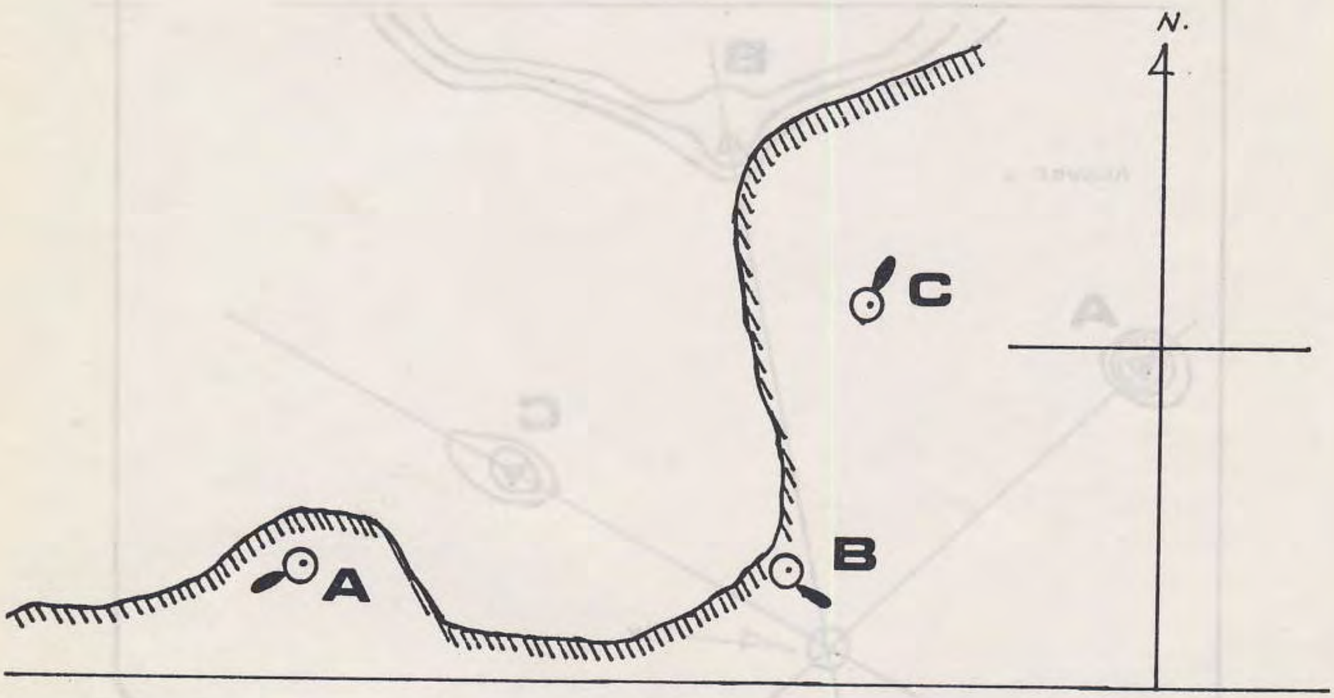


Fig. 204 Fix position on the above chart.

An observation boat measured the included angle between A and B as 60 degrees (The angle at the circumference: 60 degrees) and the bearing of C was 95 degrees.

Question:- Fix position on Figure 204.

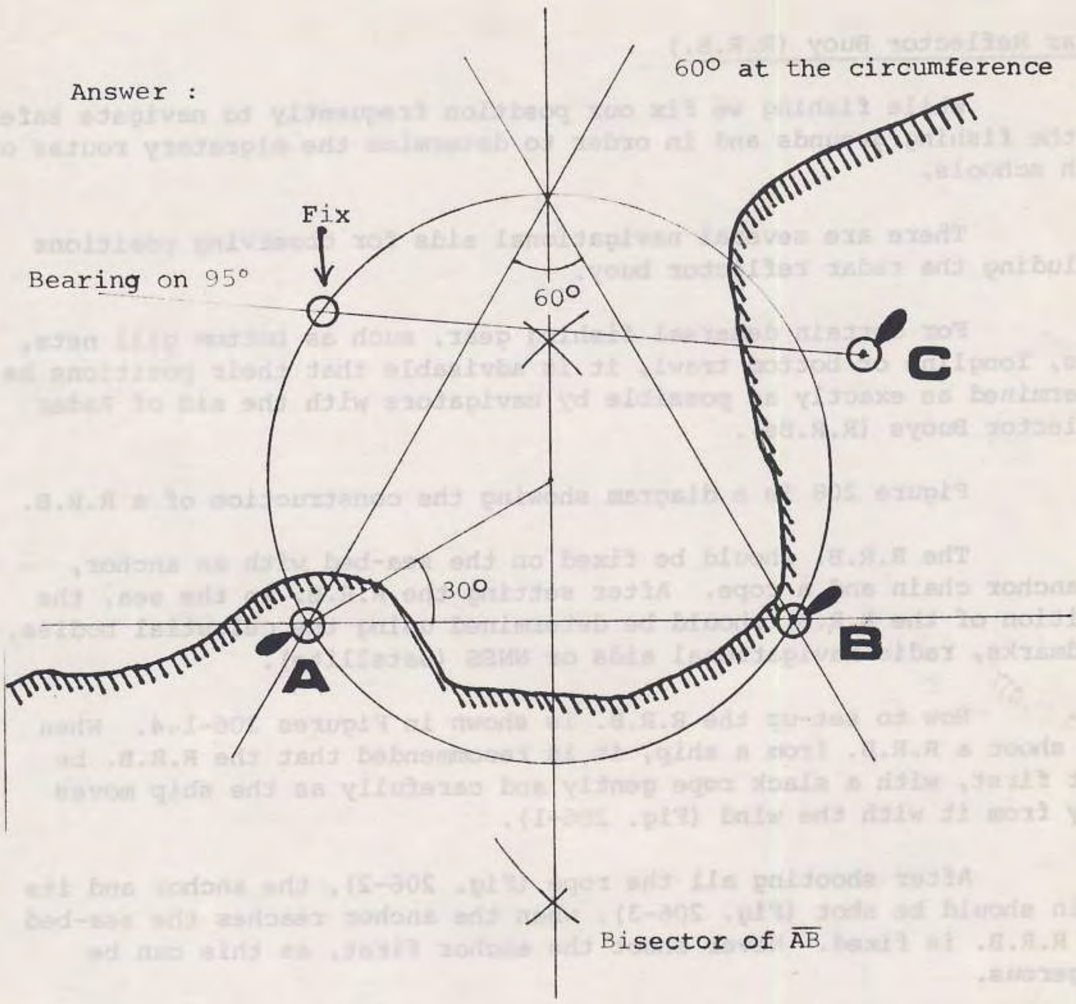


Fig. 205 A bearing line and a circle of position.

Radar Reflector Buoy (R.R.B.)

While fishing we fix our position frequently to navigate safely in the fishing grounds and in order to determine the migratory routes of fish schools.

There are several navigational aids for observing positions including the radar reflector buoy.

For certain demersal fishing gear, such as bottom gill nets, pots, longline or bottom trawl, it is advisable that their positions be determined as exactly as possible by navigators with the aid of Radar Reflector Buoys (R.R.Bs).

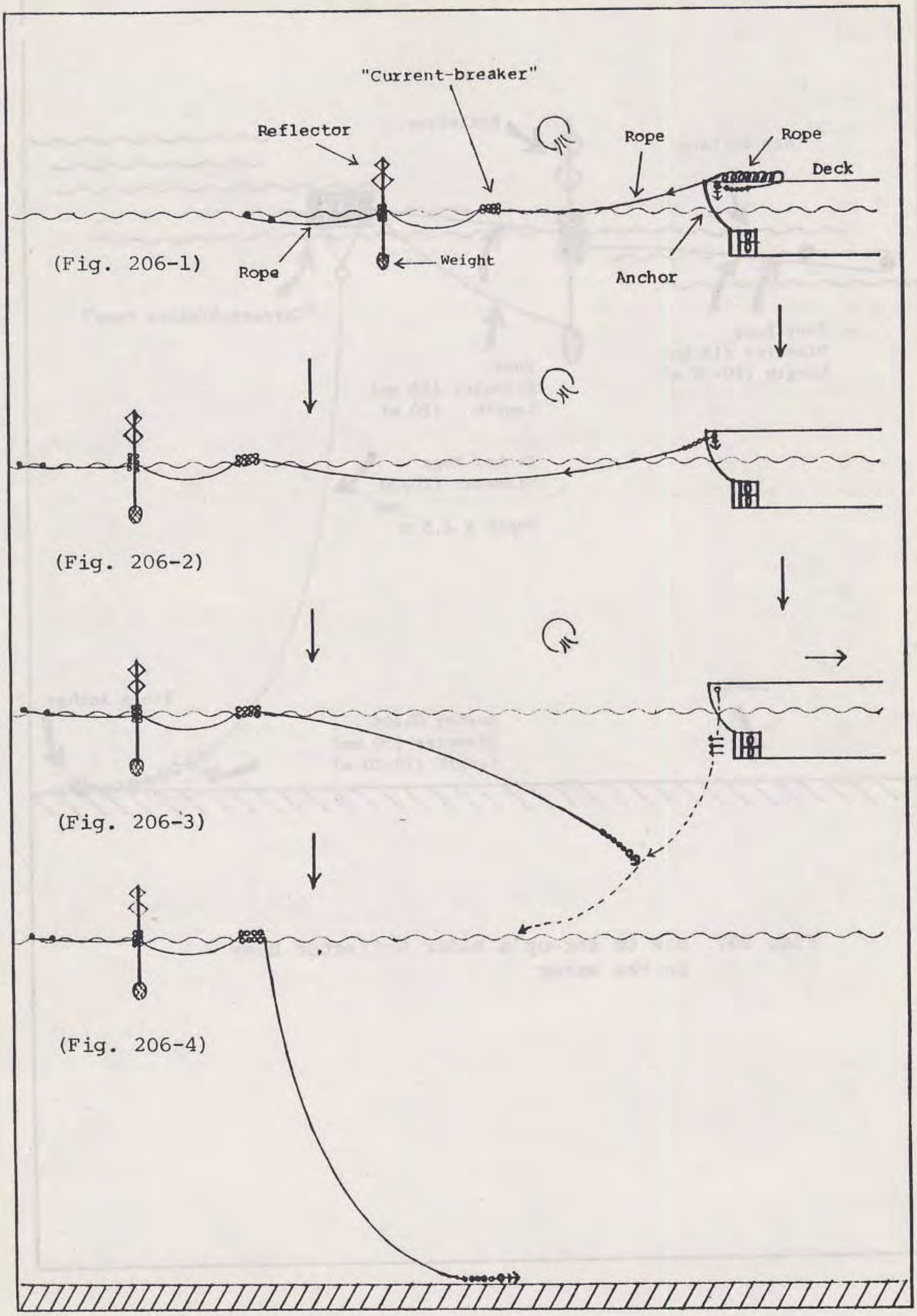
Figure 208 is a diagram showing the construction of a R.R.B.

The R.R.B. should be fixed on the sea-bed with an anchor, an anchor chain and a rope. After setting the R.R.B. in the sea, the position of the R.R.B. should be determined using the celestial bodies, landmarks, radio navigational aids or NNSS (Satellite).

How to set-up the R.R.B. is shown in Figures 206-1~4. When you shoot a R.R.B. from a ship, it is recommended that the R.R.B. be shot first, with a slack rope gently and carefully as the ship moves away from it with the wind (Fig. 206-1).

After shooting all the rope (Fig. 206-2), the anchor and its chain should be shot (Fig. 206-3), when the anchor reaches the sea-bed the R.R.B. is fixed. Never shoot the anchor first, as this can be dangerous.

How to shoot a Radar Reflector Buoy



(Fig. 206-1)

(Fig. 206-2)

(Fig. 206-3)

(Fig. 206-4)

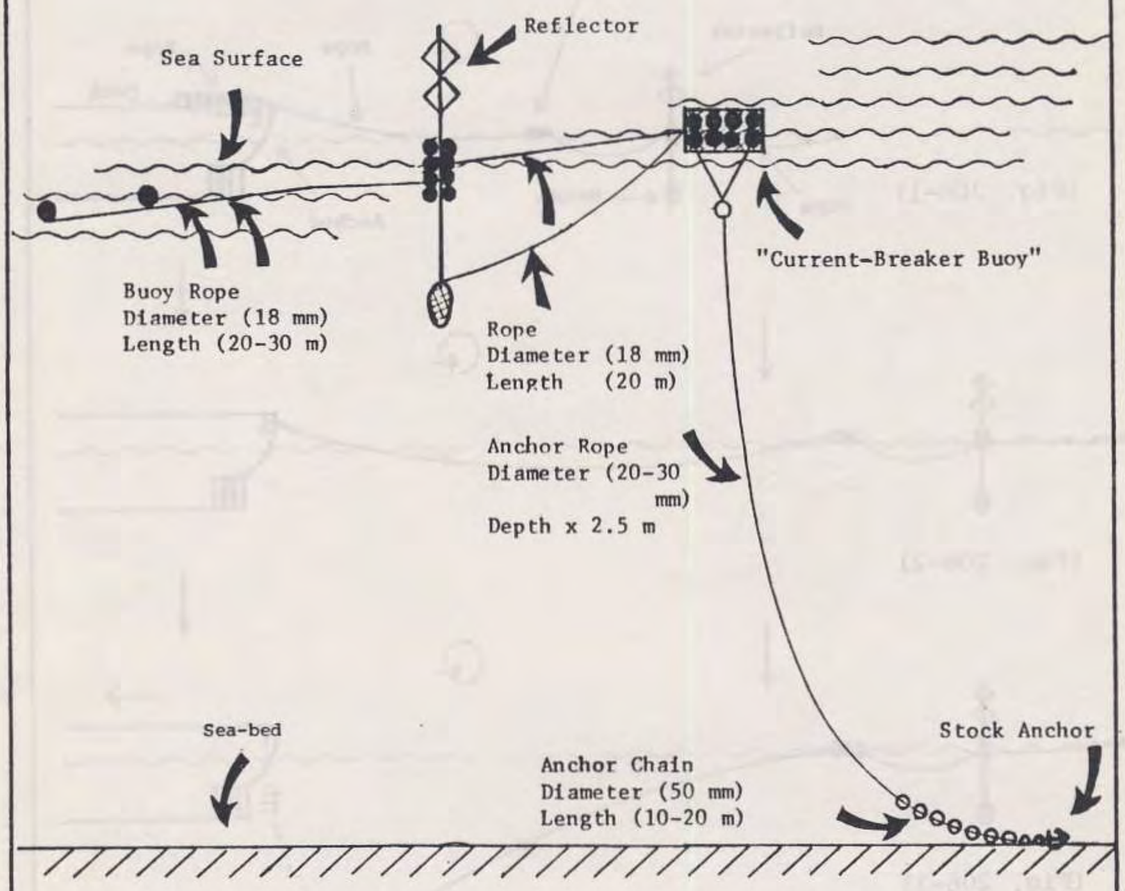


Fig. 207 How to set-up a Radar Reflector Buoy in the water

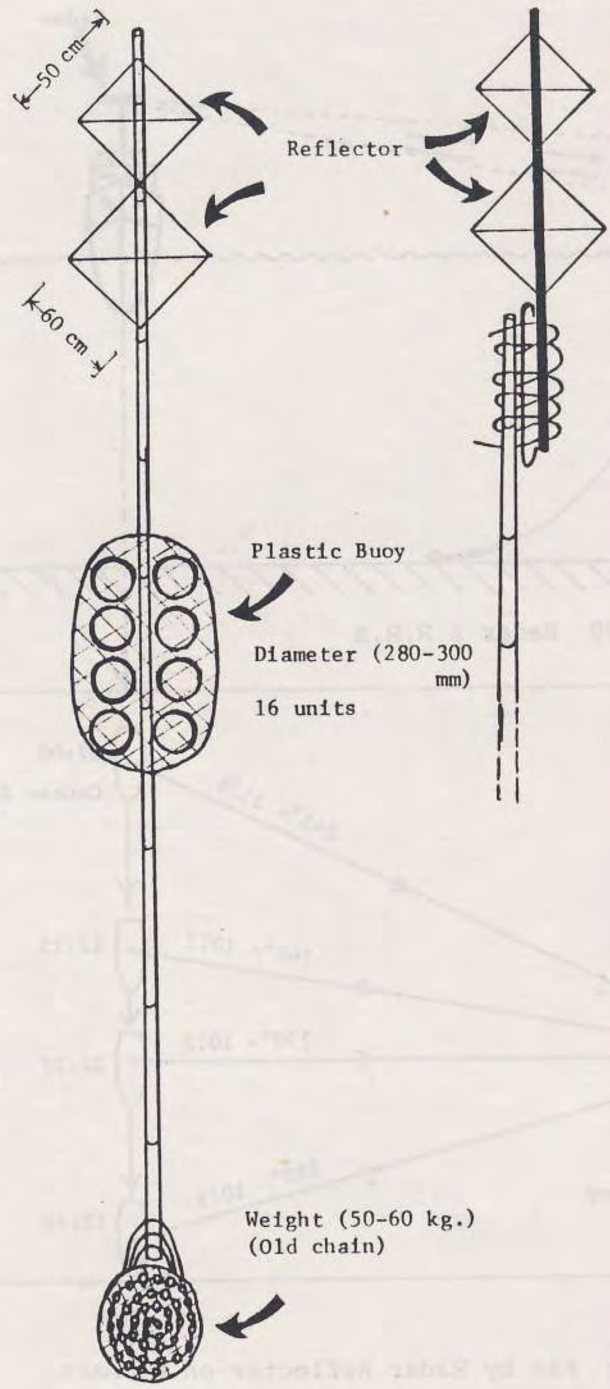


Fig. 208 Construction of a Radar Reflector Buoy

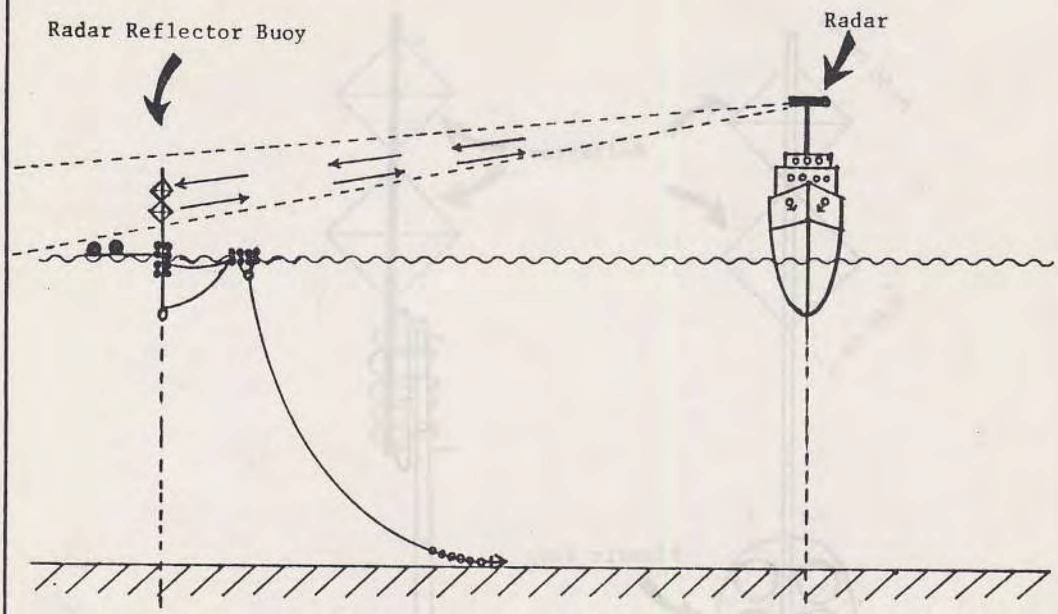


Fig. 209 Radar & R.R.B

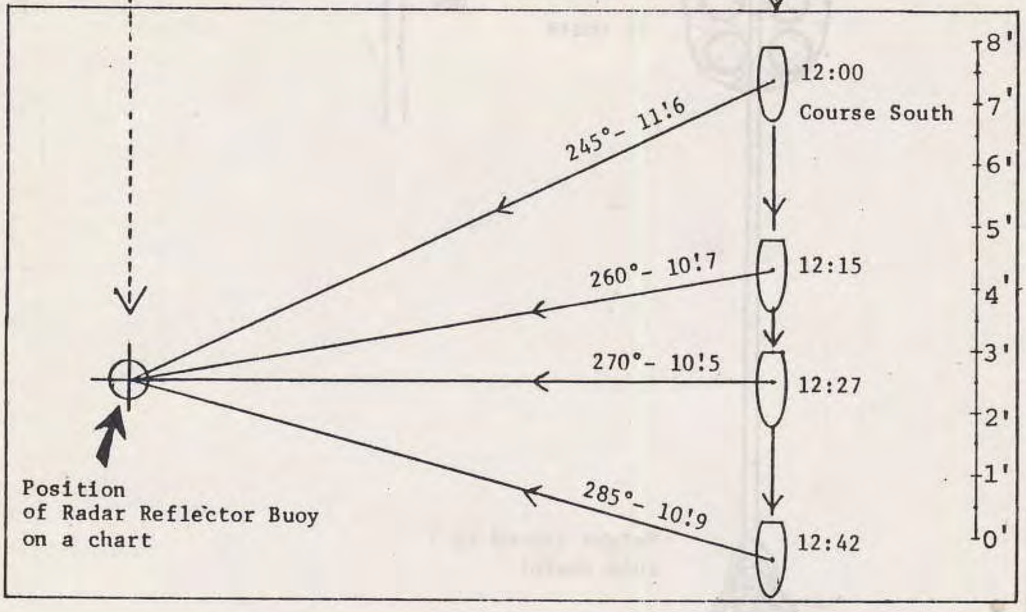


Fig. 210 Fix by Radar Reflector on a chart.

Example

B'g, Dist. and Depth by R.R.B.

No.	Time	True B'g	Dist	Depth	P^n by Radar Reflector
1	12:00	245°	11!6	30 m	12° 25!30N, 100°- 43!0E
2	12:15	260°	10!7	30 m	12° 22!26N, 100°- 43!0E
3	12:17	270°	10!5	29 m	12° 20!46N, 100°- 43!0E
4	12:42	285°	10!9	32 m	12° 17!60N, 100°- 43!0E
...	....	....	....	....	....
...	....	....	....	....	....

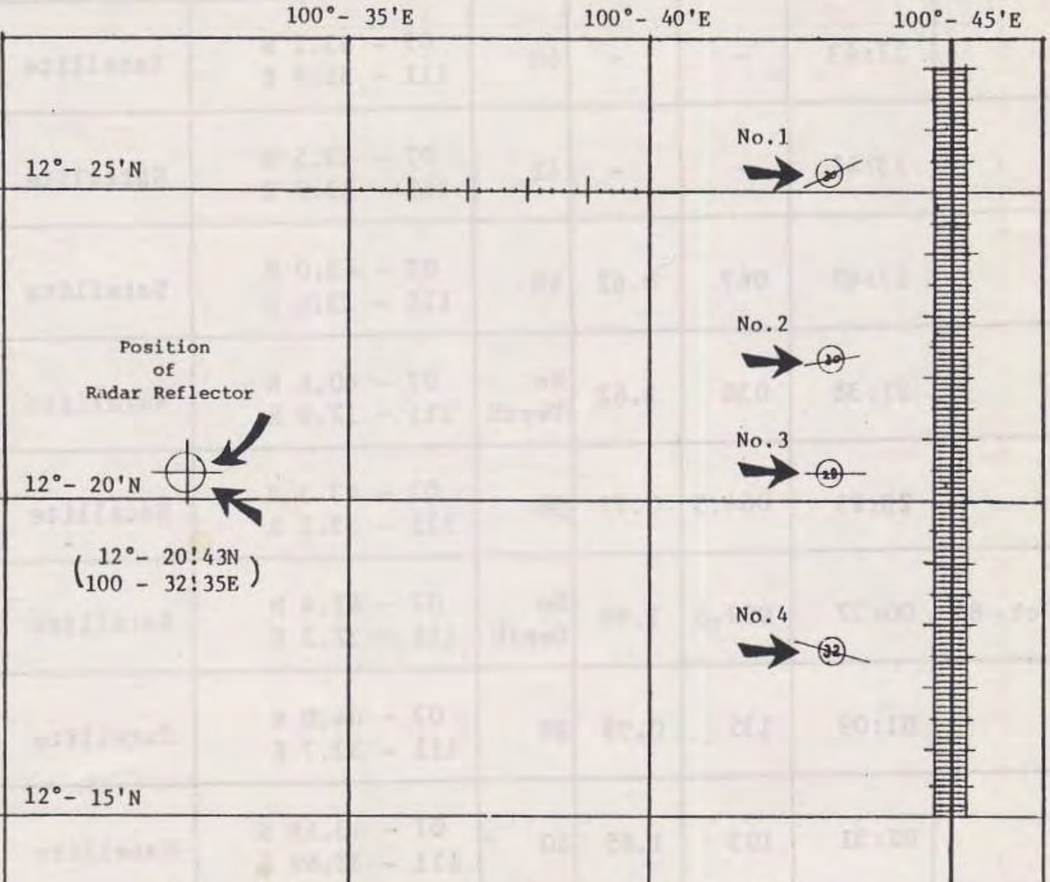


Fig. 211 Positions & Depths on a chart

Bearing & Distance of Radar Reflector Buoy

Fishing Ground: Rifleman Bank in the South China Sea

Zone Time: 07h 30m

M.V. PAKNAM Cruise: 69-4/1984

No.	Date	L.M.T.	True B'g	Dist.	Depth	Position	P^n by means of
1.	2 Oct. 84	11:29	118°	0!80	47 m	07 - 43.8 N 111 - 34.6 E	Radar Reflector
2.		11:53	110	0.82	48	07 - 43.37 N 111 - 33.01 E	Satellite
3.		12:43	-	-	60	07 - 43.1 N 111 - 31.9 E	Satellite
4.		13:41	-	-	42	07 - 43.5 N 111 - 33.9 E	Satellite
5.		17:49	067	0.62	58	07 - 43.0 N 111 - 33.6 E	Satellite
6.		21:38	038	3.62	No Depth	07 - 40.6 N 111 - 32.0 E	Satellite
7.		23:25	064.5	0.77	58	07 - 43.2 N 111 - 33.2 E	Satellite
8.	3 Oct. 84	00:37	069.5	1.99	No Depth	07 - 42.4 N 111 - 32.2 E	Satellite
9.		01:09	135	0.98	20	07 - 44.0 N 111 - 33.7 E	Satellite
10.		02:31	103	1.65	50	07 - 43.18 N 111 - 32.69 E	Satellite

No.	Date	L.M.T	True B'g	Dist	Depth	Position	P^n by means of
11.	3 Oct. 84	04:51	022	2.96	16	07 - 39.86 N 111 - 35.19 E	Radar Reflector
12.		05:07	030	2.70	46	07 - 40.29 N 111 - 35.19 E	Radar Reflector
13.		05:45	128	1.17	16	7 - 43.09 N 111 - 35.74 E	Radar Reflector
14.		07:35	035.5	2.66	87	07 - 41.36 N 111 - 32.8 E	Satellite
15.		08:44	-	-	16	07 - 43.7 N 111 - 34.1 E	Dead Reckoning
16.		09:26	133	1.74	94	07 - 44.4 N 111 - 33.2 E	Satellite
17.		12:06	-	-	72	07 - 50.4 N 111 - 41.0 E	Dead Reckoning
18.		12:21	-	-	76	07 - 50.5 N 111 - 37.8 E	Satellite
19.		12:38	-	-	78	07 - 49.9 N 111 - 36.9 E	Dead Reckoning
20.		12:50	-	-	74	07 - 50.6 N 111 - 37.6 E	Satellite
21.		13:10	-	-	66	07 - 51.8 N 111 - 39.7 E	Dead Reckoning
22.		13:18	212	9.4	74	07 - 52.25 N 111 - 40.45 E	Radar Reflector
23.		14:11	200.5	7.3	82	07 - 49.84 N 111 - 36.68	Satellite

No.	Date	L.M.T.	True B'g	Dist.	Depth	Position	P^n by means of
24.	4 Oct. 84	09:00			68	07 - 42.7 N 111 - 33.9 E	Radar Reflector
25.		09:43			24	07 - 38.6 N 111 - 39.7 E	Satellite

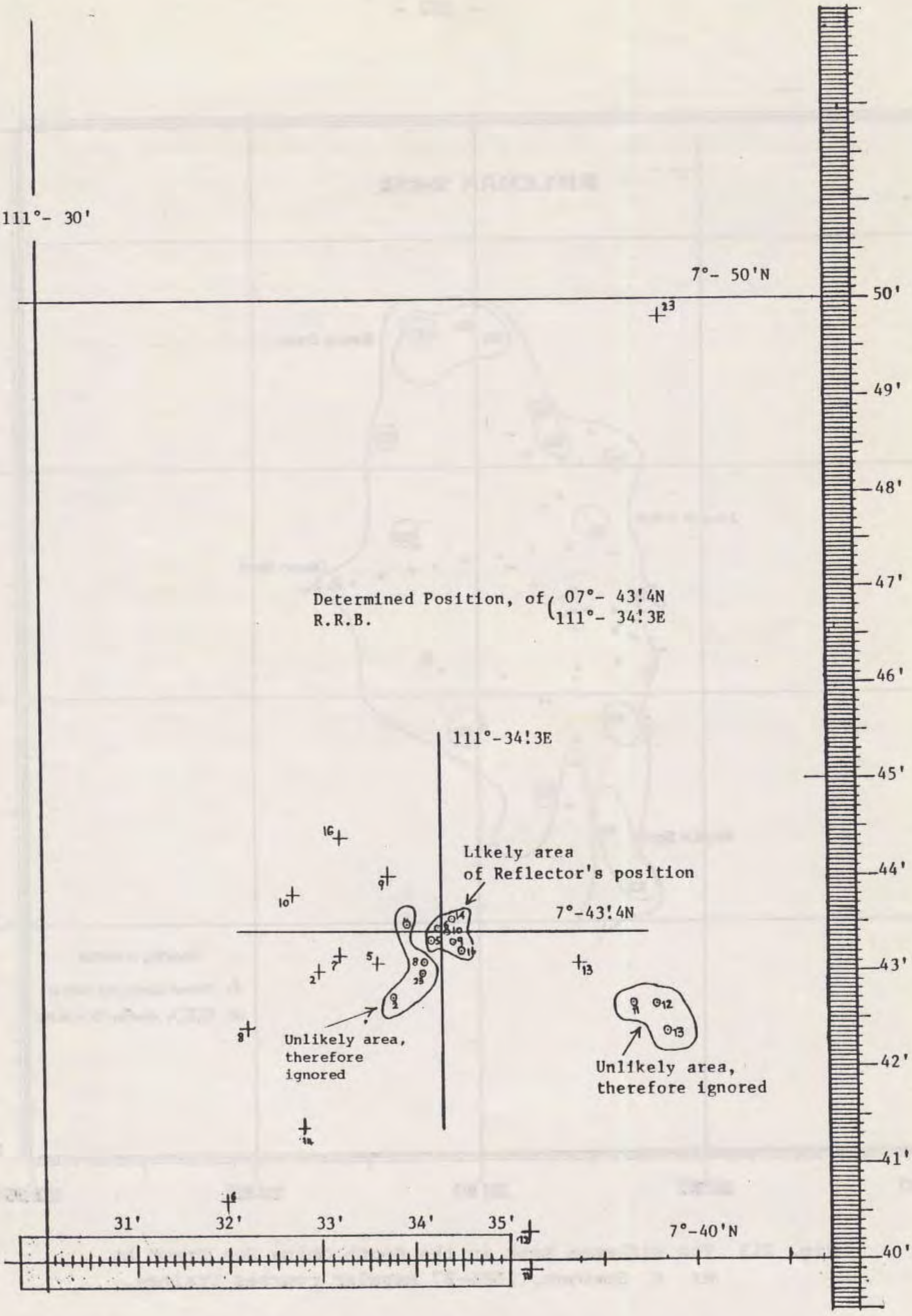


Fig. 212 How to determine the position of the Radar Reflector Buoy by satellite.

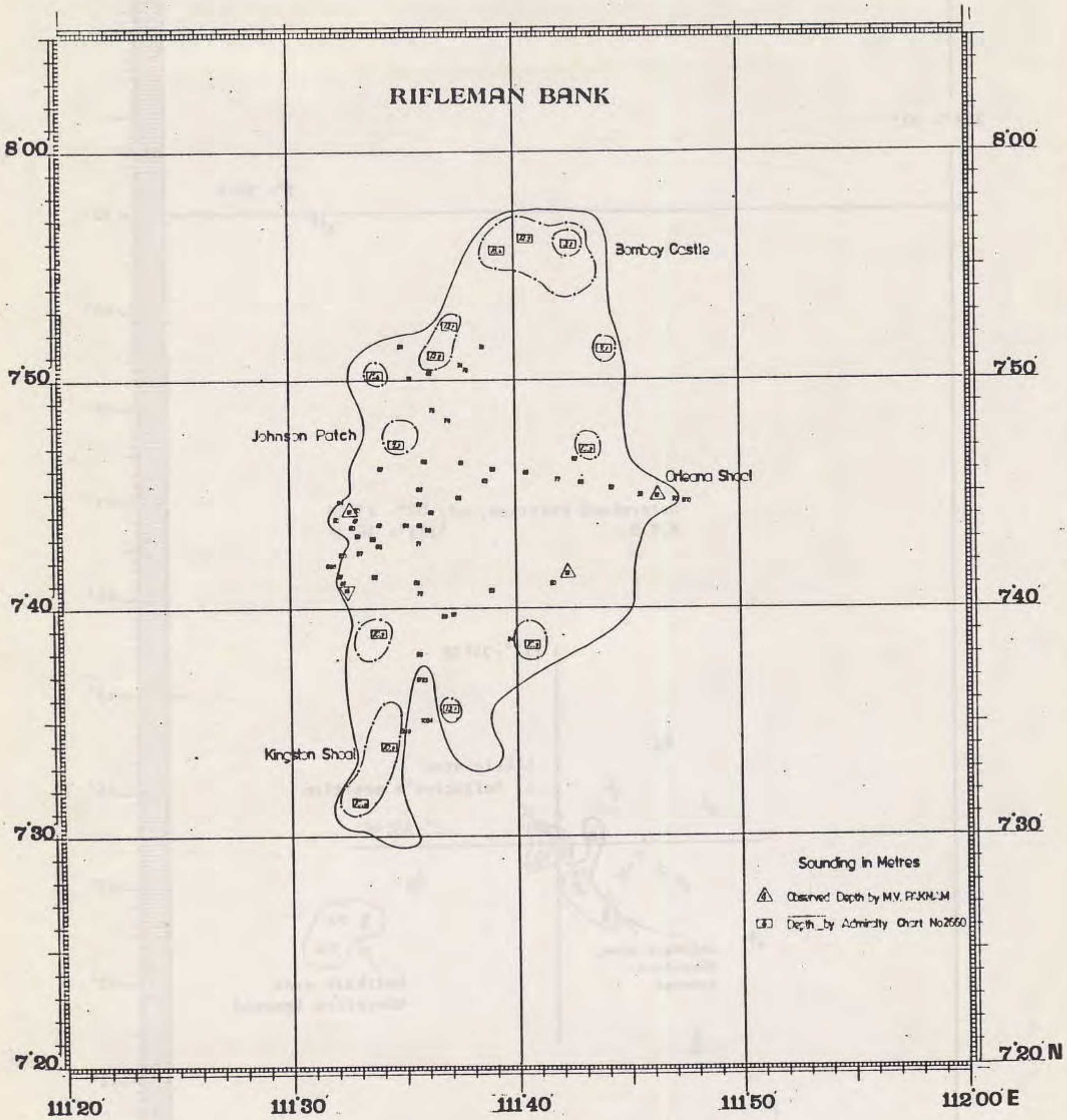


Fig. 213 The Rifleman Bank in the South China Sea Drawn by Mr. S. Somboon, 1986-87 Regular courses Trainee.

Fig. 213-1 Radar Reflector on board M.V. PAKNAM in the South China Sea, 24/2/1987

Fig. 213-3 Observing Included Angles between Samui and Phangan Island 22 Sept. 1987 M.V. PAKNAM.

Observer : Mr. Krishananansamy s/o Arunasalam a 1985-1987 Fishing Technology Course Trainee.

Fig. 213-4 Fix by Sextant & Three-arm protractor 22 Sept. 1987 M.V. PAKNAM

Three-arm protractor and a sextant

A navigator observes the included angles between the landmarks A and B, and B and C as 20° 08'.2 and 32° 15'.6 respectively with a sextant.

In the case of Fig. 214, the observer's position by a three-arm protractor is P.

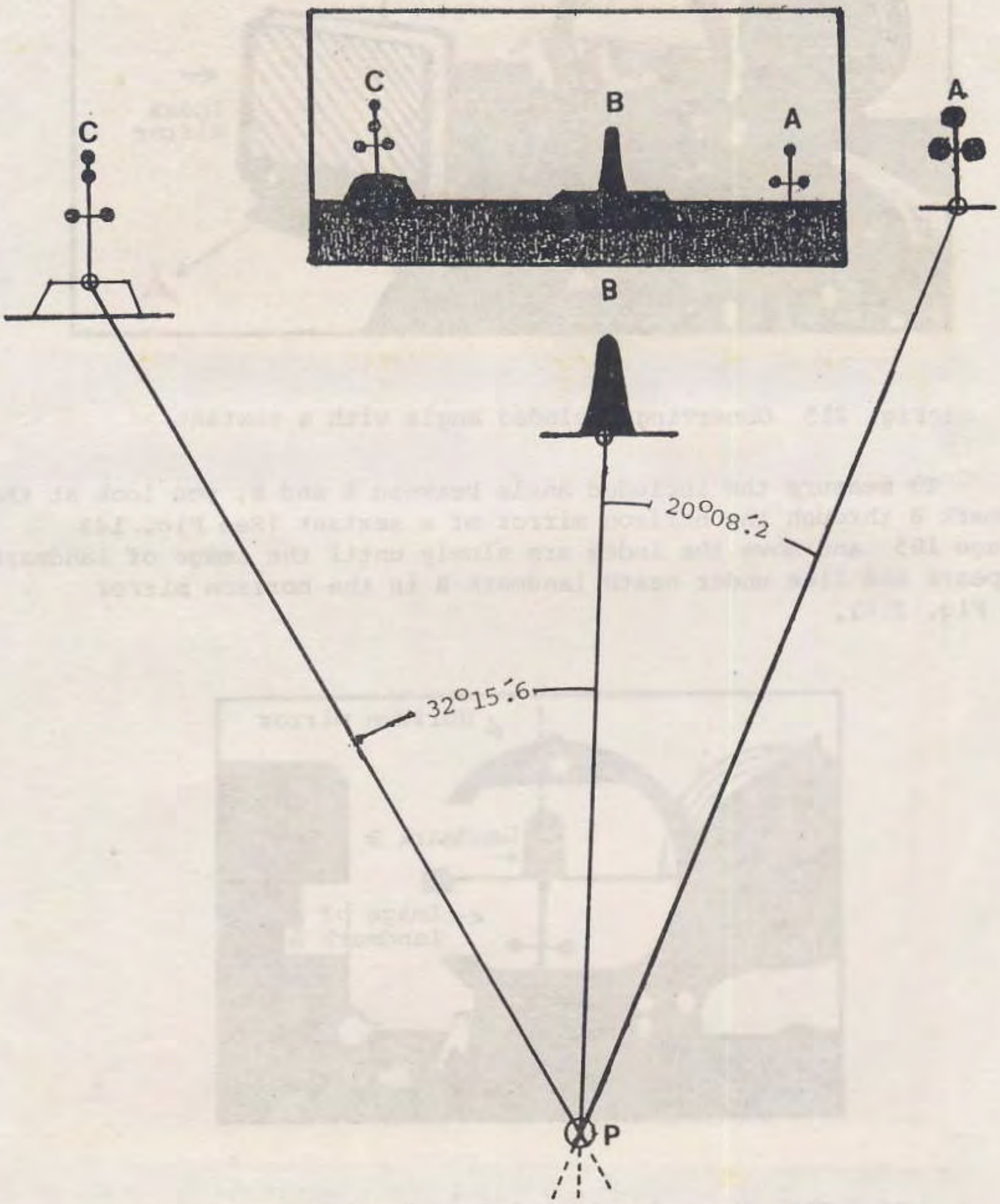


Fig. 214 Fix by Three-arm protractor

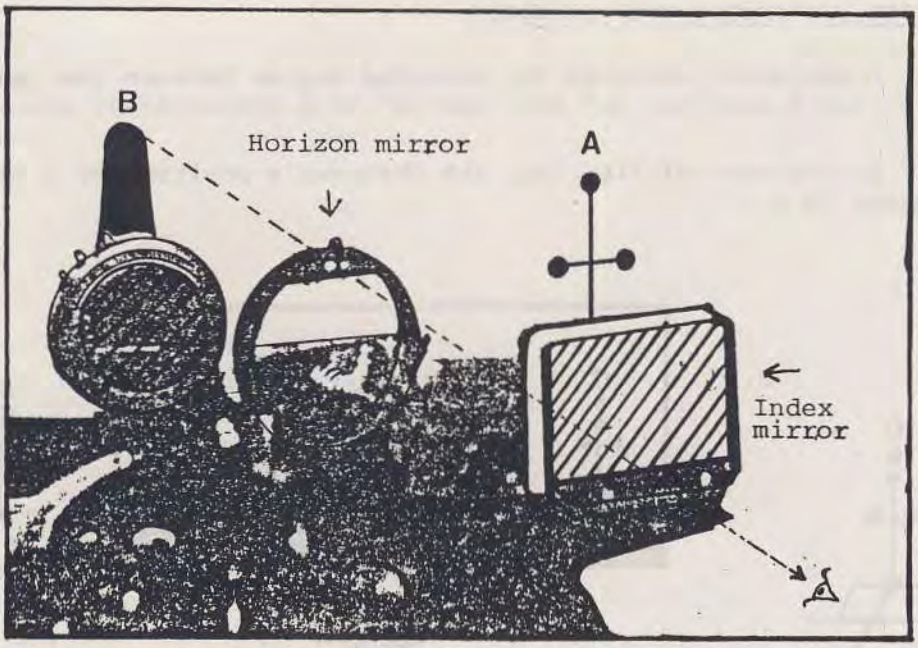


Fig. 215 Observing included angle with a sextant

To measure the included angle between A and B, you look at the landmark B through the horizon mirror of a sextant (See Fig. 143 on page 185 and move the index arm slowly until the image of landmark A appears and lies under neath landmark B in the horizon mirror (See Fig. 216).

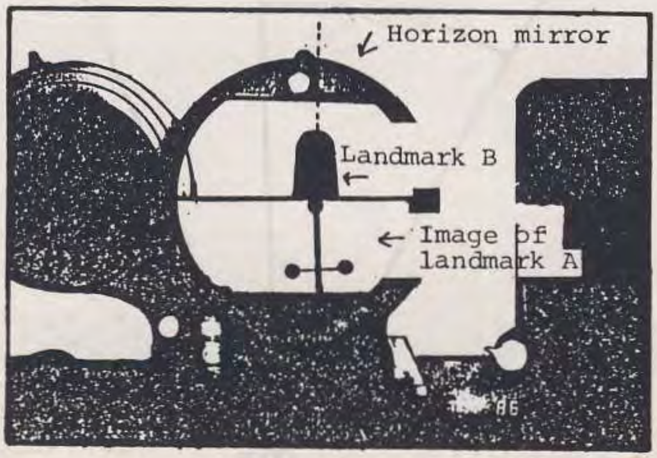


Fig. 216 Observing included angle with a sextant.

The landmark B and the image of landmark A should be in a line vertically.

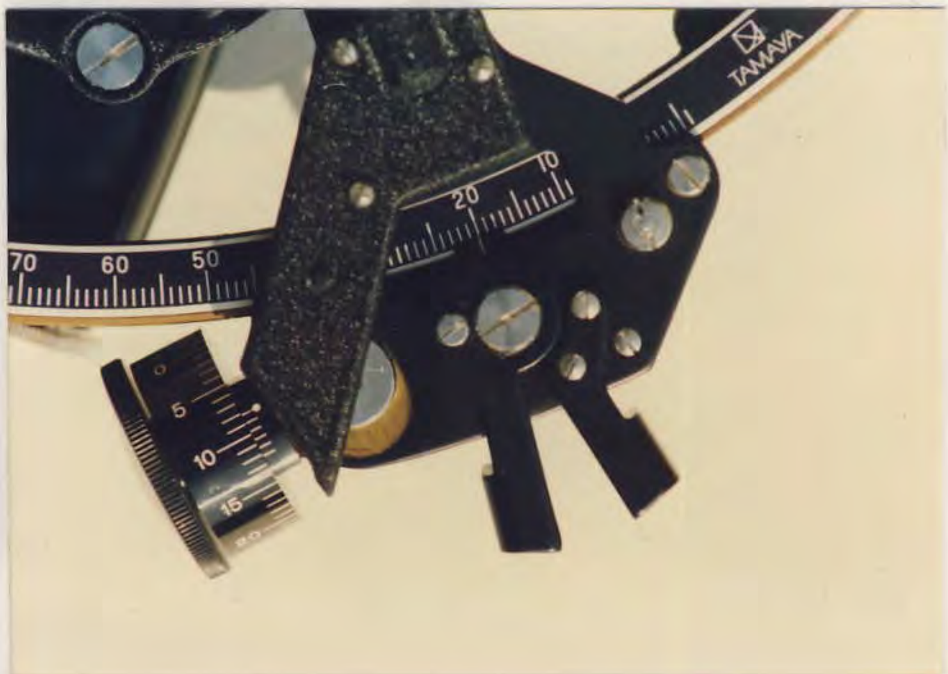


Fig. 217 The reading 20°08'.2 is the included angle between landmarks A and B.

Next, you look at landmark C through the index mirror (See Fig. 218) and, move the index arm from zero degree reading clockwise until the image of landmark B appears and lies underneath landmark C (See Fig. 219).

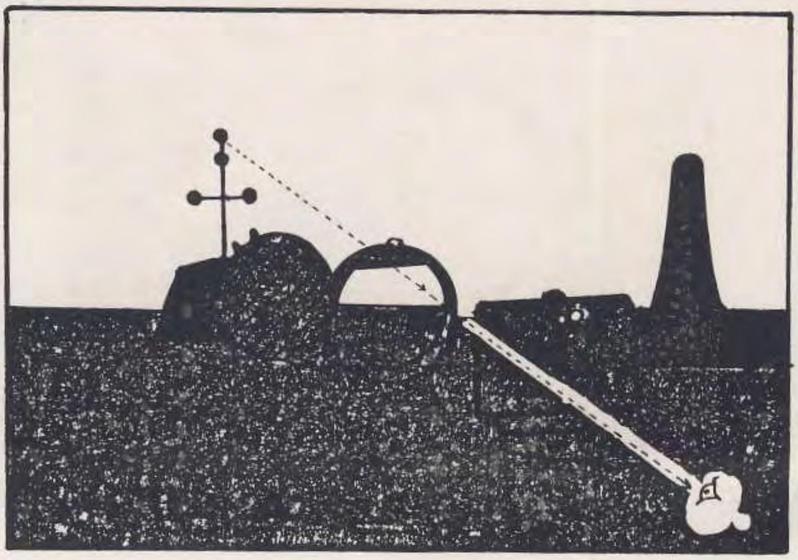


Fig. 218 Observing included angle with a sextant

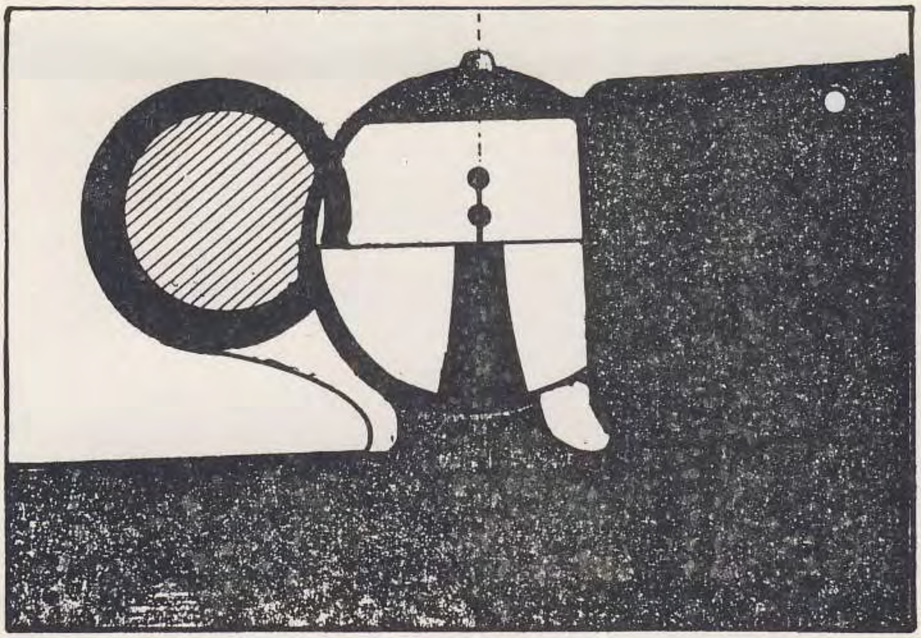


Fig. 219 Observing included angle with a sextant

Landmark C and the image of landmark B are in a line vertically.

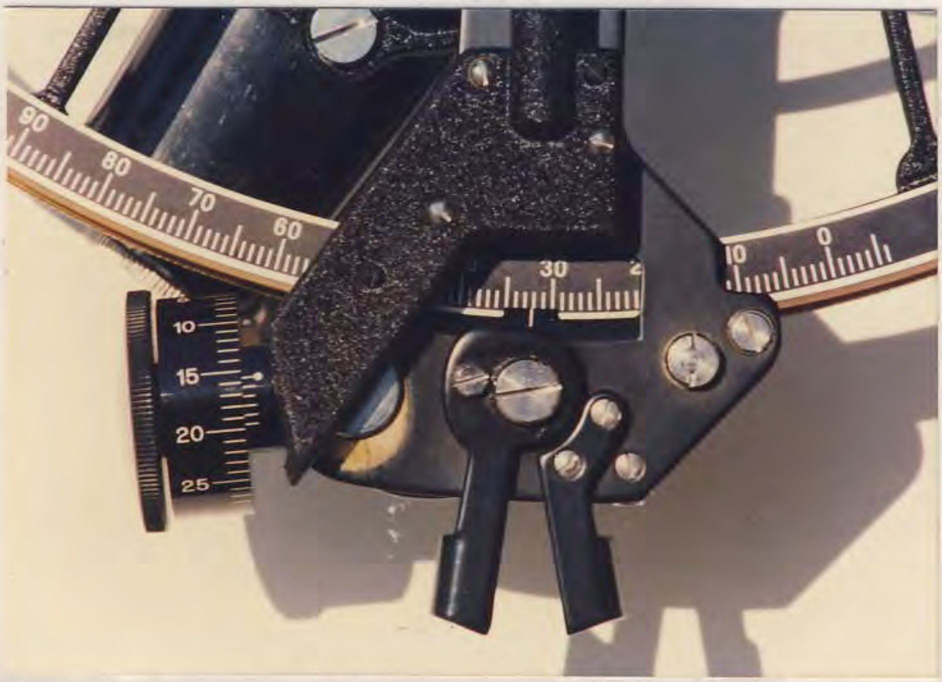


Fig. 220 The reading 32°15.'6 is the included angle between landmarks B and C.

After reading the two included angles between A and B, and B and C, (∠ APB, & ∠ BPC in Fig. 214), the two included angles measured are plotted with a three-arm protractor. This device has a circular scale representing degrees and minutes and three arms are attached. The central arm is fixed and the others are rotatable. These three arms can be set and locked at angles to the central arm in accordance with the angles observed by a sextant.

Fig. 221 Three-arm protractor

The right movable arm of the protractor is set to the included angle 20 degrees and 08.2 minutes between landmarks A and B.

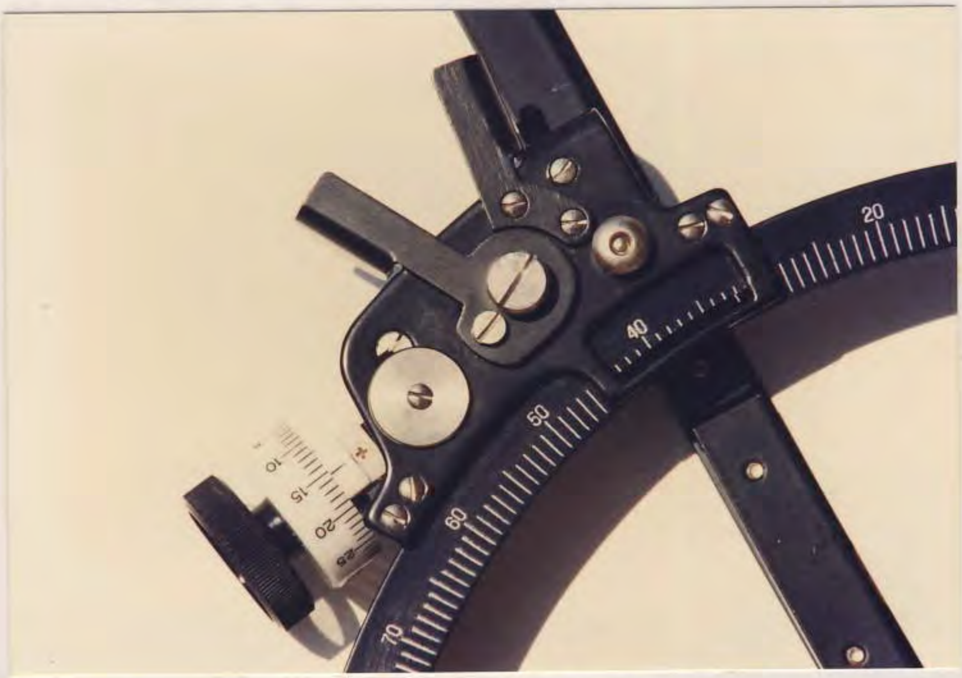


Fig. 222 Three-arm protractor

The left movable arm of the same protractor is set to the included angle 32 degrees and 15.6 minutes between landmarks B and C.

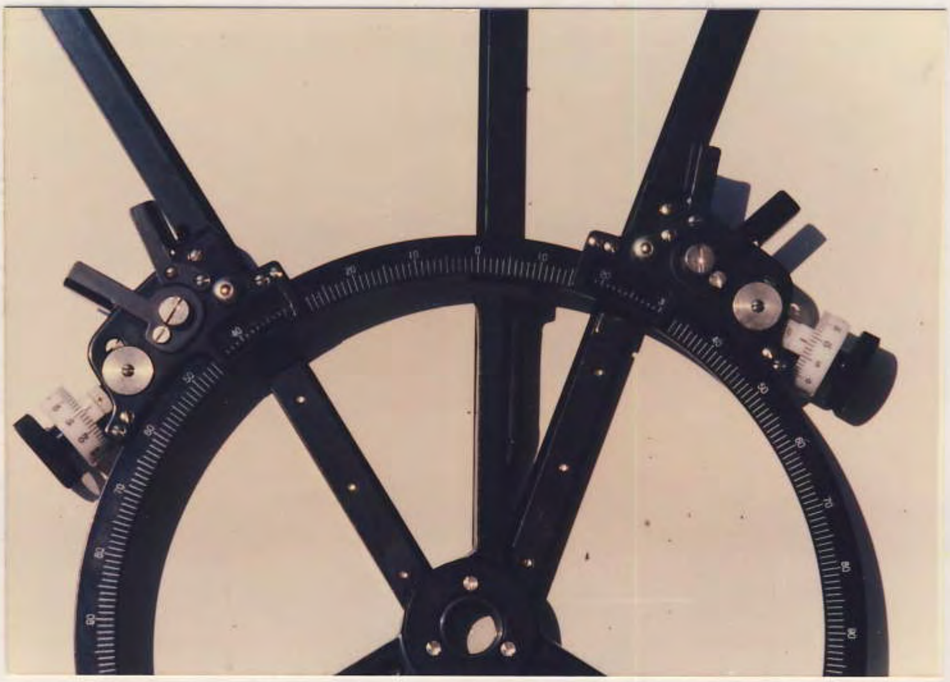


Fig. 223 Three-arm protractor

The movable arms are set to the angles, $20^{\circ}08'.2$ (right) and $32^{\circ}15'.6$ (left), then locked.

Fig. 224 Three-arm protractor

Complete view of the three-arm protractor in position. The two movable arms are set to the included angles, 32°15.'6 and 20°08.'2.

To fix the position on the mercator chart (nautical chart), the three-arm protractor is placed on the chart, with the central arm passing through the center object (landmark B). Then the protractor is moved across the chart until the three arms are aligned with the three objects (See Fig. 226). Then ship's position may be the protractor hole (See Fig. 227).

Note : To fix an exact position, the central landmark (B) should be closer to the ship's estimated position than the right (A) and left (C) landmarks. This is because when three landmarks and ship's position lie on the same circumference of a circle, no fix is obtainable.

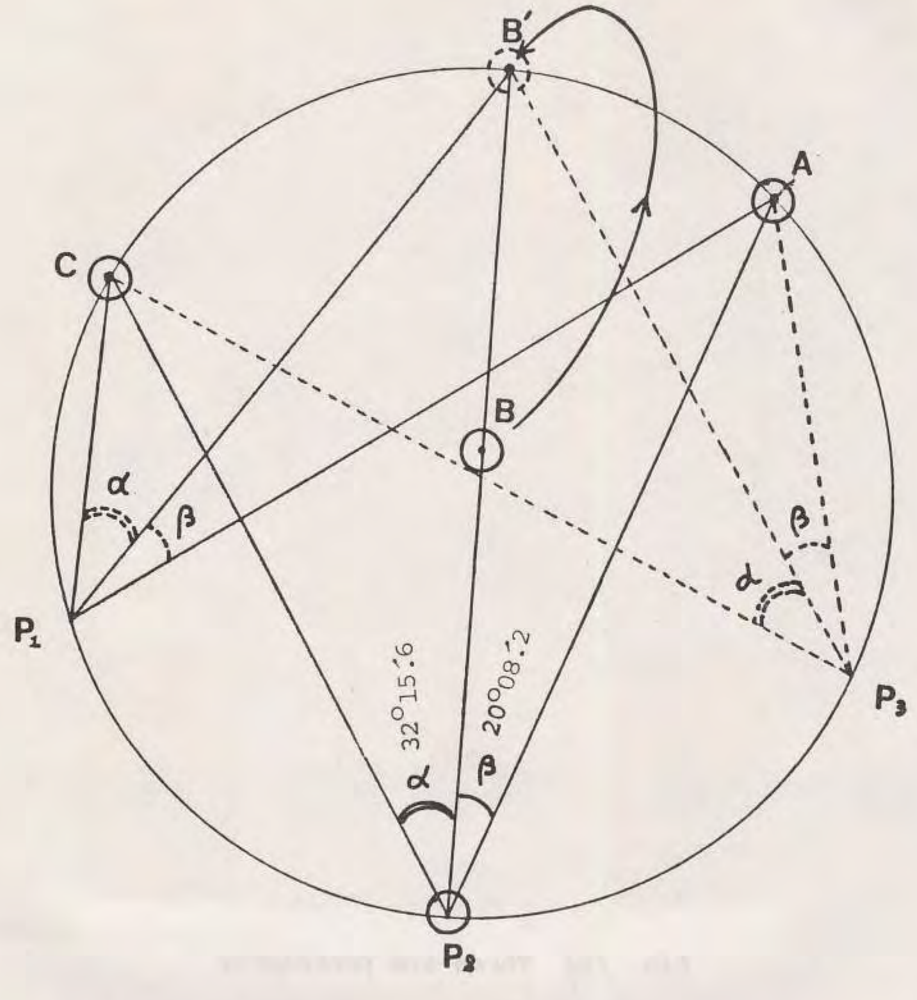


Fig. 225 No fix is available

In Fig. 225, the included angle between A and B is $\beta = 20^{\circ}08'.2$, and between B and C it is $\alpha = 32^{\circ}15'.6$. So ship's position may be P_2. When the landmark B lies on the circumference of a circle on the line P_2, B and B´ , three landmarks A, B´, C and ship's position P_2 are lying on the same circumference A, P_3, P_2, P_1, C and B´

In this case, the angle $\angle CP_1B'$, $\angle CP_2B'$ and $\angle CP_3B'$ have the same value as $32^{\circ}15'.6$, also the angle $\angle B'P_1A$, $\angle B'P_2A$ and $\angle B'P_3A$ have the same value as $20^{\circ}08'.2$. So ship's position is somewhere on the circumference P_3, P_2, P_1, and no fix is available.

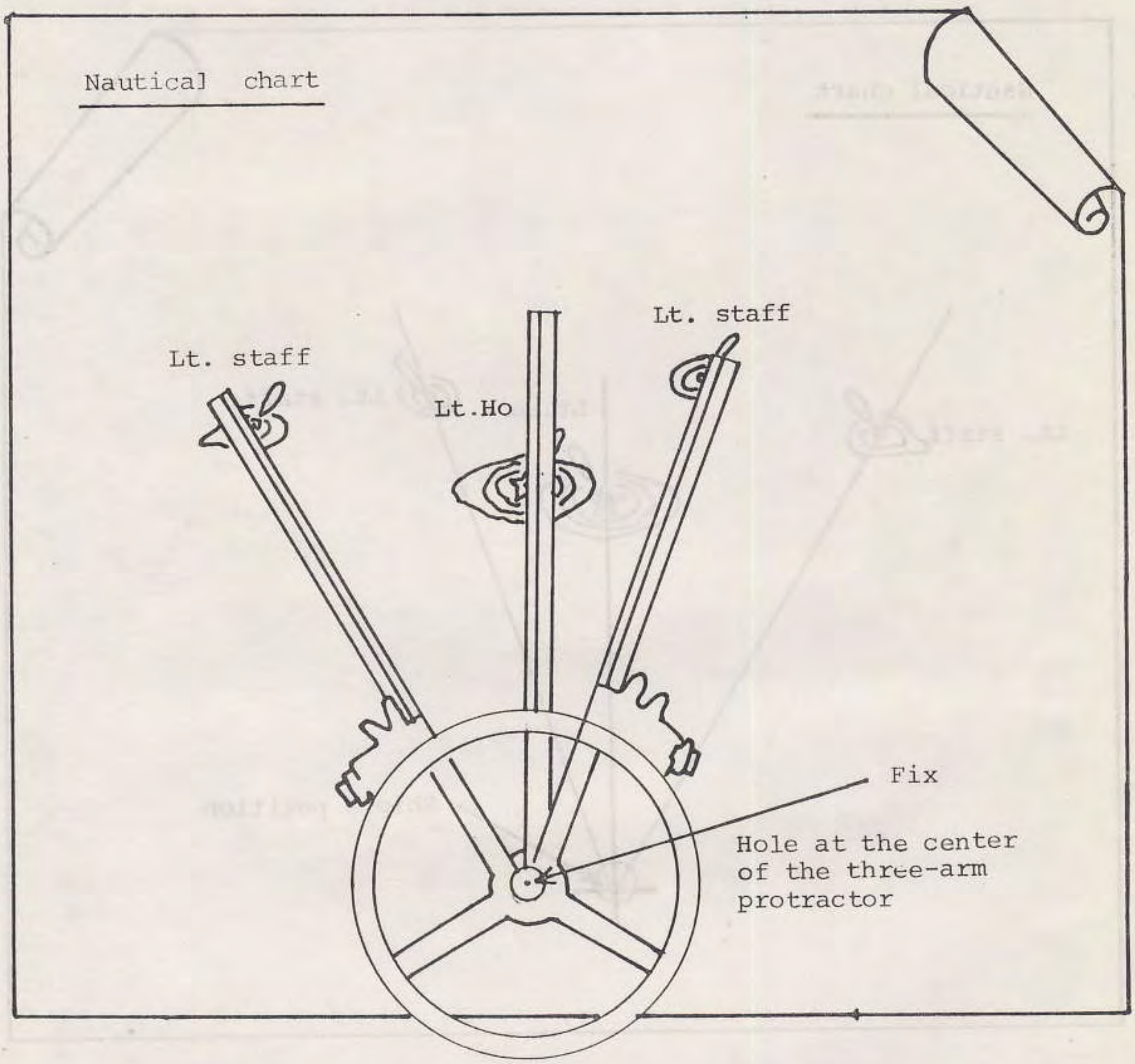


Fig. 226 Fix by a three-arm protractor

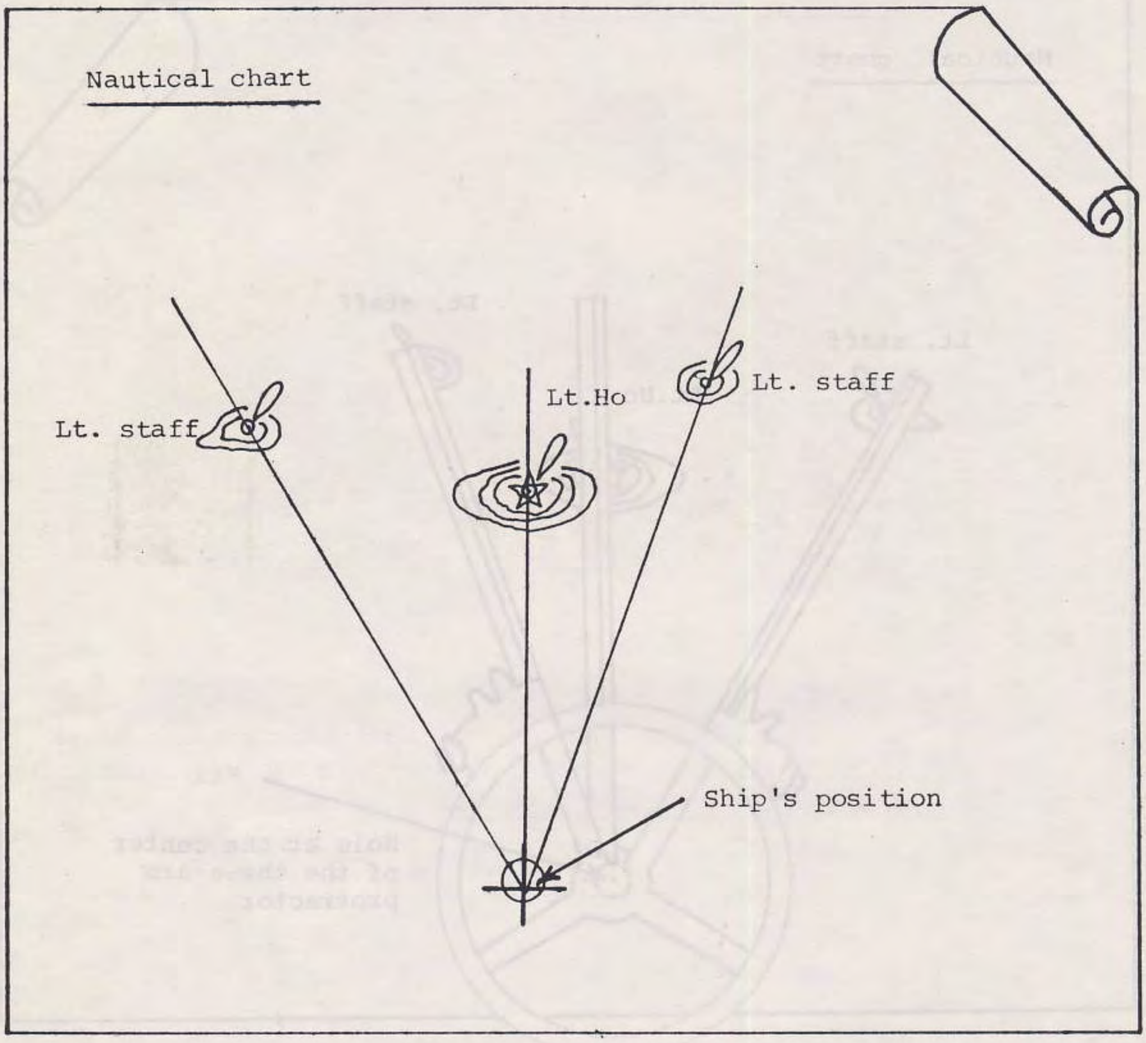


Fig. 227 Fixed

To draw a circle of position by included angle of two landmarks

Example

A Marine Fisheries Department officer observes an included angle between the landmarks D and E in Fig. 228.

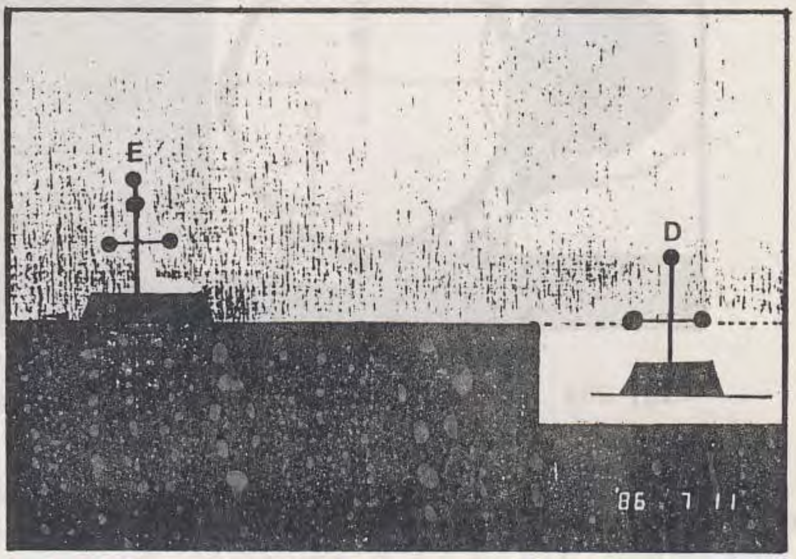


Fig. 228 Landmarks D and E

He looks at landmark E through the horizon mirror and moves the index arm clockwise (See Fig. 229) until the image of landmark D appears. (See Fig. 230)

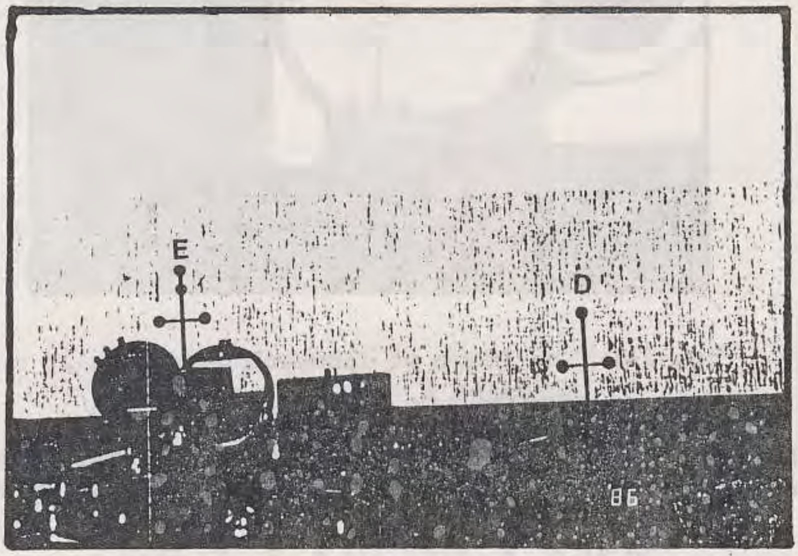


Fig. 229 Looking at the landmark E through the horizon mirror

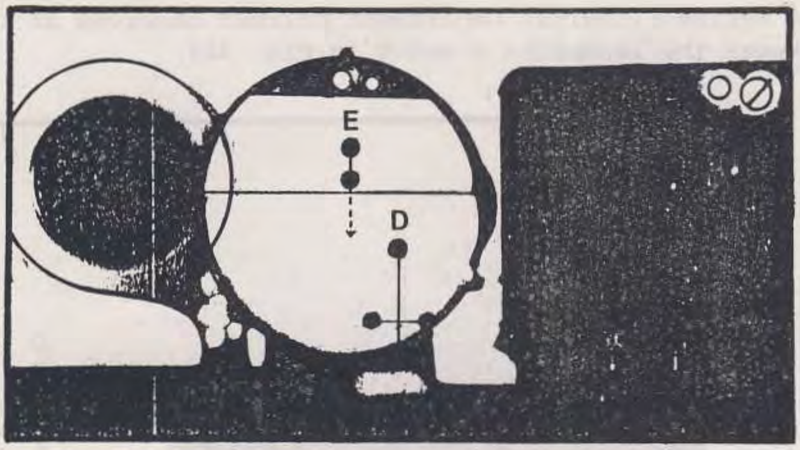


Fig. 230 Landmark D appearing in the horizon mirror and moving from right to left.

When the image of landmark D is exactly underneath landmark E in the mirror, he stops moving the index arm (See Fig. 231).

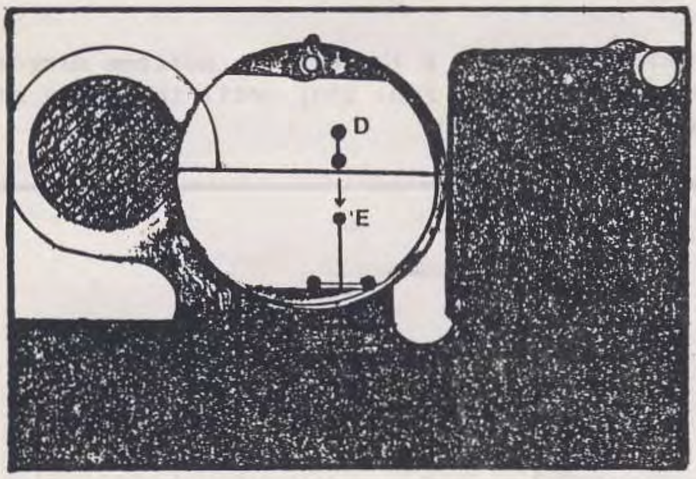


Fig. 231 The image of landmark D vertically underneath landmark E.

The reading on the sextant is $50^\circ 41'.4$ (See Fig. 232), but this sextant has an index effor of ⊕ $2'.6$. So the true reading is:

$$\underline{50^\circ 41'.4 \;+\; \text{I.E.}\;\; 2'.6 \;=\; 50^\circ 44'.0}$$

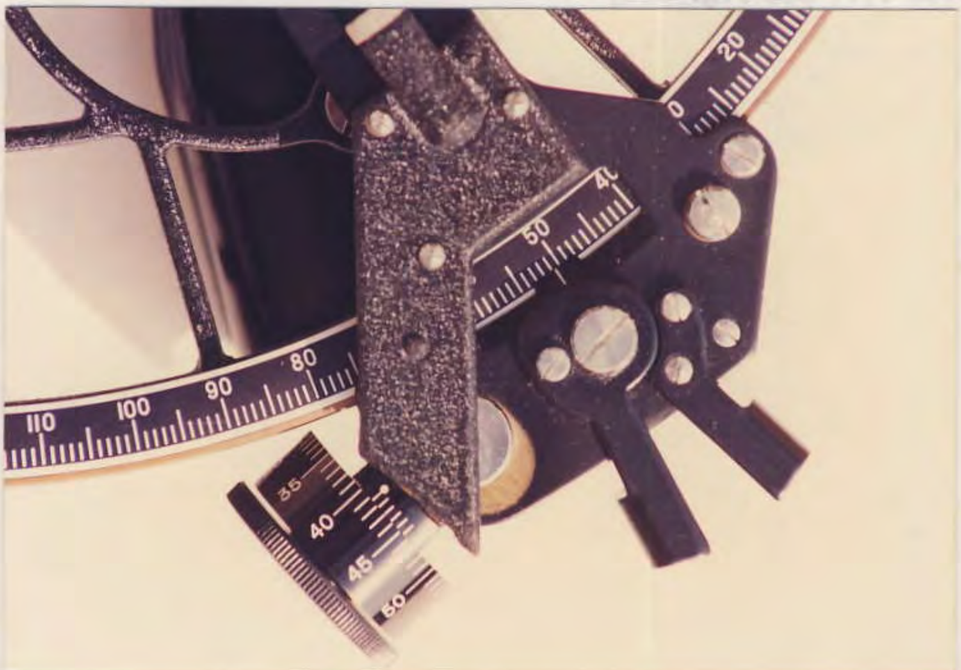


Fig. 232 The reading is $50^{\circ}41\overset{'}{.}4$ excluding index error

Solution

True included angle between D and E = $50^{\circ}44'$

$50^{\circ}44' \times 2 = 101^{\circ}28'$ (→ the central angle of a position circle)

$(180^{\circ} - 101^{\circ}28')/2 = 39^{\circ}16'$

Ref. $39^{\circ}16' + 39^{\circ}16' + 101^{\circ}28' = 180^{\circ}00'$

※ The three interior angles of a triangle add up to 180 degrees.

In Fig. 233 below, ΔCDE is an equilateral triangle, so $\angle DEC$ = $\angle EDC = 39^{o}16'$ and the central angles $\angle ECD = 101^{o}28'$

Answer .... See Fig. 233.

The circle of position is the line passing through D, E, P_1 and P_2

·※· The angles, $39^{o}16'$ and $101^{o}28'$ were set-up and drawn by a three-arm protractor.

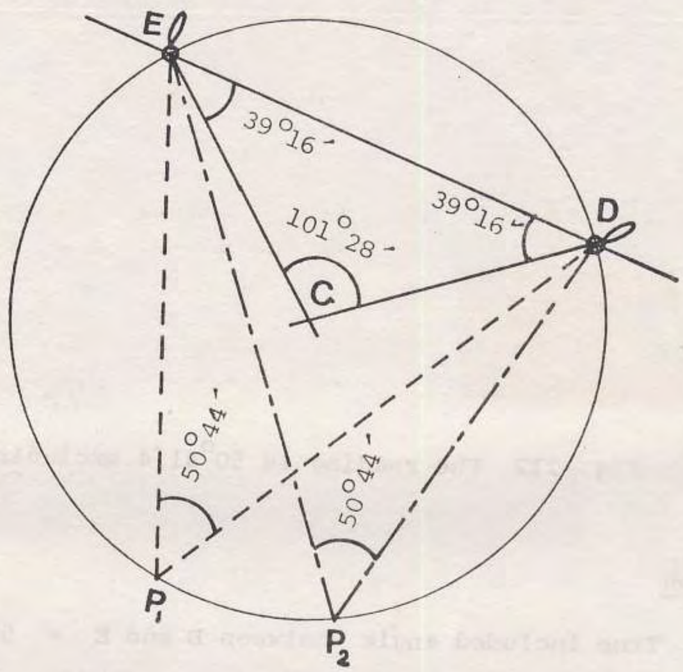


Fig. 233 Circle of Position

Theodolite Distance Meter

When it is decided to set-up a stationary trapnet in a fishing ground in the vicinity of beach, or establish a conservation zone or artificial fish shelter (reef), the areas must be surveyed before setting up or establishing them.

Important survey points to be covered are:

1. Position of the area
2. Depth of water - Isobath
3. Featurs of sea-bed - Gradient of sea-bed
4. Bottom composition
5. Transparency of water - Visibility
6. Current - Direction & speed

The data; included (horizontal) angles, distances from point A to the survey boat and depths of water, are shown in the Record Table (Fig. 234).

These data were obtained by theodolite Distance Meter and Portable Echosounder.

On the shore, the line passing through points A and B (Fig. 235) is the baseline for measurement of the positions, ①②③ .... and (35) in Fig. 234.

Marine Fisheries Department Officers intend to establish an artificial reef in the vicinity of the shore, shown in Fig. 235.

The material of the artificial reef will be reinforced concrete blocks, these blocks are to be dropped onto the specified sea-bed indicated by the serial numbers from No.1 to No.35 on the chart.

Before dropping the blocks onto the sea-bed, a survey boat must check the condition of the sea-bed using a portable echosounder and set up seamarkers on each position shown on the chart i.e. ① , ② , ③ , ... and (35) .

The procedures of such a survey are carried out as follows:

1) A small survey boat is employed to survey and load the seamarkers and instruments into the boat by the boat crew, including:

1. Single or triple prism holder of Theodolite Distance meter (See Fig. 126 on page 173)

2. Portable echosounder and spare batteries
3. Hand lead (a line with a lead weight at the end of it)
4. Secchi Disk
5. Current meter
6. Magnetic compass
7. Transceiver (portable radio telephone) and spare batteries
8. Seamarkers (Bamboo stick with flags, floats, and sinkers)
9. Small anchors or canvas bags with sand
10. Ropes
11. Binoculars

2) The shore staff carry a Theodolite Distance Meter, portable radio telephones, binoculars and a pole with a flag.

3) The survey boat leaves port and proceeds to the position No.1 (①) while the shore staff one observing the included angle between the baseline $\overrightarrow{AB}$ and the boat, also the distance from the point A by Theodolite Distance Meter.

4) When the boat reaches position ①, the observer informs the boat crew to stop there, drop a seamarker, check the depth of water, current speed and direction, measure the transparency of water by Secchi Disk, sample the sea-bed material, etc.. while the shore staff record the included angle and the distance to the boat from point A in a note book.

5) After the completion of the survey at position ①, the boat proceeds to position ② to carry out the same measurements and the shore staff record the data.

6) Thus the boat carries out the survey at all the positions from ① to ㉟ and records information and data. The shore staff observe the included angles and the distances by Theodolite Distance Meter when the boat drops the seamarkers and records data.

7) After recording the data, the boat proceeds from Point 1 to Point 35 at a constant speed and takes a sounding to record the topography of the sea-bed on the recording paper of the portable echosounder.

8) Each position observed by the survey boat should be marked on a nautical chart, according to the Record Table in Fig. 234.

Example

To fix the position of Point 1 in Fig. 235, a three-arm protractor should be placed on the nautical chart, then the included angle ($52^{\circ}23'$) to the baseline and the distance (1970 metres) should be measured (see Fig. 235).

In the case of the Point 7, the three-arm protractor should be placed on the nautical chart. The hole at the center of the three-arm protractor should be on Point A and the central or index arm should be aligned with 31°- 20'. The distance (1410 metres - 141.0 millimetres) can then be measured.

So Point 7 is 141.0 millimetres from Point A and the included (or horizontal) angle between the baseline and Point 7.

Thus every point can be marked on the chart following the aforementioned method and also the depths of water at each point.

After marking the positions and depths on the nautical chart, ithobathic lines should be drawn (See Fig. 236).

Record Table

Point	Included angles (Between the baseline and the survey boat) in degrees	Distances (From Point A to the survey boat) in metres	Depth of water in metres
1	52° - 23′	1970	55
2	49° - 48′	1860	50
3	46° - 54′	1760	45
4	43° - 42′	1660	40
5	39° - 58′	1570	35
6	35° - 53′	1480	30
7	31° - 20′	1410	25
8	34° - 46′	1290	30
9	39° - 22′	1370	35
10	43° - 38′	1460	38
11	47° - 21′	1560	40
12	50° - 33′	1670	48
13	53° - 20′	1770	55
14	55° - 52′	1890	60
15	59° - 38′	1810	65
16	57° - 12′	1690	57
17	54° - 31′	1580	49
18	51° - 23′	1470	45
19	47° - 42′	1360	40
20	43° - 30′	1260	37
21	38° - 42′	1170	34
22	43° - 22′	1070	32
23	48° - 10′	1160	38
24	52° - 18′	1270	42
25	55° - 52′	1380	45
26	58° - 48′	1500	50
27	61° - 21′	1620	57
28	63° - 31′	1740	65
29	67° - 58′	1680	63
30	66° - 00′	1550	57
31	63° - 48′	1430	50
32	61° - 08′	1310	40
33	57° - 50′	1190	43
34	53° - 57′	1070	37
35	49° - 17′	970	31

Fig. 234 Recording of survey

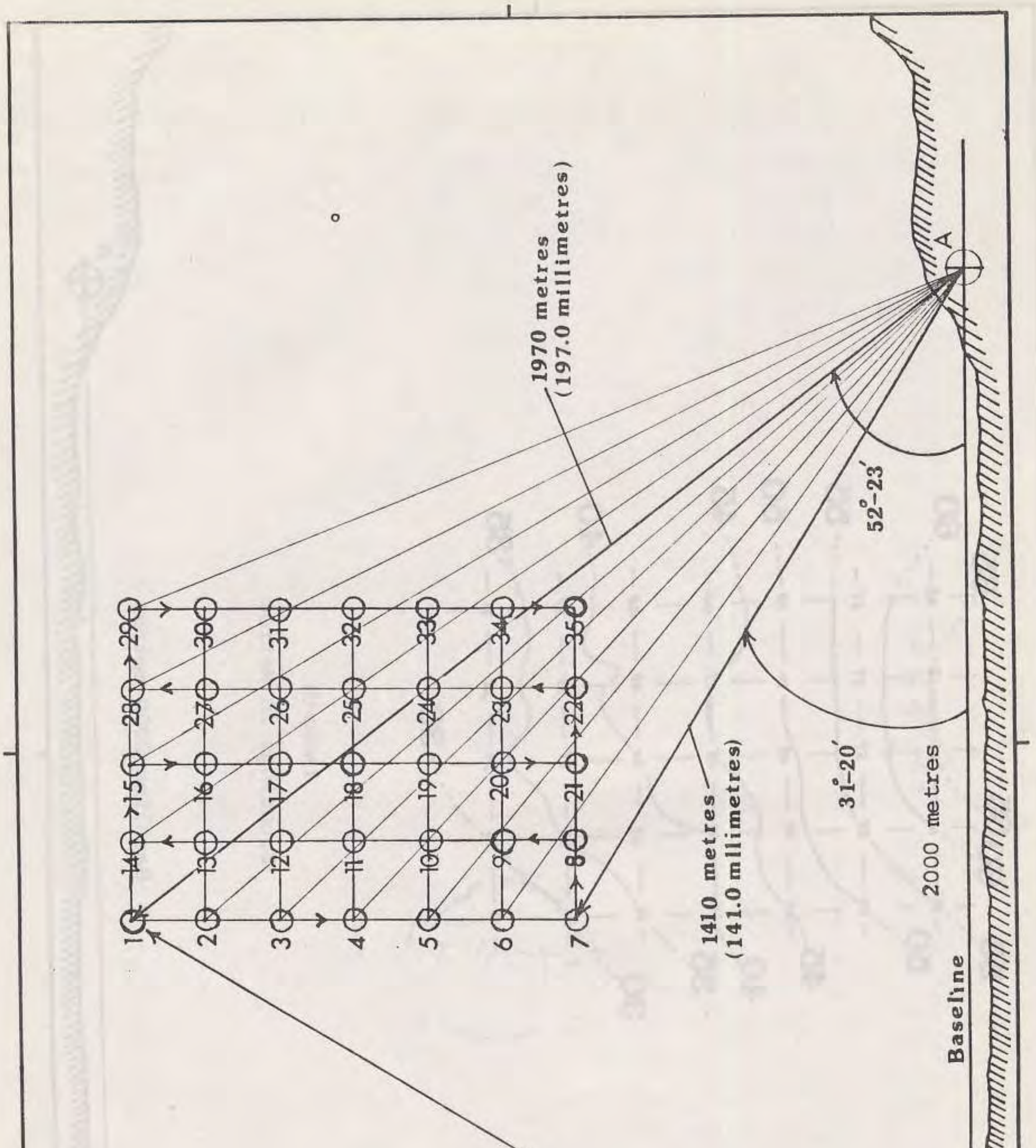


Fig. 235 Fishing Ground to set-up artificial reef

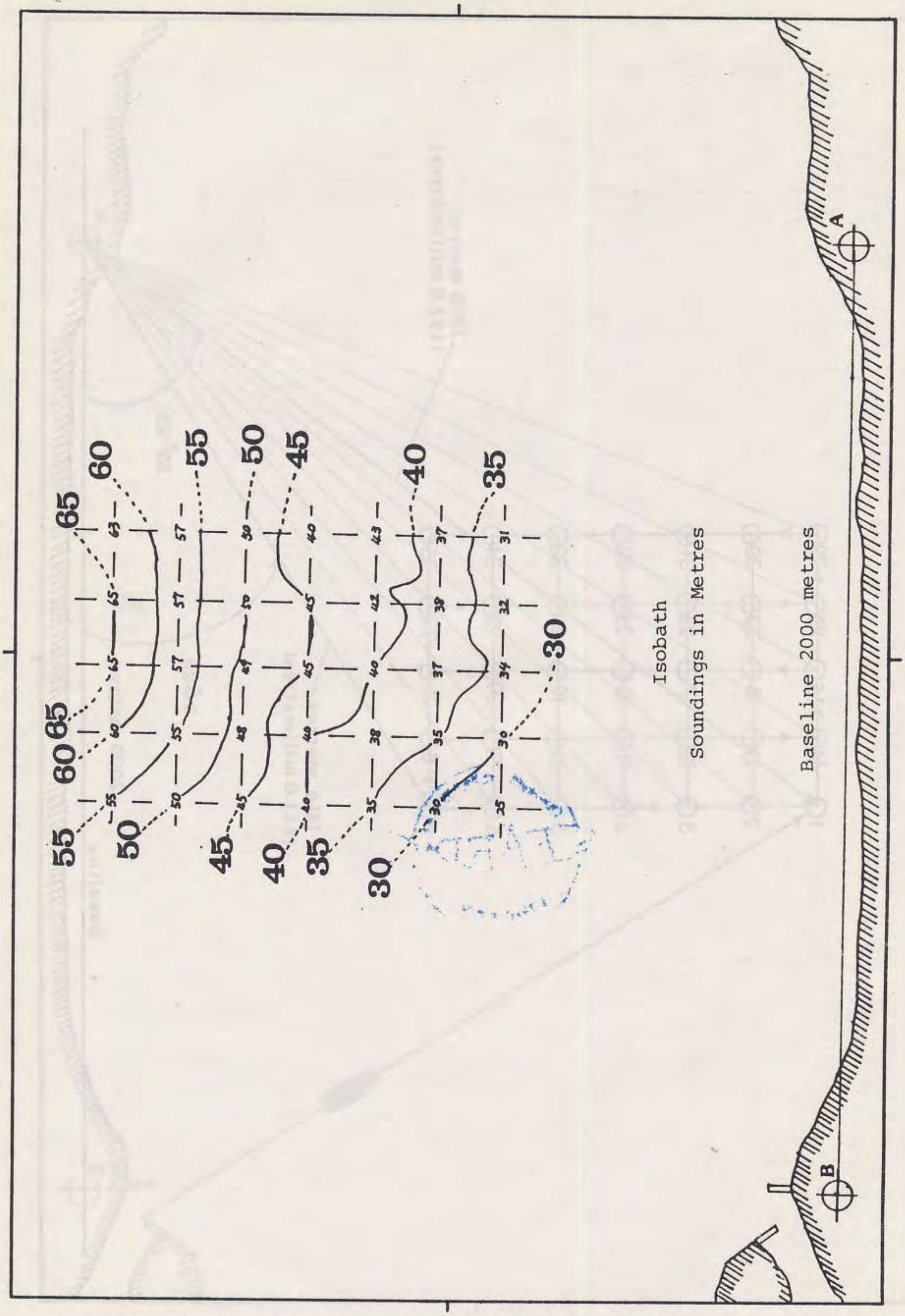


Fig. 236 Fishing Ground Nautical Chart with isobaths

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